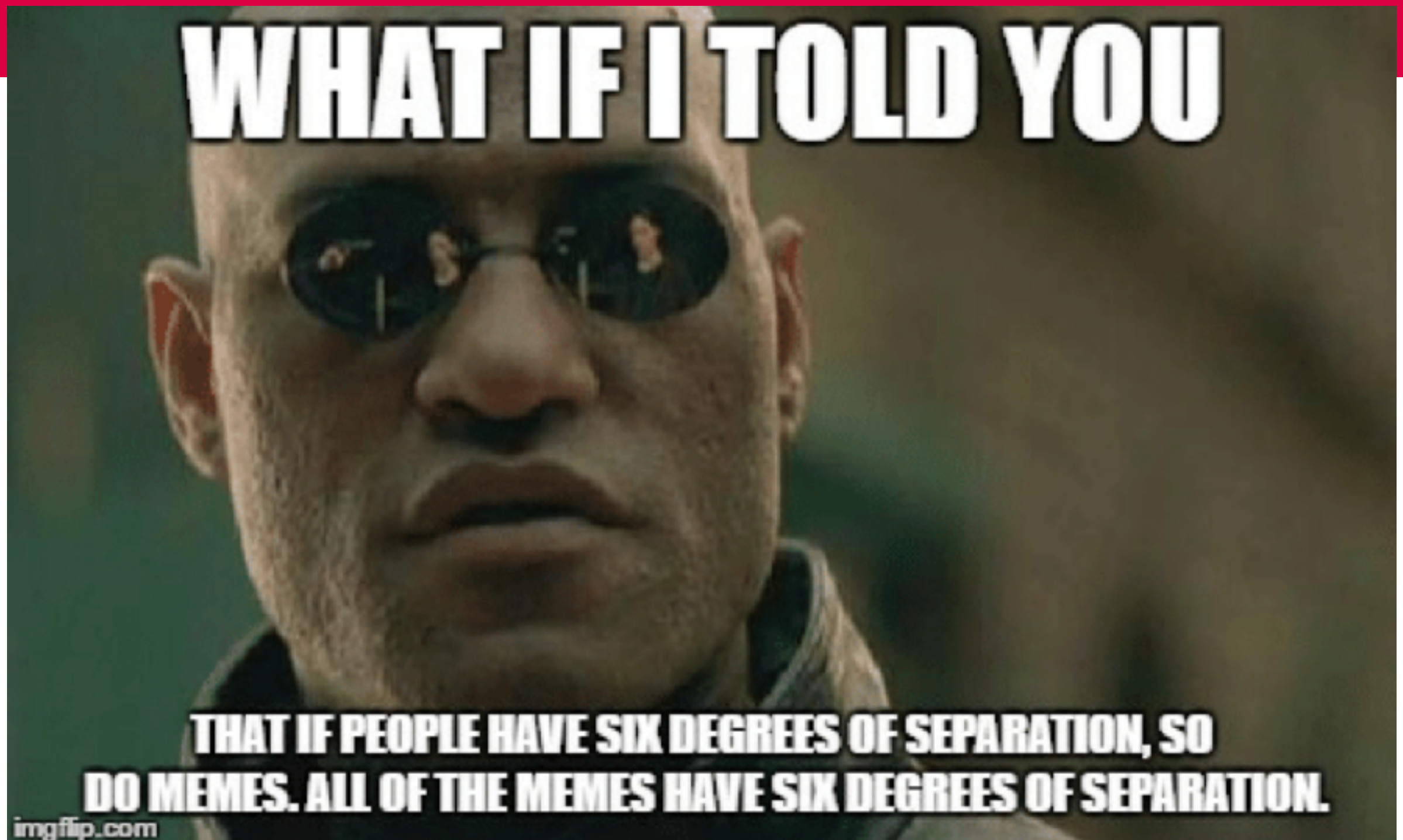


# Network elements



# LEARNING OUTCOMES

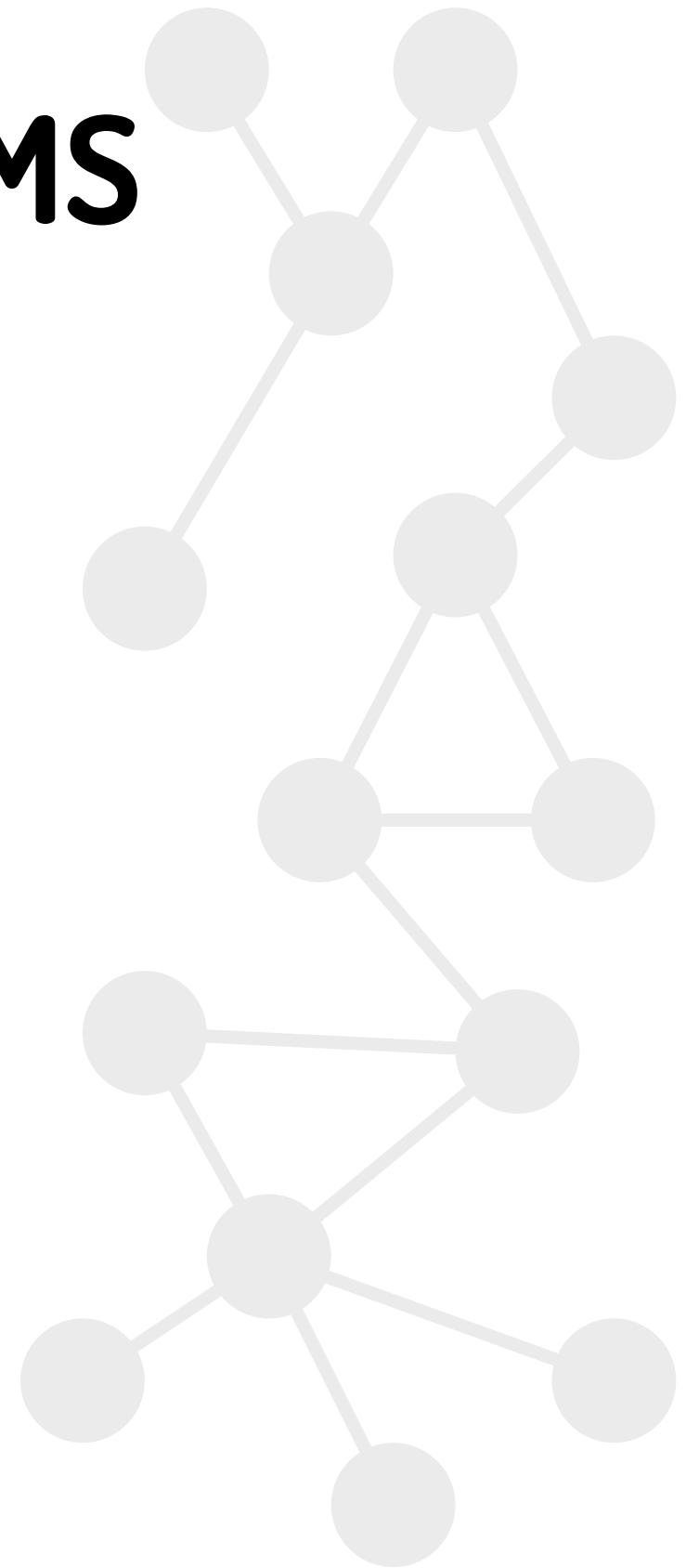
Get to know network **terminology**

Explore the different **types of networks**

See some network **applications**

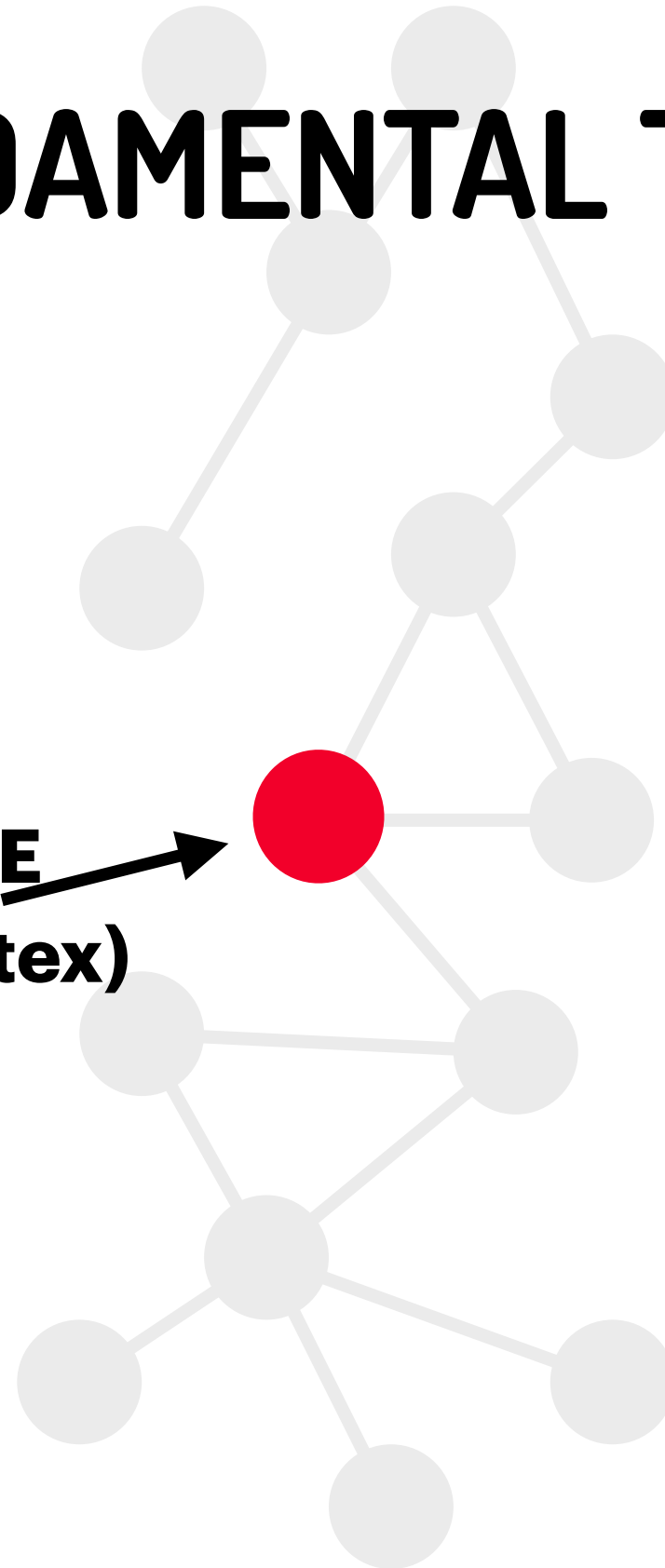


