

This homework runs from Thursday 31 October 2024 until 12 noon on Thursday 7 November 2024. Submission is to Gradescope Homework 4.

You should aim to write out solutions that someone who does not already know the answer could follow and understand. **Just writing down an answer will not get you any marks.**

Please remember the good scholarly practice requirements of the University regarding work for credit. You can find guidance at the School page <https://web.inf.ed.ac.uk/infweb/admin/policies/academic-misconduct>. This also has links to the relevant University pages.

### Question 1

- (a) In how many ways can the letters of the word INFORMATION be arranged?

**Note: you may leave factorials in your answer**

[1 mark]

- (b) If the letters of the word MEMORANDUM are arranged in a random sequence, what is the probability that the arrangement begins with MMM?

[1 mark]

### Question 2

Four sim cards are taken at random from a box of 9 sim cards and inspected as a quality assurance test in a factory.

- (a) How many possible selections of the four sim cards are there?

The box is rejected if **more than 1 sim card** in the four that are selected is found to fail automated tests which makes the sim card poor quality.

- (b) If the box actually has 3 poor quality sim cards, find the probability that the box is accepted.

[3 marks]

### Question 3

Two whiteboard pens are drawn at random without replacement from a box containing 4 red pens and 6 black pens.

Let  $X$  be the random variable “the number of red pens drawn”.

- (a) Fill in the probability distribution for  $X$  in the table.

$x$	0	1	2
$P(X = x)$			

Work out

- (b) the expected number of red pens  $E(X)$ ;  
(c) the standard deviation of  $X$ .

[5 marks]

**Solution 1**

- (a) There are 11 letters in INFORMATION, with I, N, and O each appearing twice. This makes  $\frac{11!}{2!2!2!}$  distinct ways to order them.
- (b) Choosing the letters in sequence, we combine the probabilities that the first, second, and third letters in turn are all M to get  $\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = \frac{1}{120}$  as the overall probability.

**Solution 2**

- (a) There are  $\binom{9}{4} = 126$  possible ways to choose 4 sim cards from 9.
- (b) Case 1 there are no poor quality sim cards in the selection: ways of choosing 4 good quality sim cards  $\binom{6}{4} = 15$ .

Case 2 there is one poor quality sim card in the selection: 1 poor quality and 3 good quality  $\binom{3}{1} \times \binom{6}{3} = 60$

Let  $E$  be the event that the number of poor quality sim cards is 0 or 1 then  $n(E) = 15 + 60 = 75$  and  $P(E) = \frac{75}{126} = \frac{25}{42} = 0.595$  (3 d.p.).

**Solution 3**

- (a) Let  $B_i$  and  $R_i$ , for  $i \in \{1, 2\}$  be the event that the  $i$ th pen drawn is blue or red, respectively.

$$P(X = 0) = P(B_1 \cap B_2) = \left(\frac{6}{10}\right) \left(\frac{5}{9}\right) = \frac{30}{90} = \frac{1}{3}$$

$$\begin{aligned} P(X = 1) &= P(B_1 \cap R_2) + P(R_1 \cap B_2) \\ &= \left(\frac{6}{10}\right) \left(\frac{4}{9}\right) + \left(\frac{4}{10}\right) \left(\frac{6}{9}\right) = \frac{48}{90} = \frac{8}{15} \end{aligned}$$

$$P(X = 2) = P(R_1 \cap R_2) = \left(\frac{4}{10}\right) \left(\frac{3}{9}\right) = \frac{12}{90} = \frac{2}{15}$$

$x$	0	1	2
$P(X = x)$	$\frac{1}{3}$	$\frac{8}{15}$	$\frac{2}{15}$

- (b) We calculate the expected value using the probabilities in the table.

$$E(X) = \sum_{all\ x} x \cdot P(X = x) = 0 \left(\frac{1}{3}\right) + 1 \left(\frac{8}{15}\right) + 2 \left(\frac{2}{15}\right) = \frac{4}{5}$$

So the expected number of red pens is  $\frac{4}{5}$ .

- (c) The standard deviation of  $X$  is calculated as the square root of the variance.

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$E(X^2) = \sum_{all\ x} x^2 \cdot P(X = x) = 0 \left(\frac{1}{3}\right) + 1 \left(\frac{8}{15}\right) + 4 \left(\frac{2}{15}\right) = \frac{16}{15}$$

$$\text{Var}(X) = \frac{16}{15} - \left(\frac{4}{5}\right)^2 = \frac{32}{75}$$

$$\text{SD}(X) = \sqrt{\text{Var}(X)} = \sqrt{\frac{32}{75}} = 0.65 \quad (2 \text{ d.p.})$$