

Elements of Programming Languages

Lecture Notes: L_{Arith}

1 Abstract Syntax

$Expr \ni e ::= n \in \mathbb{N} \mid e_1 + e_2 \mid e_1 \times e_2 \quad L_{\text{Arith}}$

$Type \ni \tau ::= \text{int} \quad L_{\text{Arith}}$

$Value \ni v ::= n \in \mathbb{N} \quad L_{\text{Arith}}$

2 Evaluation

$e \Downarrow v$ for L_{Arith}

$$\frac{}{v \Downarrow v} \quad \frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1 + e_2 \Downarrow v_1 +_{\mathbb{N}} v_2} \quad \frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1 \times e_2 \Downarrow v_1 \times_{\mathbb{N}} v_2}$$

3 Types

$\vdash e : \tau$ for L_{Arith}

$$\frac{}{\vdash n : \text{int}} \quad \frac{\vdash e_1 : \text{int} \quad \vdash e_2 : \text{int}}{\vdash e_1 + e_2 : \text{int}} \quad \frac{\vdash e_1 : \text{int} \quad \vdash e_2 : \text{int}}{\vdash e_1 \times e_2 : \text{int}}$$