# FUNDAMENTAL TERMS



# FUNDAMENTAL TERMS

**NODE**  
**(Or vertex)**



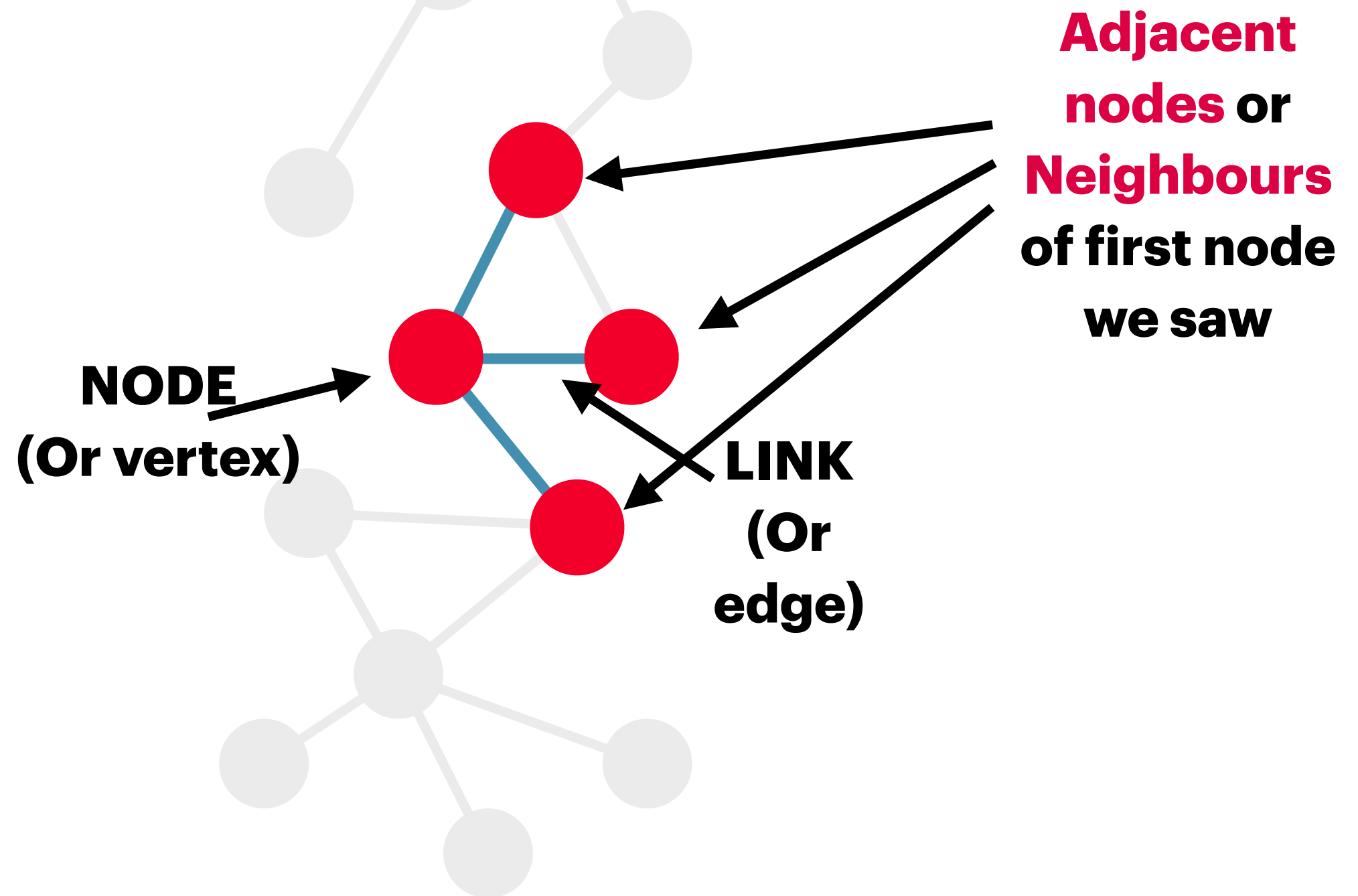
# FUNDAMENTAL TERMS

**NODE**  
(Or vertex)

**LINK**  
(Or  
edge)



# FUNDAMENTAL TERMS

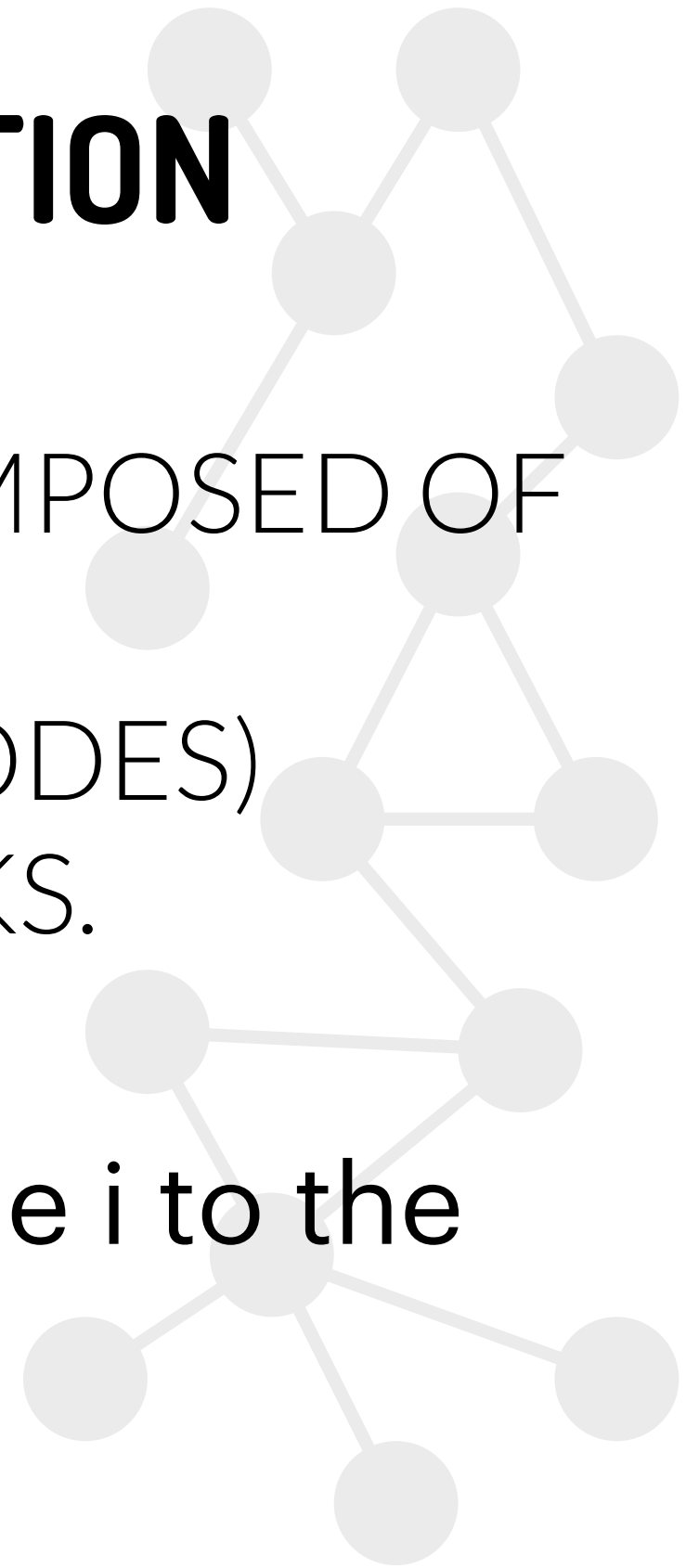


# FUNDAMENTAL NOTATION

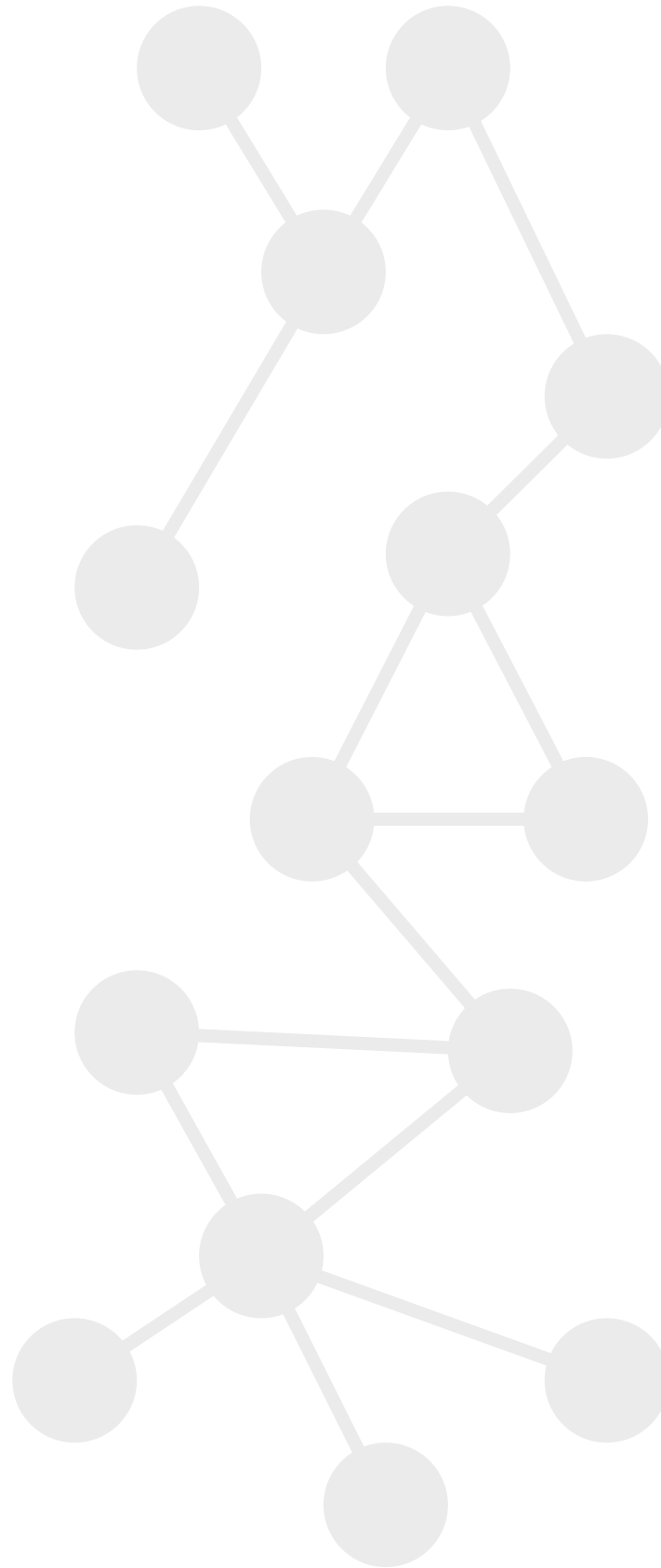
A NETWORK **G** IS A SYSTEM COMPOSED OF  
TWO PARTS:

