Elements of Programming Languages
Tutorial 6: Classes, subtyping, and comprehensions
Week 8 (November 6–10, 2023)

Starred exercises are more challenging. Please try all unstarred exercises before the tutorial meeting.

1. **Covariant and contravariant type parameters**

   In Scala, a type parameter in a definition can be marked *covariant* by prefixing it with $+$ and *contravariant* by prefixing it with $-$.

   Consider the following Scala code:

```
abstract class Super
class Sub1(n: Int) extends Super
class Sub2(b: Boolean) extends Super
class Box1[+A] // covariant
class Box2[-A] // contravariant

def g1(x: Box1[Super]) = x
def g2(x: Box1[Sub1]) = x
def h1(x: Box2[Super]) = x
def h2(x: Box2[Sub1]) = x
```

   Suppose that `A` is replaced with one of the types `Any`, `Nothing`, `Super`, `Sub1`, or `Sub2`. For which values of `A` do the following calls typecheck:

   - `g1(new Box1[A])`
   - `g2(new Box1[A])`
   - `h1(new Box2[A])`
   - `h2(new Box2[A])`

   (It may help to draw a matrix with rows labeled by the function names and columns by the five possible types for `A`). You can type all of these expressions into Scala to find this out. What is the pattern?

2. **Parameterized traits**

   Traits can also be parameterized by types. The builtin trait `Ordered[T]` is an example:

```
trait Ordered[T] {
 def compare(that: T): Int
 def < (that: T): Boolean = ???
 def <= (that: T): Boolean = ???
}
```

Here, the type parameter `T` is needed to name the type of other elements to which **`this`** will be compared. The **`this`**`.compare(that)` operation returns a negative integer if **`this`** is less than `that`, zero if they are equal and a positive integer if **`this`** is greater than `that`.

Based on this specification, fill in the `???` regions in the above code snippet with code that defines standard comparison operators such as `<` in terms of `compare`. Define the remaining operations `>`, `>=` and `equalTo`, `nequalTo`. (The operator names `==` and `!=` are already defined in Scala and can't be overridden at a different type in this trait.)

3. **List comprehensions**

   Using the desugaring rules for list comprehensions described in Lecture 11, give the resulting list and convert the following list comprehension expressions to plain Scala code.

   (a) `for (x <- List(1,2,3)) yield (x + 1)`

   (b) `for (x <- List(1,2,3); if (x % 2 == 0)) yield (x / 2)`

   (c) ($\star$) `for (x <- List(1,2,3); y <- List(1,2,3); if (x < y)) yield (x, y)`

4. ($\star$) **Covariant lists**

   Covariance and subtyping allows us to define lists more cleanly:

```
abstract class List[+A]
case object Nil extends List[Nothing]
case class Cons[+A](head: A, tail: List[A]) extends List[A]
```

   Now `Nil` is a case object, so it doesn't need a parameter list, and it extends `List[Nothing]`, so it doesn't need a type parameter. (Case objects with only a type parameter and no value parameters are not allowed in Scala.) This is much closer to the way lists are actually defined in Scala.

   (a) Define a list expression that has type `List[Any]` and would not typecheck in the absence of subtyping.

   (b) Define a `List[A]` member `append` that allows lists of different types as arguments, provided the two element types have a common supertype. (**Hint:** Scala type parameters can be given both lower and upper *type bounds*, e.g. **`def`** `foo[A, B >: A, C <: A]` says that `foo` has three type parameters, `A`, `B` which must be a supertype of `A`, and `C` which must be a subtype of `A`.)