

Elements of Programming Languages

Lecture Notes: L_{While}

1 Abstract Syntax

$$\begin{aligned}
 Expr \ni e & ::= n \in \mathbb{N} \mid e_1 + e_2 \mid e_1 - e_2 \mid e_1 \times e_2 && \text{Numbers} \\
 & \mid b \in \mathbb{B} \mid e_1 == e_2 && \text{Booleans} \\
 Value \ni v & ::= n \mid b \\
 Stmt \ni s & ::= \text{skip} \mid s_1; s_2 \mid x := e \\
 & \mid \text{if } e \text{ then } s_1 \text{ else } s_2 \mid \text{while } e \text{ do } s \\
 Env \ni \sigma & ::= [x_1 = v_1, \dots, x_n = v_n]
 \end{aligned}$$

2 Evaluation

$\sigma, e \Downarrow v$

$$\begin{array}{l}
 \frac{n \in \mathbb{N}}{\sigma, n \Downarrow n} \quad \frac{\sigma, e_1 \Downarrow v_1 \quad \sigma, e_2 \Downarrow v_2}{\sigma, e_1 + e_2 \Downarrow v_1 +_{\mathbb{N}} v_2} \quad \frac{\sigma, e_1 \Downarrow v_1 \quad \sigma, e_2 \Downarrow v_2}{\sigma, e_1 - e_2 \Downarrow v_1 -_{\mathbb{N}} v_2} \quad \frac{\sigma, e_1 \Downarrow v_1 \quad \sigma, e_2 \Downarrow v_2}{\sigma, e_1 * e_2 \Downarrow v_1 \times_{\mathbb{N}} v_2} \\
 \\
 \frac{b \in \mathbb{B}}{\sigma, b \Downarrow b} \quad \frac{\sigma, e_1 \Downarrow v \quad \sigma, e_2 \Downarrow v}{\sigma, e_1 == e_2 \Downarrow \text{true}} \quad \frac{\sigma, e_1 \Downarrow v_1 \quad \sigma, e_2 \Downarrow v_2 \quad v_1 \neq v_2}{\sigma, e_1 == e_2 \Downarrow \text{false}}
 \end{array}$$

$\sigma, s \Downarrow \sigma'$

$$\begin{array}{l}
 \frac{}{\sigma, \text{skip} \Downarrow \sigma} \quad \frac{\sigma, s_1 \Downarrow \sigma' \quad \sigma', s_2 \Downarrow \sigma''}{\sigma, s_1; s_2 \Downarrow \sigma''} \\
 \\
 \frac{\sigma, e \Downarrow \text{true} \quad \sigma, s_1 \Downarrow \sigma'}{\sigma, \text{if } e \text{ then } s_1 \text{ else } s_2 \Downarrow \sigma'} \quad \frac{\sigma, e \Downarrow \text{false} \quad \sigma, s_2 \Downarrow \sigma'}{\sigma, \text{if } e \text{ then } s_1 \text{ else } s_2 \Downarrow \sigma'} \\
 \\
 \frac{\sigma, e \Downarrow \text{true} \quad \sigma, s \Downarrow \sigma' \quad \sigma', \text{while } e \text{ do } s \Downarrow \sigma''}{\sigma, \text{while } e \text{ do } s \Downarrow \sigma''} \\
 \\
 \frac{\sigma, e \Downarrow \text{false}}{\sigma, \text{while } e \text{ do } s \Downarrow \sigma} \quad \frac{\sigma, e \Downarrow v}{\sigma, x := e \Downarrow \sigma[x := v]}
 \end{array}$$