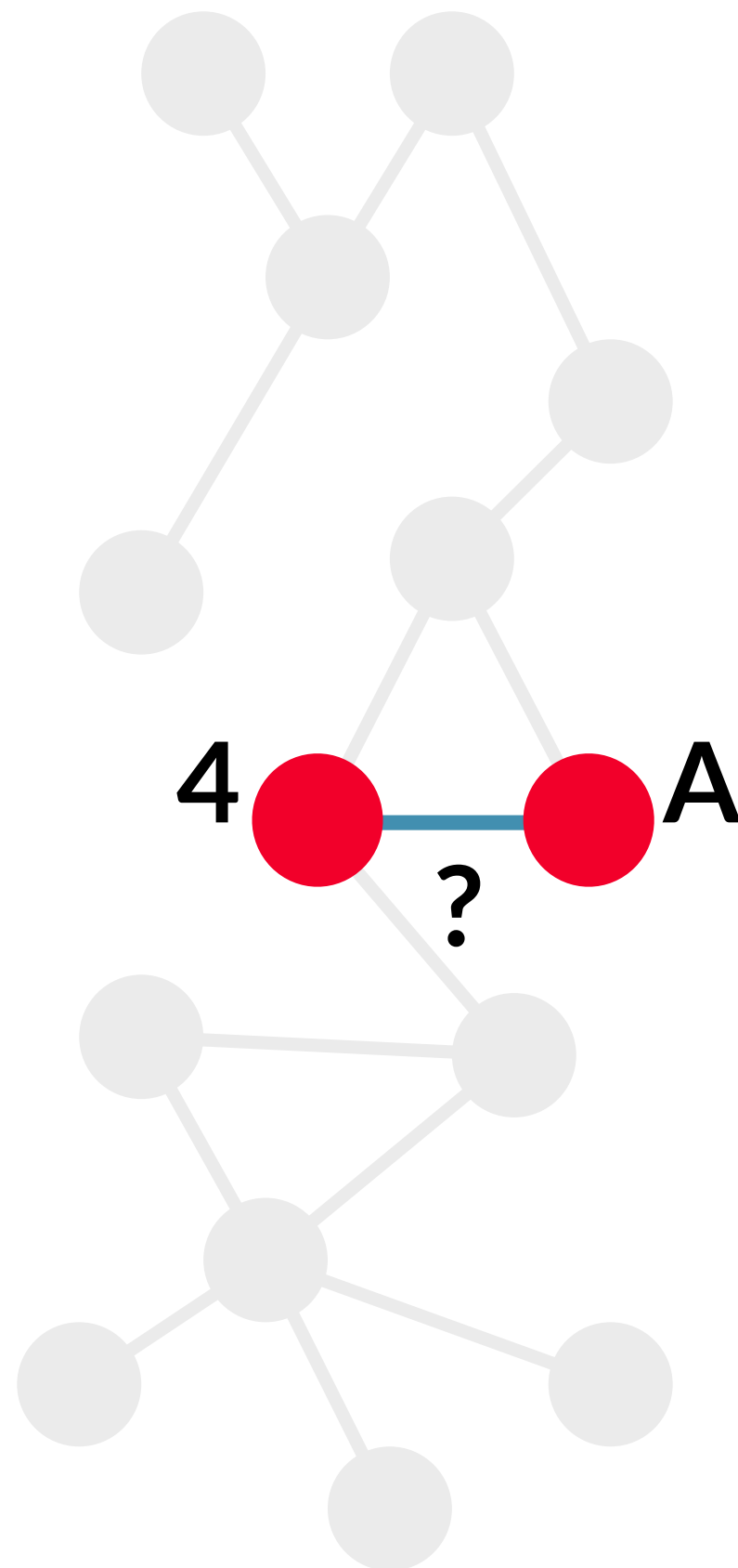
A SET **N** OF ELEMENTS (NODES)  
CONNECTED BY **L** LINKS.

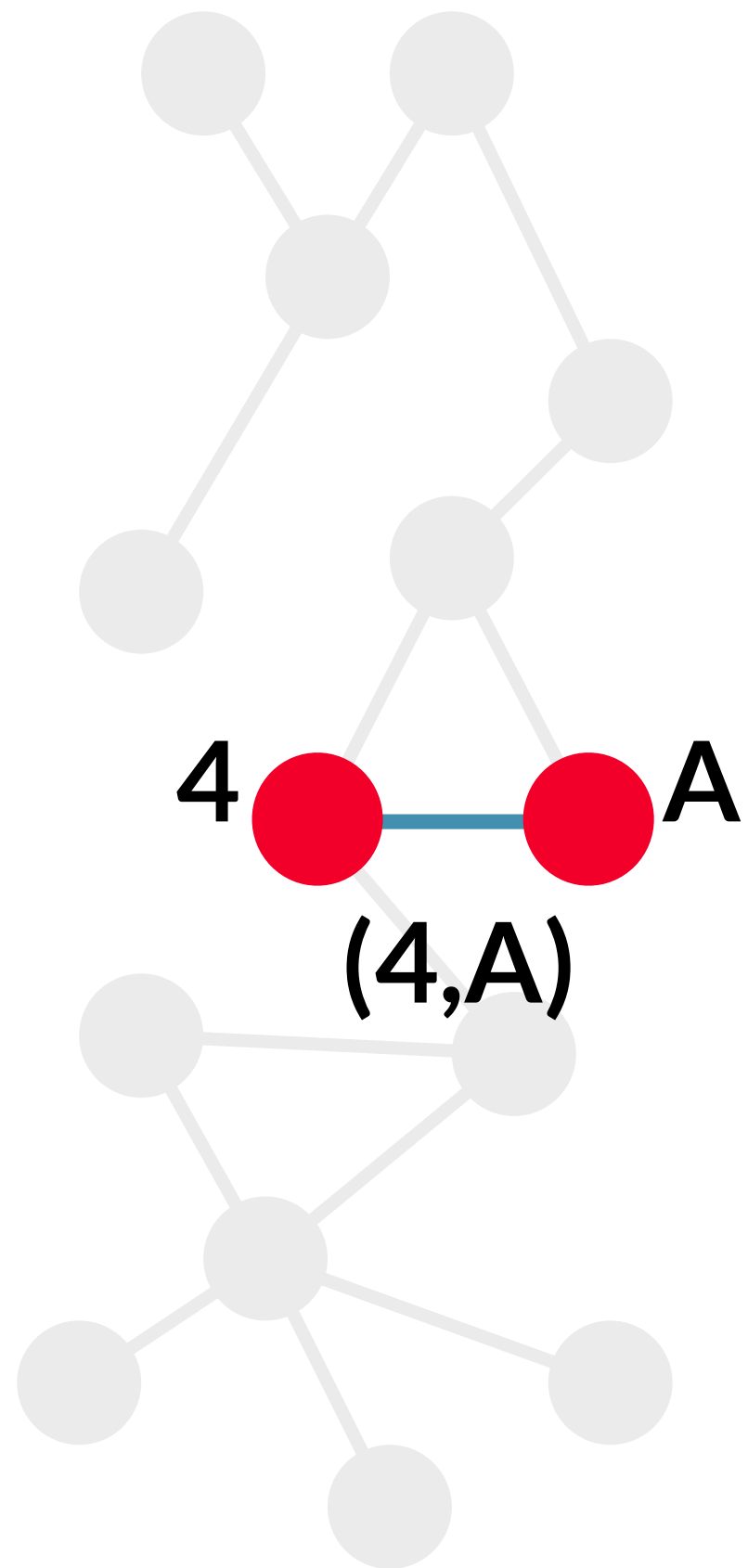
The link **(i,j)** connects the node **i** to the  
node **j**



