Retrieve your submissions from Homework 1 in Week 2 as well as the solution notes on the course website. Compare solutions around the group. Ask your tutor for help if there is anything you do not understand.

What counterexample did you pick in Question 1(b)? Can you find others? Do all counterexamples n have $(n^2 + 4)$ a multiple of 5?

Now work together as a group on each of the following tasks.

Task A

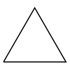
Below are two constructions of numerical sequences. For each one carry out the following steps.

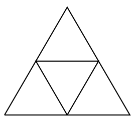
- **(a)** Extend the sequence given for $n = 1, 2, 3, 4,$ and 5.
- **(b)** Write down a conjecture for what you think the nth term of the sequence would be.
- **(c)** Prove your conjecture correct or find a counterexample.

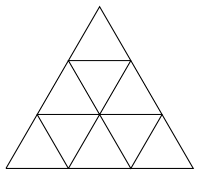
For both of these examples there is an obvious conjecture for how the sequence continues: one of those “obvious” conjectures is right and one is wrong.

Triangulation

The sequence $t_1, t_2, \ldots, t_n, \ldots$ counts how many small triangles make a big triangle with n to a side, as shown below.

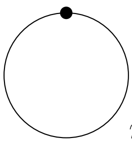

$t_1 = 1$

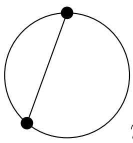

$t_2 = 4$

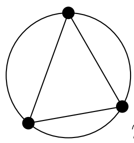

$t_3 = 9$

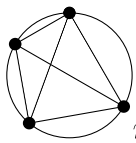
Circle Subdivision

The sequence $r_1, r_2, \ldots, r_n, \ldots$ counts the largest number of regions a circle can be divided into by straight lines joining n points on the circumference, as shown below.


$r_1 = 1$


$r_2 = 2$


$r_3 = 4$


$r_4 = 8$

Task B

Use mathematical induction to prove that for any integer $n \geq 0$, $7^n - 2^n$ is divisible by 5.

Task C

Suppose that $f_0, f_1, f_2, \ldots$ is a sequence defined as follows:

$$f_0 = 5 \qquad f_1 = 16 \qquad f_k = 7f_{(k-1)} - 10f_{(k-2)} \quad \text{for any integer } k \geq 2.$$

Prove by mathematical induction that $f_n = 3 \cdot 2^n + 2 \cdot 5^n$ for all non-negative integers n.

This is Question 6 from Exercise Set 5.4 in the Epp textbook. Does your solution use strong induction or not? How do you tell?

Task D

Sequence $c_0, c_1, c_2, \ldots$ is defined below.

$$c_0 = 2 \quad c_1 = 2 \quad c_2 = 6$$
$$c_k = 3c_{k-3} \quad \text{for every integer } k > 2.$$

Prove that all elements of the sequence are even.

Task E

Calculate 9^k for $k = 0, 1, 2, 3, 4$, and 5. Use these results to make a conjecture relating the parity of n to the units digit of 9^n for non-negative integers n.

Use mathematical induction to prove your conjecture.