Task A

You decide to have a dinner party. Even though you are incredibly popular and have 14 different friends, you only have enough chairs to invite 6 of them.

(a) How many choices do you have for which 6 friends to invite?

(b) What if you need to decide not only which friends to invite but also where to seat them along your long table? How many choices do you have then?

(c) Discuss how your answers to the first two parts relate to each other and whether this makes good sense.

(d) How many ways could your group sit at your tutorial table?

You can leave factorials in your answers.

Task B

Seven women and nine men are in an Institute in the Informatics department at a university.

(a) How many ways are there to select a committee of five members from the Institute if at least one woman must be on the committee?

(b) How many ways are there to select a committee of five members from the Institute if at least one woman and at least one man must be on the committee?

Task C

A telegraph sends out three symbols on the communication line.

(a) Using 1 for symbol received and 0 for a missed symbol. List all the possible outcomes for the three symbols as a string of 3 bits.

(b) Represent the following events in a single Venn diagram:

$$A_1 = \{\text{only the first symbol is received}\}$$
$$A_2 = \{\text{at least one symbol is received}\}$$
$$A_3 = \{\text{exactly two symbols are received}\}$$
$$A_4 = \{\text{less than two symbols are received}\}$$
$$A_5 = \{\text{exactly one symbol is received}\}$$

NB: You may find it helpful to change the binary string into an integer for your Venn diagram.

Task D

Five cards are numbered as 1, 2, 3, 4, and 5. Three cards are randomly selected from the set and are lined up next to each other to form a 3-digit number x. Find the probabilities of the following events:

(a) $A = \{x = 123\}$

(b) $B = \{x$ does not contain the digit $4\}$

(c) $C = \{x$ is even$\}$

(d) $D = \{x$ contains at least one of the digits 1, 2$\}$

Task E

(a) How many functions $f{:}\{1, 2, 3, 4, 5\} \to \{a, b, c, d\}$ are there?

(b) How many functions $f{:}\{1, 2, 3, 4, 5, 6, 7, 8\} \to \{1, 2, 3, 4, 5, 6, 7, 8\}$ are bijective?

(c) How many functions $f{:}\{1, 2, 3\} \to \{1, 2, 3, 4, 5, 6, 7, 8\}$ are injective?

Task F

In how many ways can you order the elements of the set $\{1, 2, \ldots, 2n\}$ so that every even number is at an even position?