Discrete Mathematics and Probability **Discrete Probability**
Session 2026/27, Semester 1 Week 8 Homework 4 with Solution Notes

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This homework runs from Thursday 5 November 2026 until 12 noon on Thursday 12 November 2026. Submission is to Gradescope Homework 4.

You should aim to write out solutions that someone who does not already know the answer could follow and understand. A number on its own without explanation receives no marks.

Answers with unevaluated factorials like $n!$ or $p!/q!$ are acceptable but you should avoid leaving in unexpanded binomial coefficients such as $\binom{n}{m}$ or ${}_nC_m$.

You will receive marks and feedback on this homework, but this is formative assessment and does not affect your final course grade. You may discuss your work with others, including with AI, and can ask questions about the homework in lectures, to your tutors, and on Ed: please do.

Question 1

(a) In how many different ways can you arrange the letters of the word INFORMATION, assuming that the various duplicate letters are indistinguishable from each other?

(b) If the letters of the word MEMORANDUM are arranged in a random sequence, what is the probability that the arrangement begins with MMM?

[*2 marks*]

Question 2

A factory manufacturing sim cards produces them in boxes of nine. For quality assurance, four cards are taken at random from the box and inspected.

(a) How many possible selections of the four sim cards are there?

If more than one sim card of the four selected fails the inspection then the whole box is rejected.

(b) If the box actually has 3 poor quality sim cards that would fail inspection, find the probability that the box ends up being accepted anyway.

[*3 marks*]

Question 3

Two whiteboard pens are drawn at random without replacement from a box containing 4 red pens and 6 black pens. Let X be the random variable "the number of red pens drawn".

(a) Compute the probability distribution for X by filling in the following table.

x	$P(X = x)$
0	
1	
2	

Calculate the following, showing your working in each case.

(b) The expected number of red pens $\mathrm{E}(X)$.

(c) The standard deviation of X.

[*5 marks*]

Solution 1

(a) There are 11 letters in INFORMATION, with I, N, and O each appearing twice. This makes $\dfrac{11!}{2!2!2!}$ distinct ways to order them.

(b) Choosing the letters in sequence, we combine the probabilities that the first, second, and third letters in turn are all M to get $\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = \frac{1}{120}$ as the overall probability.

Solution 2

(a) There are $\binom{9}{4} = 126$ possible ways to choose 4 sim cards from 9.

(b) Case 1 there are no poor quality sim cards in the selection: ways of choosing 4 good quality sim cards $\binom{6}{4} = 15$.

Case 2 there is one poor quality sim card in the selection: 1 poor quality and 3 good quality $\binom{3}{1} \times \binom{6}{3} = 60$

Let E be the event that the number of poor quality sim cards is 0 or 1 then $n(E) = 15 + 60 = 75$ and $P(E) = \frac{75}{126} = \frac{25}{42} = 0.595$ (3 d.p.).

Solution 3

(a) Let B_i and R_i, for $i \in \{1, 2\}$ be the event that the ith pen drawn is blue or red, respectively.

$$\begin{aligned}
P(X = 0) &= P(B_1 \cap B_2) = \left(\frac{6}{10}\right)\left(\frac{5}{9}\right) = \frac{30}{90} = \frac{1}{3} \\
P(X = 1) &= P(B_1 \cap R_2) + P(R_1 \cap B_2) \\
&= \left(\frac{6}{10}\right)\left(\frac{4}{9}\right) + \left(\frac{4}{10}\right)\left(\frac{6}{9}\right) = \frac{48}{90} = \frac{8}{15} \\
P(X = 2) &= P(R_1 \cap R_2) = \left(\frac{4}{10}\right)\left(\frac{3}{9}\right) = \frac{12}{90} = \frac{2}{15}
\end{aligned}$$

x	$P(X = x)$
0	$\frac{1}{3}$
1	$\frac{8}{15}$
2	$\frac{2}{15}$

(b) We calculate the expected value using the probabilities in the table.

$$\mathrm{E}(X) = \sum_{all\ x} x \cdot P(X = x) = 0\left(\frac{1}{3}\right) + 1\left(\frac{8}{15}\right) + 2\left(\frac{2}{15}\right) = \frac{4}{5}$$

So the expected number of red pens is $\frac{4}{5}$.

(c) The standard deviation of X is calculated as the square root of the variance.

$$\begin{aligned}
\mathrm{Var}(X) &= \mathrm{E}(X^2) - (\mathrm{E}(X))^2 \\
\mathrm{E}(X^2) &= \sum_{\text{all } x} x^2 \cdot P(X = x) = 0\left(\frac{1}{3}\right) + 1\left(\frac{8}{15}\right) + 4\left(\frac{2}{15}\right) = \frac{16}{15} \\
\mathrm{Var}(X) &= \frac{16}{15} - \left(\frac{4}{5}\right)^2 = \frac{32}{75} \\
\mathrm{SD}(X) &= \sqrt{\mathrm{Var}(X)} = \sqrt{\frac{32}{75}} = 0.65 \quad (2\,\text{d.p.})
\end{aligned}$$