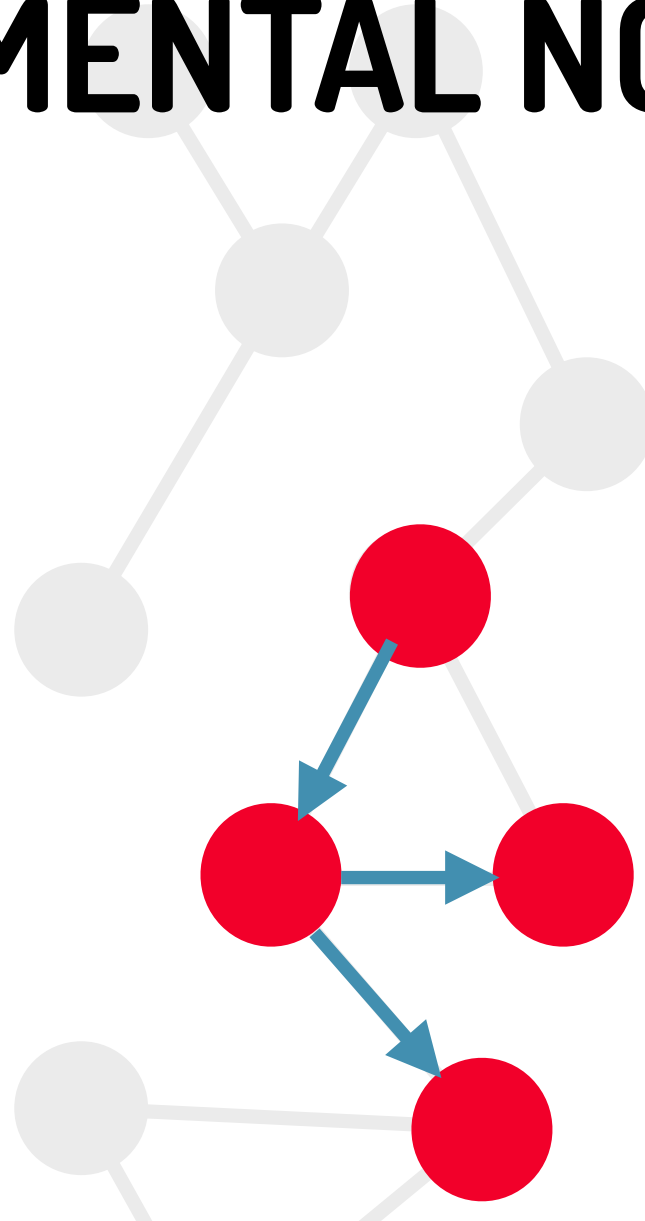
**$|N|=14$**   
 **$|L|=15$**



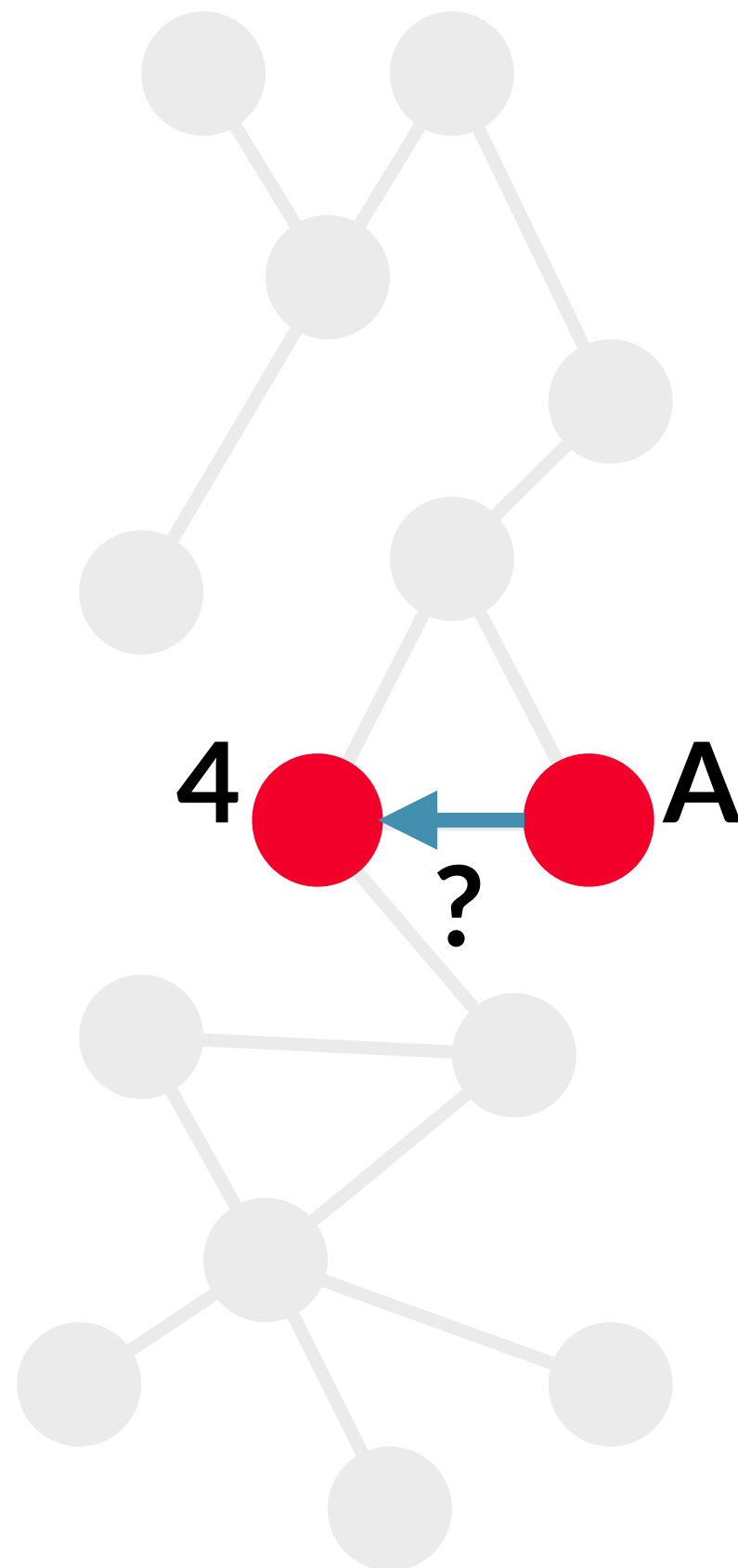


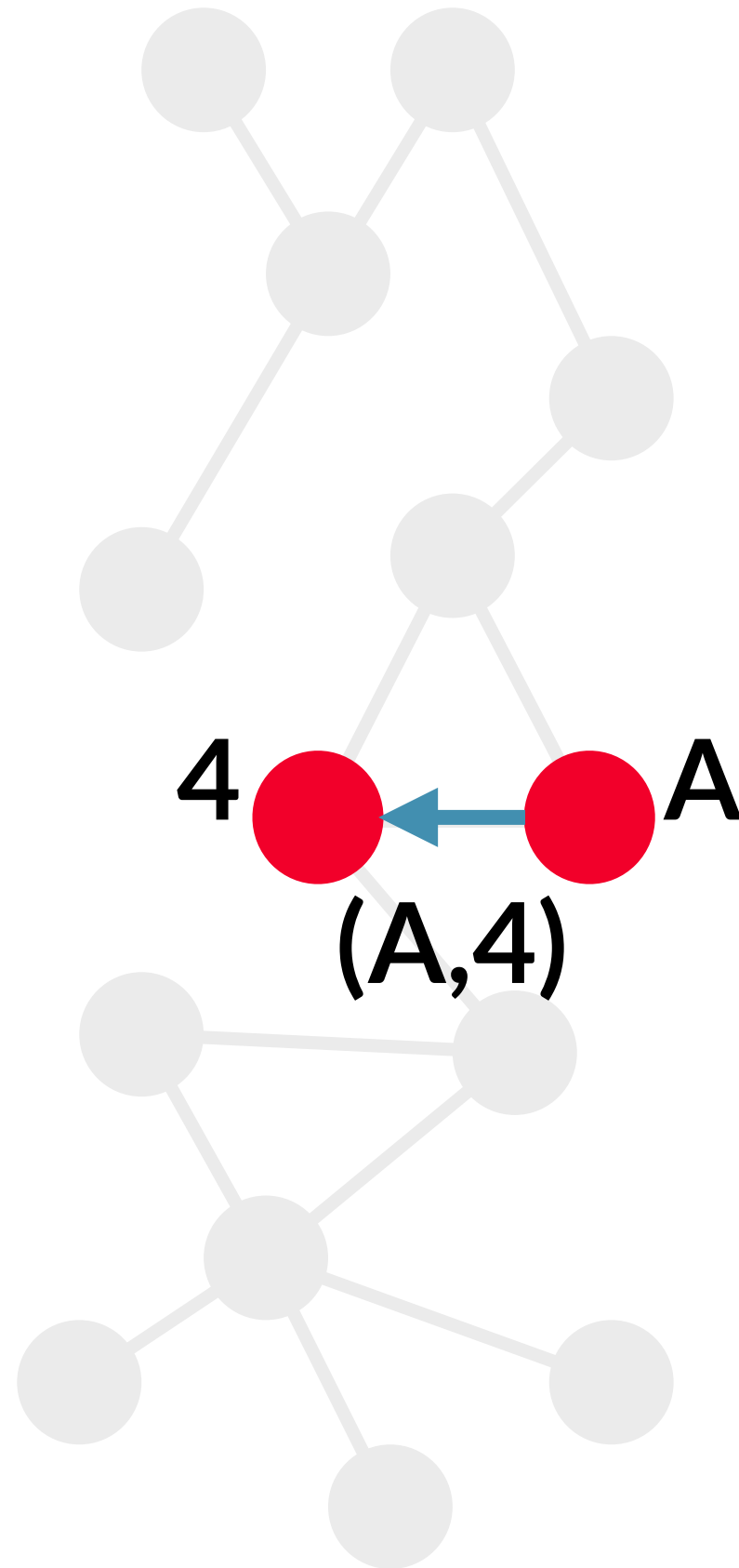


# FUNDAMENTAL NOTATION

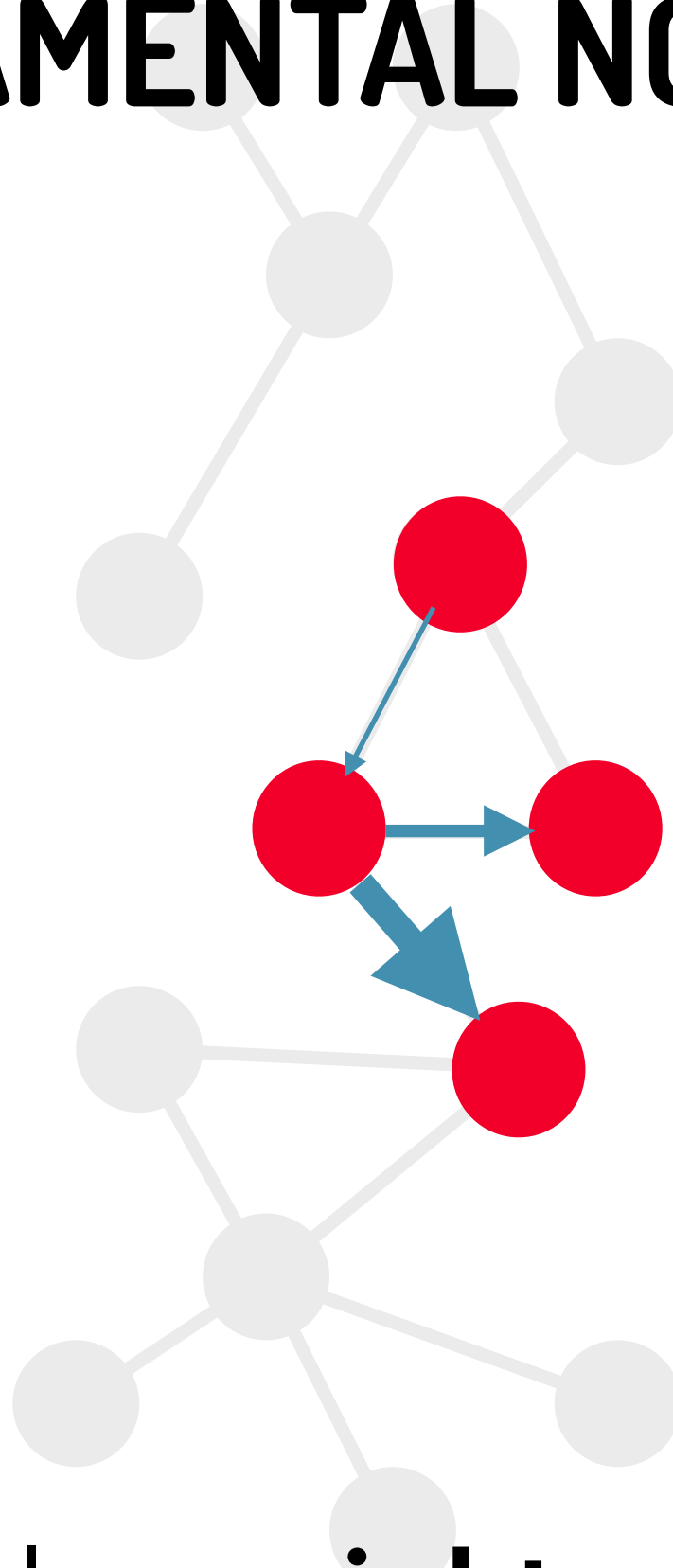


A network  $\mathbf{G}$  can be directed or undirected. A directed network has directed links.



