

Task A

Discuss the difference between ‘mutually exclusive’ and ‘independent’ events.

You could find it helpful to explore this example.

Two dice are rolled and there are three events which we consider:

- A first die lands 1;
- B second die shows larger number than first die;
- C both dice show same number.

Which events are independent and which are mutually exclusive?

Task B

A random variable X has the following probability distribution:

x_i	1	3	4	6
p_i	0.25	0.3	0.3	0.15

Compute the cumulative distribution function (CDF) and from this calculate the probability $P(2 \leq X \leq 5)$.

Plot the probability mass function (PMF) and the cumulative distribution function.

Task C

An exam has 4 questions. Each question has 4 answers, of which exactly 1 is correct. The exam is given to 256 students. Each student answers each question randomly. What is the expected number of exam scripts with no correct answers? With one correct answer? With 2, with 3, with 4?

Consider the expected number of exam scripts with all four answers correct. What is the probability that the actual number of all-correct scripts is exactly that? What is the probability that it is lower? Higher? that?

Task D

Alice and Bob play take turns throwing a six-sided die. The first one to throw a 5 or 6 wins. Alice starts. What are the probability of the events $A = \{\text{Alice wins}\}$ and $B = \{\text{Bob wins}\}$?

(You may use the fact that $\frac{1}{1-x^2} = 1 + x^2 + x^4 + x^6 + x^8 + \dots$)