Retrieve your submissions from Homework 1 in Week 2 as well as the solution notes on the course website. Compare solutions around the group.

What counterexample did you pick in Question 1(b)? Can you find others? Do all counterexamples n have $(n^2 + 4)$ a multiple of 5?

Now work together as a group on each of the following tasks.

Task A

Below are two constructions of numerical sequences. For each one carry out the following steps.

(a) Extend the sequence given for $n = 1, 2, 3, 4$, and 5.

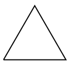
(b) Write down a conjecture for what you think the nth term of the sequence would be.

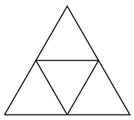
(c) Prove your conjecture correct or find a counterexample.

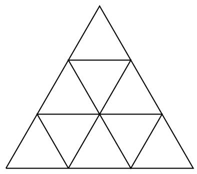
For both of these examples there is an obvious conjecture for how the sequence continues: one of those "obvious" conjectures is right and one is wrong.

Triangulation

The sequence $t_1, t_2, \ldots, t_n, \ldots$ counts how many small triangles make a big triangle with n to a side, as shown below.

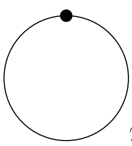

$t_1 = 1$

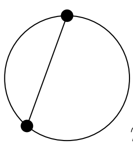

$t_2 = 4$

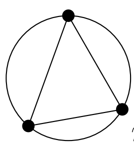

$t_3 = 9$

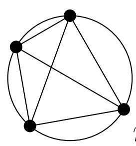
Circle Subdivision

The sequence $r_1, r_2, \ldots, r_n, \ldots$ counts the largest number of regions a circle can be divided into by straight lines joining n points on the circumference, as shown below.


$r_1 = 1$


$r_2 = 2$


$r_3 = 4$


$r_4 = 8$

Task B

Use mathematical induction to prove that for any integer $n \geq 0$, $7^n - 2^n$ is divisible by 5.

Task C

Suppose that $f_0, f_1, f_2, \ldots$ is a sequence defined as follows:

$$f_0 = 5 \qquad f_1 = 16 \qquad f_k = 7f_{(k-1)} - 10f_{(k-2)} \quad \text{for any integer } k \geq 2.$$

Prove by mathematical induction that $f_n = 3 \cdot 2^n + 2 \cdot 5^n$ for all non-negative integers n.

This is Question 6 from Exercise Set 5.4 in the Epp textbook. Does your solution use strong induction or not? How do you tell?

Task D

Sequence $c_0, c_1, c_2, \ldots$ is defined below.

$$c_0 = 2 \quad c_1 = 2 \quad c_2 = 6$$
$$c_k = 3c_{k-3} \quad \text{for every integer } k > 2.$$

Prove that all elements of the sequence are even.

Task E

Calculate 9^k for $k = 0, 1, 2, 3, 4$, and 5. Use these results to make a conjecture relating the parity of n to the units digit of 9^n for non-negative integers n.

Use mathematical induction to prove your conjecture.