Discrete Mathematics and Probability **Continuous and Joint Probability**
Session 2026/27, Semester 1 Week 10 Homework 6

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This homework runs from Thursday 19 November 2026 until 12 noon on Thursday 26 November 2026. Submission is to Gradescope Homework 6.

You should aim to write out solutions that someone who does not already know the answer could follow and understand.

You will receive marks and feedback on this homework, but this is formative assessment and does not affect your final course grade. You may discuss your work with others, including with AI, and can ask questions about the homework in lectures, to your tutors, and on Ed: please do.

Question 1

A rectangle has height and width given in metres by discrete random variables H and W, each of which can take one of two possible integer values: $H \in \{6, 9\}$ and $W \in \{3, 6\}$. These are jointly distributed with probability mass function p_{HW} as follows:

$$p_{HW}(h, w) \;=\; \begin{cases} 1/3 & h > w \quad h \in \{6,9\}, w \in \{3,6\} \\ 0 & \text{otherwise.} \end{cases}$$

(a) Draw a 2×2 table showing the joint probability mass function for all possible values of H and W. Use this table to calculate the marginal probability mass functions for H and W.

(b) Calculate the expected values of the rectangle's height, width, perimeter, and area.

[*6 marks*]

Question 2

Suppose a continuous random variable X has the following cumulative distribution function (CDF).

$$F_X(x) \;=\; \frac{1}{1 + e^{-x}} \qquad \text{for } -\infty < x < \infty$$

(a) Draw a sketch of this function, showing its behaviour when x is large and negative, zero, or large and positive.

This is known as the *logistic function* and is used in machine learning and ecological modelling.

Calculate the following, showing your working in each case.

(b) The probability that X takes a value between -1 and 1.

(c) The 90th percentile of this distribution.

(d) The probability density function (PDF) $f_X(x)$ for the random variable X.

[*4 marks*]