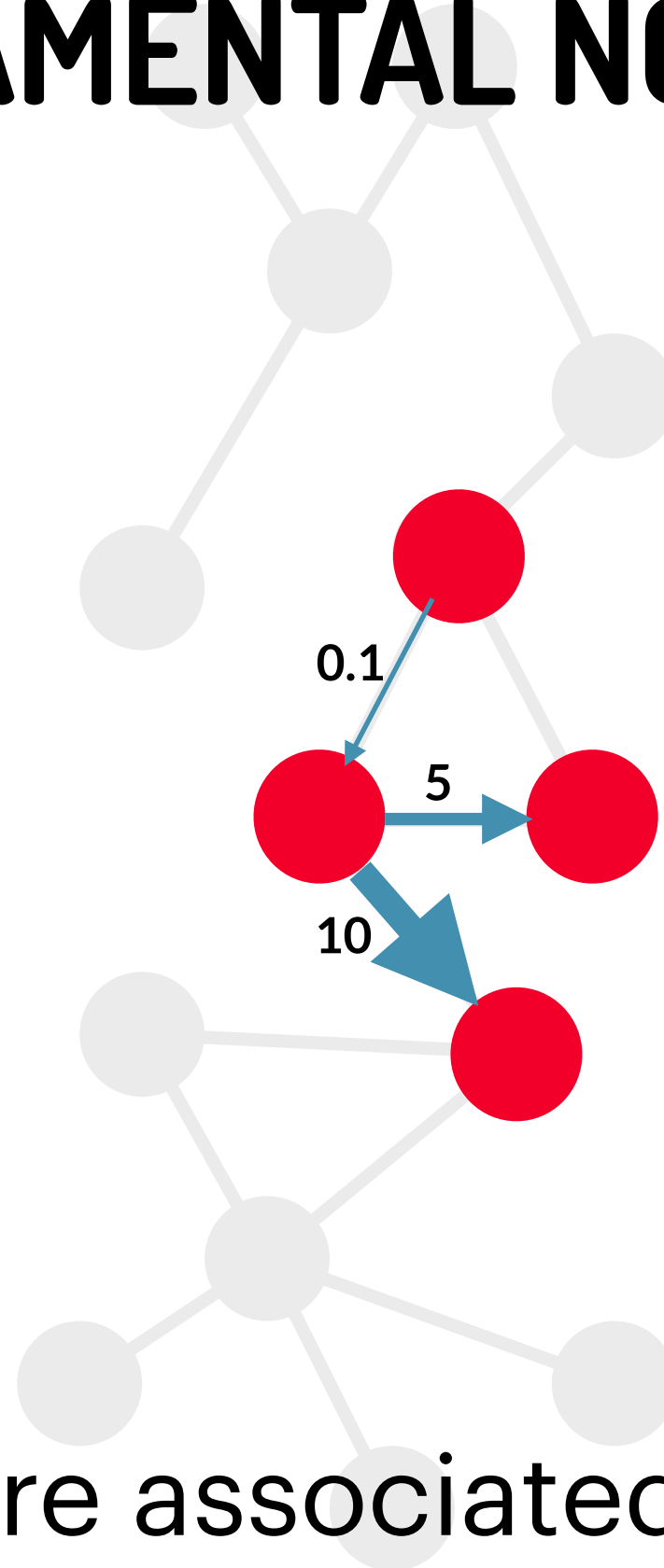


# FUNDAMENTAL NOTATION



Links can also be **weighted** or **unweighted**

# FUNDAMENTAL NOTATION



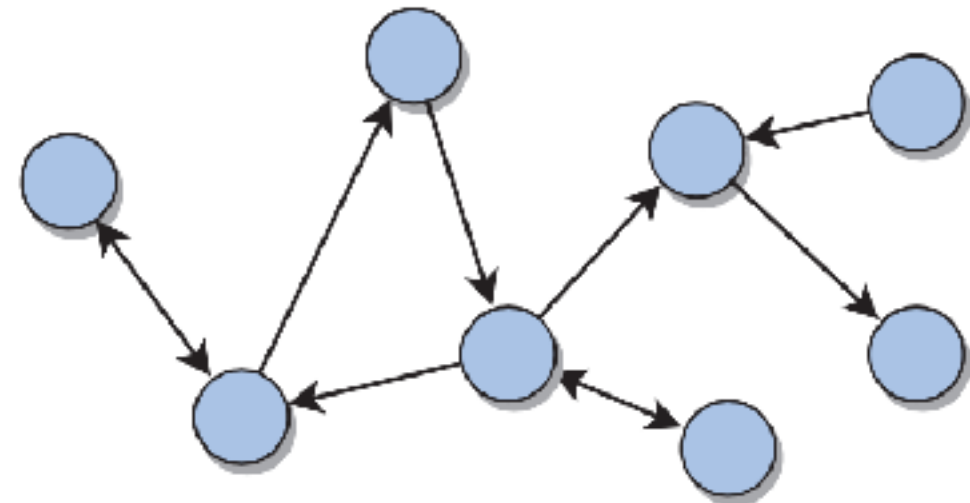
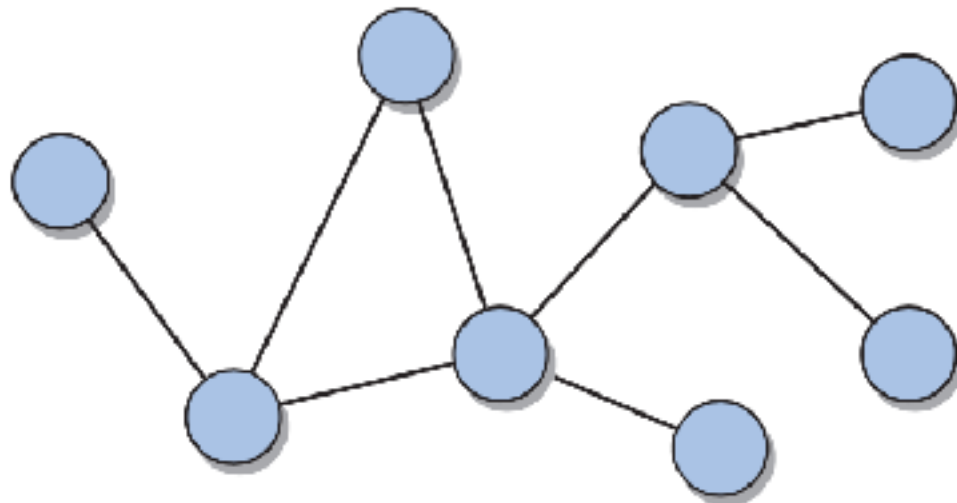
Weighted links are associated with a weight  $w$ ,  
so they are described by  $(i,j,w)$

