Discrete Mathematics and Probability **Sets and Functions**
Session 2025/26, Semester 1 Week 4 Homework 3

This homework runs from Thursday 2 October 2025 until 12 noon on Thursday 9 October 2025. Submission is to Gradescope Homework 3.

Questions marked with an asterisk $*$ may be a little harder than others. All are still within the course curriculum, though, using the methods taught in the study guides and textbook.

You should aim to write out solutions that someone who does not already know the answer could follow and understand.

You will receive marks and feedback on this homework, but this is formative assessment and does not affect your final course grade. You may discuss your work with others and can ask questions about the homework in lectures, to your tutors, and in Piazza: please do.

Question 1

Prove using the element method that for any four sets A, B, C, and D, if $A \subseteq C$ and $B \subseteq D$ then $(A \cap B) \subseteq (C \cap D)$ and $(A \cup B) \subseteq (C \cup D)$.

[*4 marks*]

Question 2

Here are three functions from $\mathbb{R}$ to $\mathbb{R}$.

(a) $f(x) = -x$.

(b) $g(x) = 2^x$.

(c) $h(x) = (x^3 - x)$.

For each of these sketch a graph of the function, say whether or not it is injective, and whether or not it is surjective.

[*3 marks*]

$*$ Question 3

Call a function $s : X \to Y$ between two sets X and Y a *section* if there is another function (known as a *retraction*) $r : Y \to X$ such that $r \circ s = I_X$. Here I_X is the identity function on X. Suppose that $f : A \to B$ and $g : A \to C$ are both sections.

(a) Is there necessarily a section $h : A \to B \cup C$?

(b) Is there necessarily a section $j : A \to B \cap C$?

(c) Is there necessarily a section $k : A \to B \times C$?

In each case either show how to construct such a section, together with a suitable corresponding retraction, or give a counterexample.

[*3 marks*]

Note: The specific concepts of "section" and "retraction" are not part of the course syllabus — you don't need to memorise their definition. However, it is part of the course for you to exercise the skill of reading the definition of a new concept and being able to apply it in practice.