Retrieve your submissions from Homework 2 in Week 3 as well as the solution notes on the course website. Compare solutions around the group.

How did you identify the (infinite) set of integers required for Question 1(a)?

For Question 3, what constraints do you need to put on placement of the n straight lines to make sure that the number of regions r_n is as large as possible? What is the *smallest* number of regions s_n that a plane can be divided into by n different straight lines?

Now work together as a group on each of the following tasks, some of which are taken from questions in the Epp textbook.

Task A

Find the mistake in the following "proof" that for all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

"**Proof:** Suppose A, B, and C are any sets such that $A \subseteq B$ and $B \subseteq C$. Since $A \subseteq B$, there is an element x such that $x \in A$ and $x \in B$, and since $B \subseteq C$, there is an element x such that $x \in B$ and $x \in C$. Hence there is an element x such that $x \in A$ and $x \in C$ and so $A \subseteq C$."

Task B

Find the mistake in the following "proof."

"Theorem:" For all sets A and B, $A^c \cup B^c \subseteq (A \cup B)^c$.

"**Proof:** Suppose A and B are any sets, and $x \in A^c \cup B^c$. Then $x \in A^c$ or $x \in B^c$ by definition of union. It follows that $x \notin A$ or $x \notin B$ by definition of complement, and so $x \notin A \cup B$ by definition of union. Thus $x \in (A \cup B)^c$ by definition of complement, and hence $A^c \cup B^c \subseteq (A \cup B)^c$."

Task C

Recall that for sets A and B if there is a bijection $f : A \to B$, a function that is both injective (one-to-one) and surjective (onto), then $|A| = |B|$. Let $E = \{0, 2, 4, \ldots\}$ be the set of non-negative even integers.

(a) Give an example of a function $g : E \to E$ that is injective but not surjective.

(b) Prove that $|\mathbb{Z}| = |E|$ by defining explicitly a bijection h from $\mathbb{Z}$ to E.

Task D

(a) For each of the following functions $f : (\mathbb{Z} \times \mathbb{Z}) \to \mathbb{Z}$ determine whether or not it is surjective.

- **(i)** $f(m, n) = m^2 + n^2$
- **(ii)** $f(m, n) = m$
- **(iii)** $f(m, n) = |n|$
- **(iv)** $f(m, n) = m - n$

(b) Given any two functions $g : A \to B$ and $f : B \to C$ prove or disprove each of the following statements.

- **(i)** If $f \circ g$ and g are injective then f is injective.
- **(ii)** If $f \circ g$ and f are injective then g is injective.

Task E

Below is a table of set identities from Chapter 6 of Epp. Use these to simplify the following set expression, for arbitrary A and B.

$$(A - (A \cap B)) \cap (B - (A \cap B))$$

Theorem 6.2.2 Set Identities

Let all sets referred to below be subsets of a universal set U.

1. *Commutative Laws:* For all sets A and B,

   (a) $A \cup B = B \cup A$ and (b) $A \cap B = B \cap A$.

2. *Associative Laws:* For all sets A, B, and C,

   (a) $(A \cup B) \cup C = A \cup (B \cup C)$ and
   (b) $(A \cap B) \cap C = A \cap (B \cap C)$.

3. *Distributive Laws:* For all sets A, B, and C,

   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and
   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

4. *Identity Laws:* For every set A,

   (a) $A \cup \varnothing = A$ and (b) $A \cap U = A$.

5. *Complement Laws:* For every set A,

   (a) $A \cup A^c = U$ and (b) $A \cap A^c = \varnothing$.

6. *Double Complement Law:* For every set A,

   $(A^c)^c = A$.

7. *Idempotent Laws:* For every set A,

   (a) $A \cup A = A$ and (b) $A \cap A = A$.

8. *Universal Bound Laws:* For every set A,

   (a) $A \cup U = U$ and (b) $A \cap \varnothing = \varnothing$.

9. *De Morgan's Laws:* For all sets A and B,

   (a) $(A \cup B)^c = A^c \cap B^c$ and (b) $(A \cap B)^c = A^c \cup B^c$.

10. *Absorption Laws:* For all sets A and B,

    (a) $A \cup (A \cap B) = A$ and (b) $A \cap (A \cup B) = A$.

11. *Complements of U and $\varnothing$:*

    (a) $U^c = \varnothing$ and (b) $\varnothing^c = U$.

12. *Set Difference Law:* For all sets A and B,

    $A - B = A \cap B^c$.