# FUNDAMENTAL NOTATION

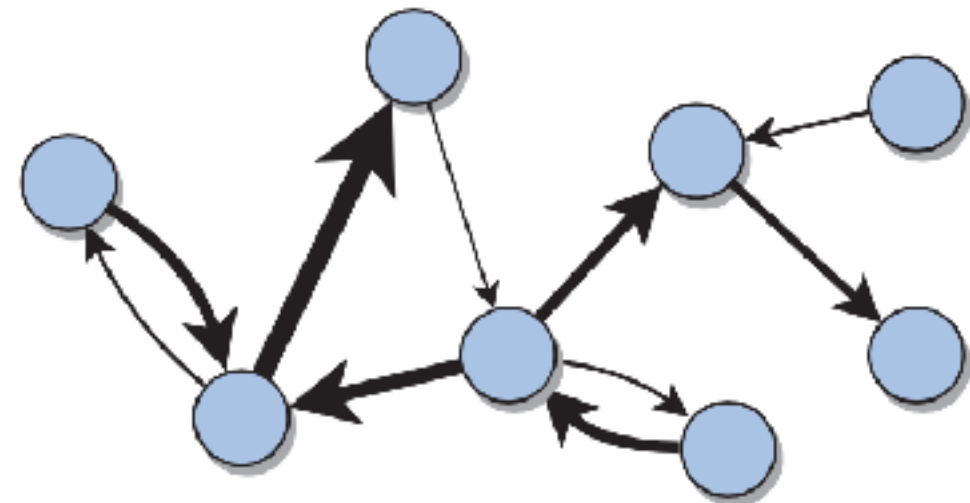
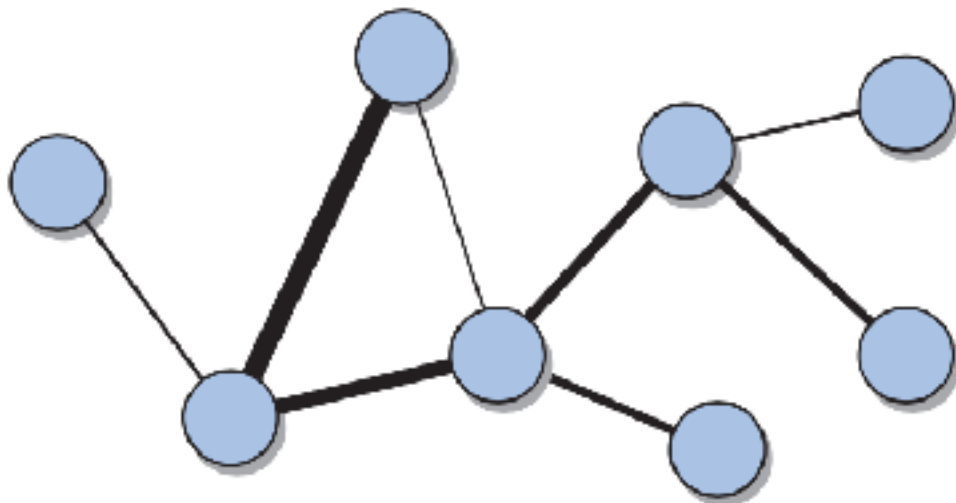
Undirected

Directed

Unweighted



Weighted



Undirected

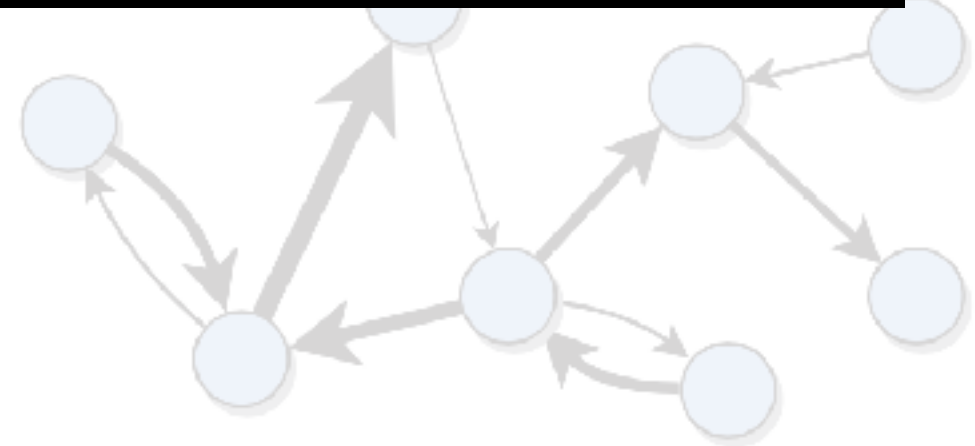
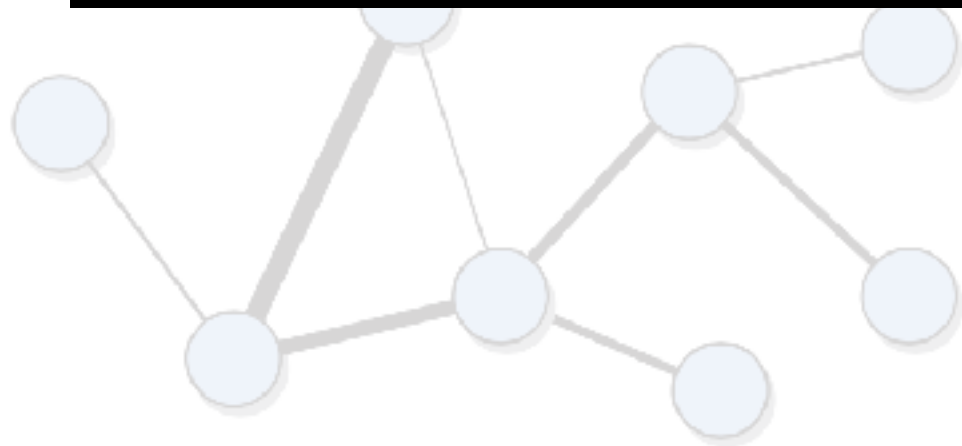
Directed

Unweighted



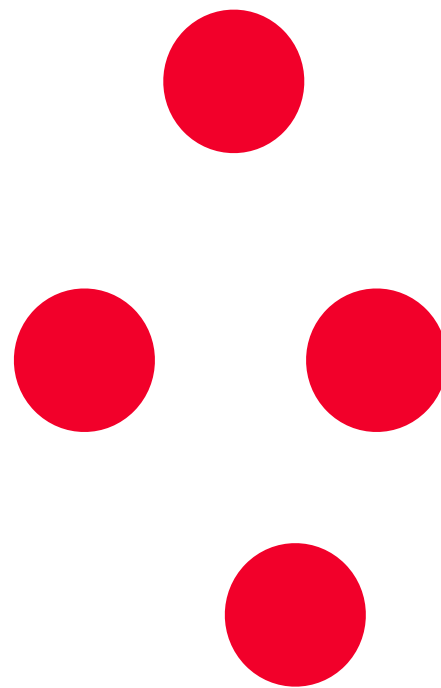
**CAN YOU MAKE EXAMPLES  
OF EACH TYPE OF NETWORK?**

Weighted



# MAXIMUM NUMBER OF LINKS

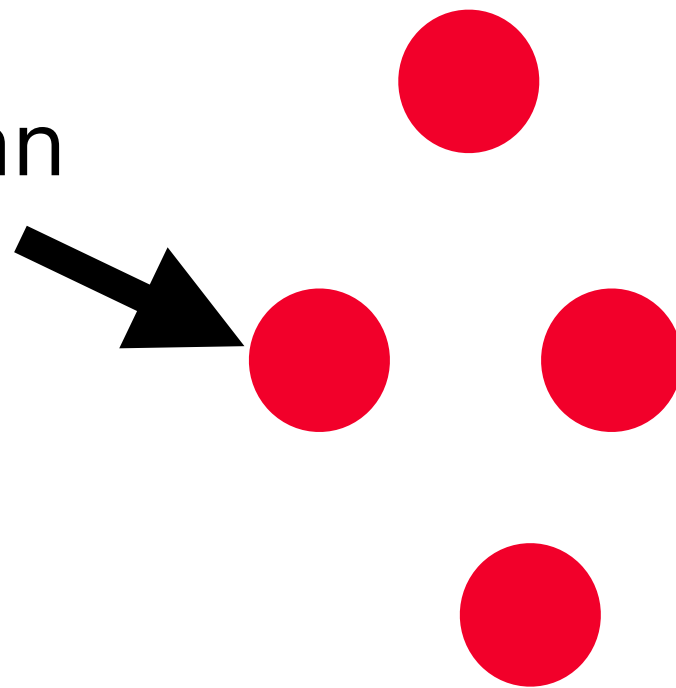
If we have 4 nodes in an undirected network, what is the **maximum possible number** of links between them?



# MAXIMUM NUMBER OF LINKS

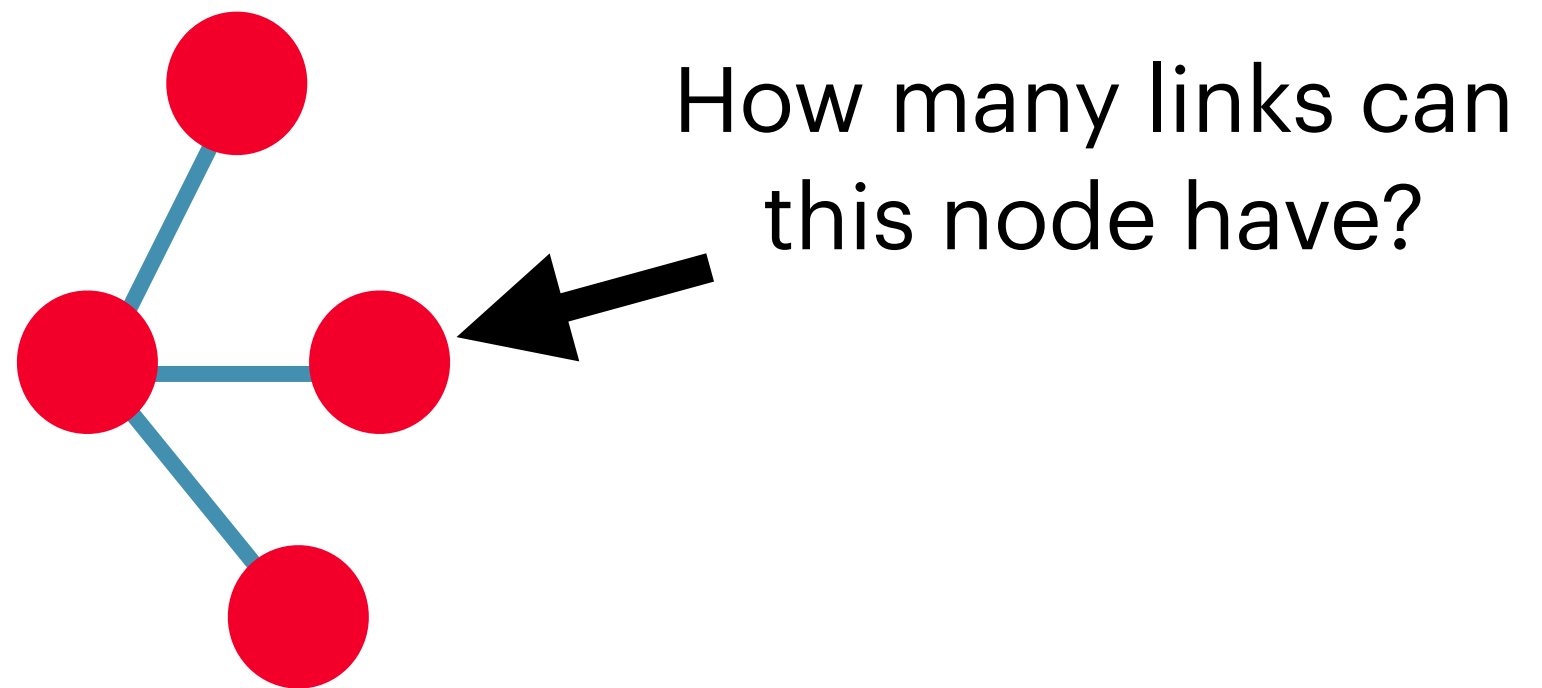
If we have 4 nodes in an undirected network, what is the **maximum possible number** of links between them?

