This homework runs from Thursday 14 November 2024 until 12 noon on Thursday 21 November 2024. Submission is to Gradescope Homework 6.

You should aim to write out solutions that someone who does not already know the answer could follow and understand.

Please remember the good scholarly practice requirements of the University regarding work for credit. You can find guidance at the School page https://web.inf.ed.ac.uk/infweb/admin/policies/academic-misconduct. This also has links to the relevant University pages.

Question 1

A sphere has radius R centimetres, where R is a continuous random variable with the following cumulative distribution function (CDF).

$$F_R(x) \; = \; \begin{cases} 0 & x < 1 \\ (x^2 - 1)/48 & 1 \leq x \leq 7 \\ 1 & 7 < x \end{cases}$$

Calculate the following, showing your working in each case.

(a) The probability the sphere has radius greater than 3cm.

(b) The median value of the continuous random variable R.

(c) The probability density function (PDF) of R.

[*5 marks*]

Question 2

The same sphere has surface area $A\,\mathrm{cm}^2$ where continuous random variable $A = 4\pi R^2$. Calculate the following, again showing your working.

(a) The probability that the sphere has surface area less than 64π square centimetres.

(b) The cumulative distribution function of A.

[*5 marks*]

These questions appeared in the December 2022 final written exam for the course.

Solution 1

(a) $P(R > 3) = 1 - F_R(3) = 5/6$.

(b) For median m we need $P(R < m) = 1/2$, thus $F_R(m) = 1/2$, from which $m^2 - 1 = 24$ giving $m = 5$.

(c) The PDF is calculated as the derivative of the CDF.

$$f_R(x) \;=\; \begin{cases} 0 & x < 1 \text{ or } x > 7 \\ x/24 & 1 < x < 7 \end{cases}$$

Note: It's important here to make clear the different values over different ranges: it's not enough just to differentiate $(x^2 - 1)/48$ since that's only the PDF for certain values of x.

Solution 2

(a) $P(A < 64\pi) = P(R < 4) = 5/16$.

Note: We can do this because $A = 4\pi R^2$ is a monotonically increasing function: so area $A < 64\pi$ if and only if radius $R < 4$.

(b) The CDF for A is calculated as a transformation of that for R, using the fact that $R = \sqrt{A/4\pi}$.

$$F_A(y) \;=\; F_R(\sqrt{y/4\pi}) \;=\; \begin{cases} 0 & y < 4\pi \\ ((y/4\pi) - 1)/48 & 4\pi < y < 196\pi \\ 1 & 196\pi < y \end{cases}$$

Note: Again, it's important to describe the CDF for the full range of y, and to convert the various subranges from x values to appropriate y values. So, for example, replacing $x < 1$ we have $y < 4\pi$ since a sphere of radius 1 has a surface area of 4π.