Discrete Mathematics and Probability **Discrete Probability**
Session 2025/26, Semester 1 Week 7 Tutorial 5

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Task A

You plan to visit some tourist attractions in Edinburgh, going to a different one every week. There are 14 different attractions that you are interested in but sadly you only have six weeks available.

(a) How many choices do you have for which six attractions to visit?

(b) Another way to approach this is to think of what your six-week plan will be, visiting each week a different attraction out of the 14. How many possible plans are there?

(c) How do your answers to the first two parts relate to each other?

(d) Suppose the numbers were the other way around: you had 14 weeks and only six attractions, so some weeks you will not go to see anything. How many different plans are possible?

You can leave factorials in your answers unevaluated, like 7! or 12!, but not binomial coefficients like $\binom{4}{2}$ or ${}_6\mathrm{C}_2$.

Task B

The *Scottish Weather Research Institute* has sixteen members and is made up of two distinct research groups: seven researchers study wind, while the other nine study rain. The institute is run by a committee of five people chosen from its members.

(a) How many ways are there to select a committee of five institute members if at least one wind researcher must be on the committee?

(b) How many ways are there to select a committee of five institute members if both wind and rain groups must have at least one member on the committee?

Task C

A telegraph sends out three symbols in succession on its communication line. Some symbols are received successfully, others are missed.

(a) Use 1 for a symbol successfully received and 0 for a missed symbol. List all possible outcomes as strings of 3 bits.

(b) Converting binary strings to decimal, list all outcomes as integers.

(c) Taking these integers as a sample space, represent the following events as sets on a single Venn diagram.

$$A_1 = \{\text{only the first symbol is received}\}$$
$$A_2 = \{\text{at least one symbol is received}\}$$
$$A_3 = \{\text{exactly two symbols are received}\}$$
$$A_4 = \{\text{less than two symbols are received}\}$$
$$A_5 = \{\text{exactly one symbol is received}\}$$

Task D

Five cards are numbered 1, 2, 3, 4, and 5. Three cards are randomly selected from these five and are lined up next to each other to form a 3-digit number x. Find the probabilities of the following events.

(a) $A = \{x = 123\}$

(b) $B = \{x$ does not contain the digit $4\}$

(c) $C = \{x$ is even$\}$

(d) $D = \{x$ contains at least one of the digits 1, 2$\}$

Task E

(a) How many functions $f{:}\{1, 2, 3, 4, 5\} \to \{a, b, c, d\}$ are there?

(b) How many functions $f{:}\{1, 2, 3, 4, 5, 6, 7, 8\} \to \{1, 2, 3, 4, 5, 6, 7, 8\}$ are bijective?

(c) How many functions $f{:}\{1, 2, 3\} \to \{1, 2, 3, 4, 5, 6, 7, 8\}$ are injective?

Task F

In how many ways can you order the elements of the set $\{1, 2, \ldots, 2n\}$ so that every even number is at an even position?