How many links can  
this node have?



# MAXIMUM NUMBER OF LINKS

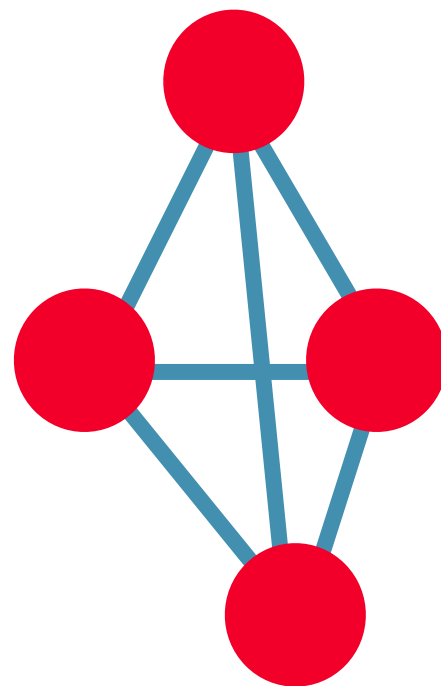
If we have 4 nodes in an undirected network, what is the **maximum possible number** of links between them?



# MAXIMUM NUMBER OF LINKS

Every node of the  $N$  nodes that we have **can connect to any other except from itself**

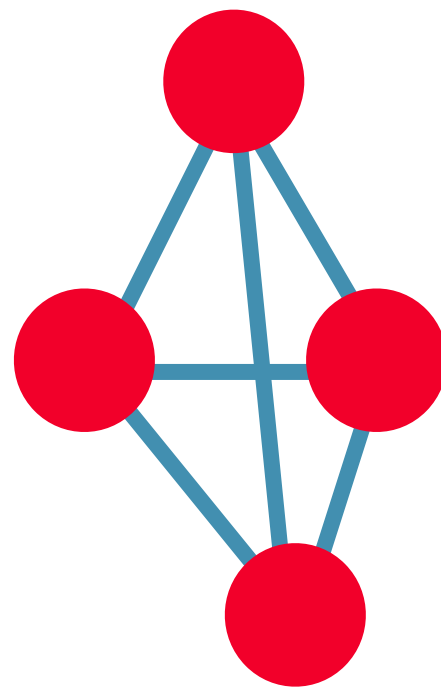
This means it can connect to  **$n-1$  nodes**



# MAXIMUM NUMBER OF LINKS

Every node of the  $N$  nodes that we have **can connect to any other except from itself**

This means it can connect to  **$n-1$  nodes**



**Then, the maximum number of links is  $N(N-1)$  right?**

**WRONG!!!**

**WE ARE COUNTING LINKS TWICE THIS  
WAY!**

**If the network is undirected, then  
the link  $(i,j)$  is equivalent to the link  $(j,i)$**

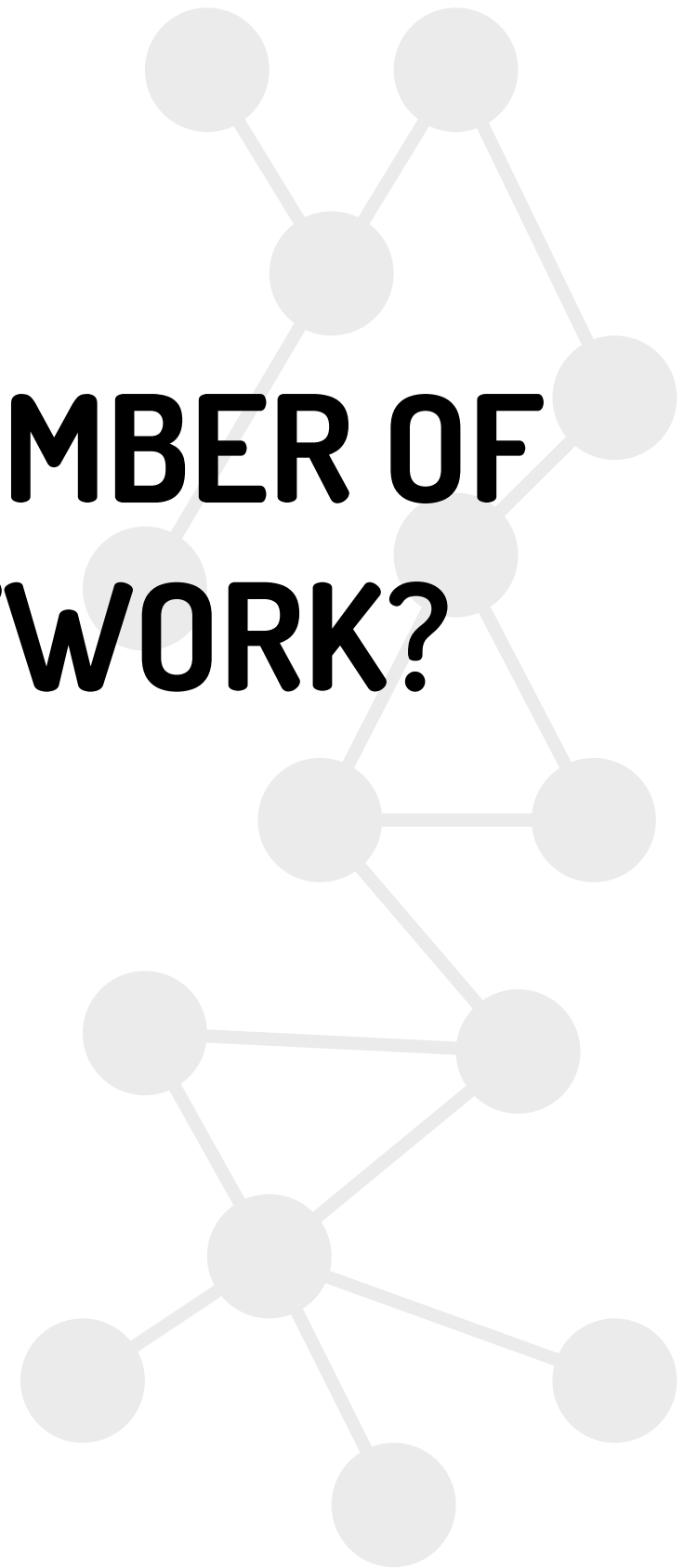
# MAXIMUM NUMBER OF LINKS

$$L_{max} = \binom{N}{2} = \frac{N(N-1)}{2}$$



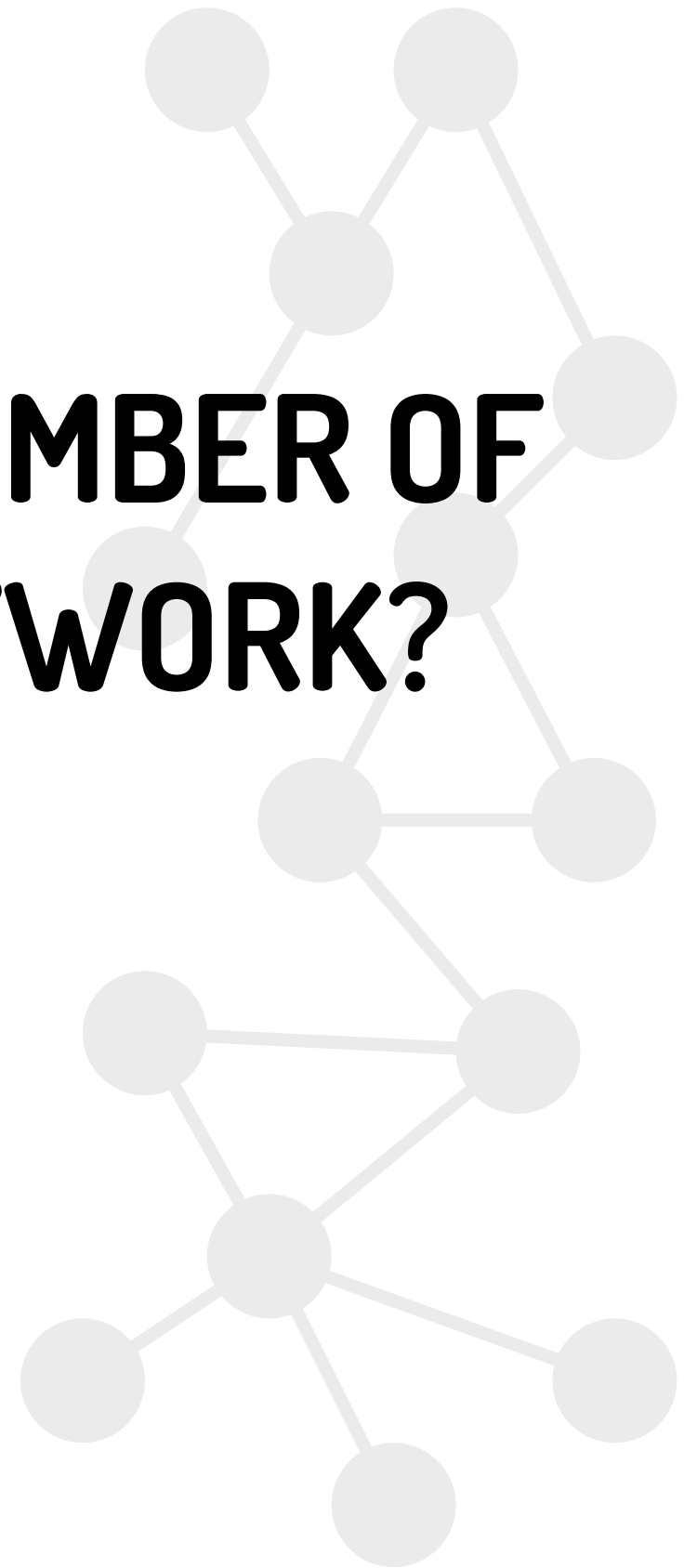
# WHAT IS THE MAXIMUM NUMBER OF LINKS IN A DIRECTED NETWORK?

$$L_{max} = ?$$



# WHAT IS THE MAXIMUM NUMBER OF LINKS IN A DIRECTED NETWORK?

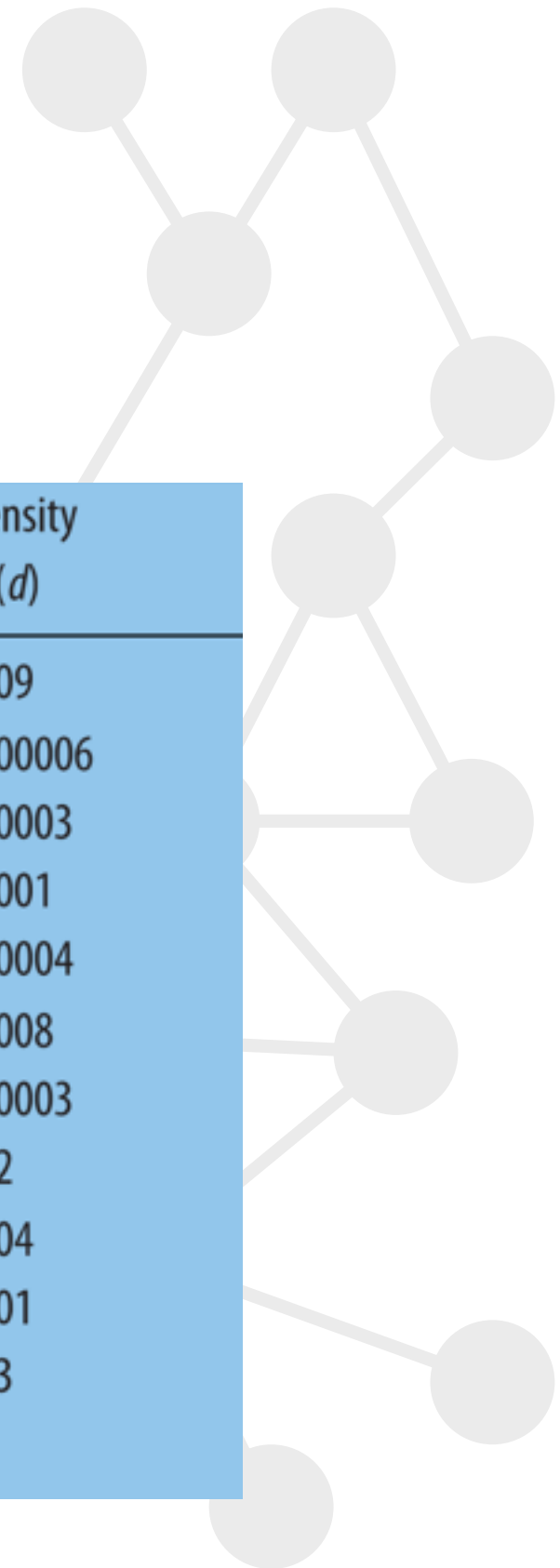
$$L_{max} = N(N - 1)$$



# DENSITY

$$d = \frac{L}{L_{max}}$$

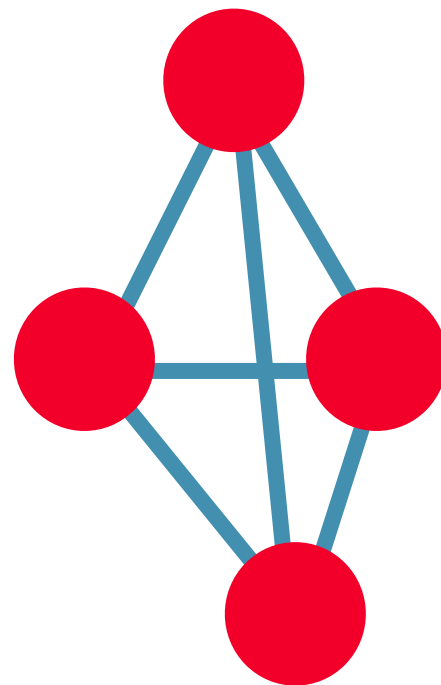
| Network                        | Type | Nodes<br>( $N$ ) | Links<br>( $L$ ) | Density<br>( $d$ ) |
|--------------------------------|------|------------------|------------------|--------------------|
| Facebook Northwestern Univ.    |      | 10,567           | 488,337          | 0.009              |
| IMDB movies and stars          |      | 563,443          | 921,160          | 0.000006           |
| IMDB co-stars                  | W    | 252,999          | 1,015,187        | 0.00003            |
| Twitter US politics            | DW   | 18,470           | 48,365           | 0.0001             |
| Enron email                    | DW   | 87,273           | 321,918          | 0.00004            |
| Wikipedia math                 | D    | 15,220           | 194,103          | 0.0008             |
| Internet routers               |      | 190,914          | 607,610          | 0.00003            |
| US air transportation          |      | 546              | 2,781            | 0.02               |
| World air transportation       |      | 3,179            | 18,617           | 0.004              |
| Yeast protein interactions     |      | 1,870            | 2,277            | 0.001              |
| <i>C. elegans</i> brain        | DW   | 297              | 2,345            | 0.03               |
| Everglades ecological food web | DW   | 69               | 916              | 0.2                |



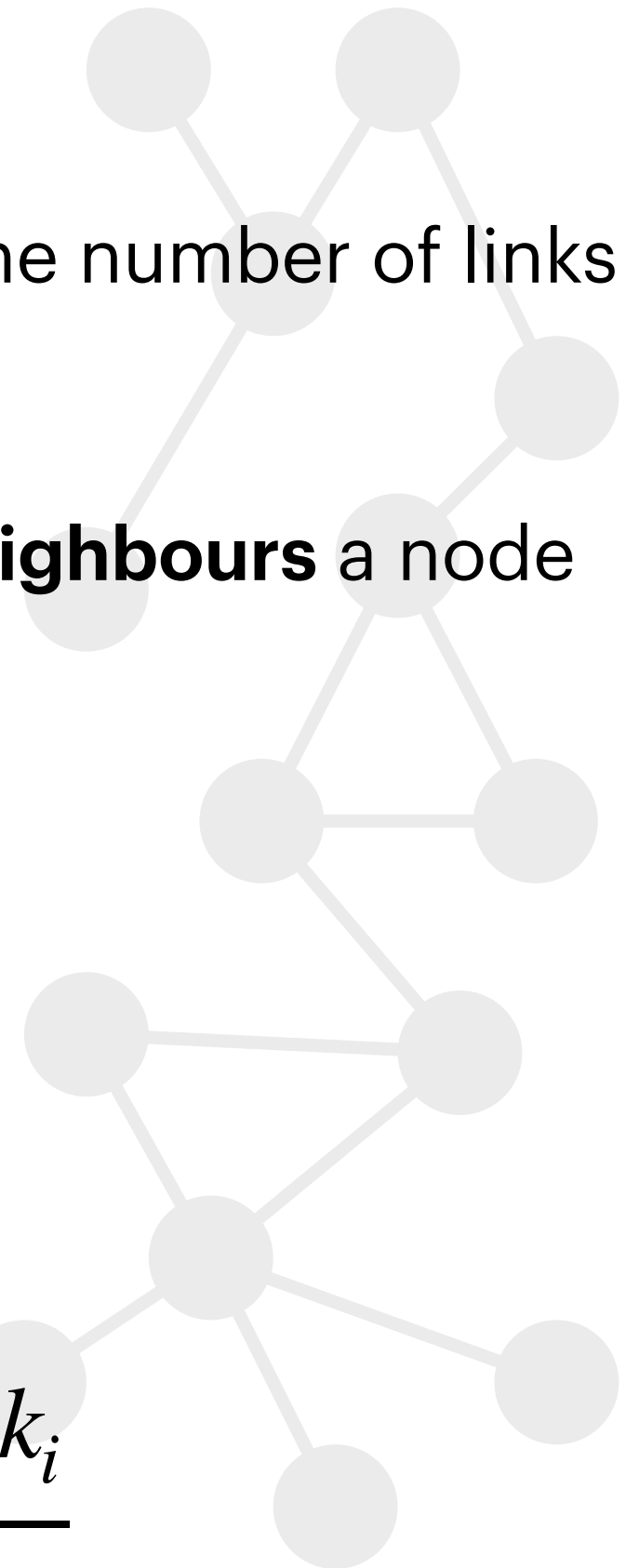
# DEGREE

In an undirected network, the degree **k** of a node is the number of links a node has

This is equivalent of saying that k is the number of **neighbours** a node has



The **average degree** of a network is  $\langle k \rangle = \frac{\sum_i k_i}{N}$



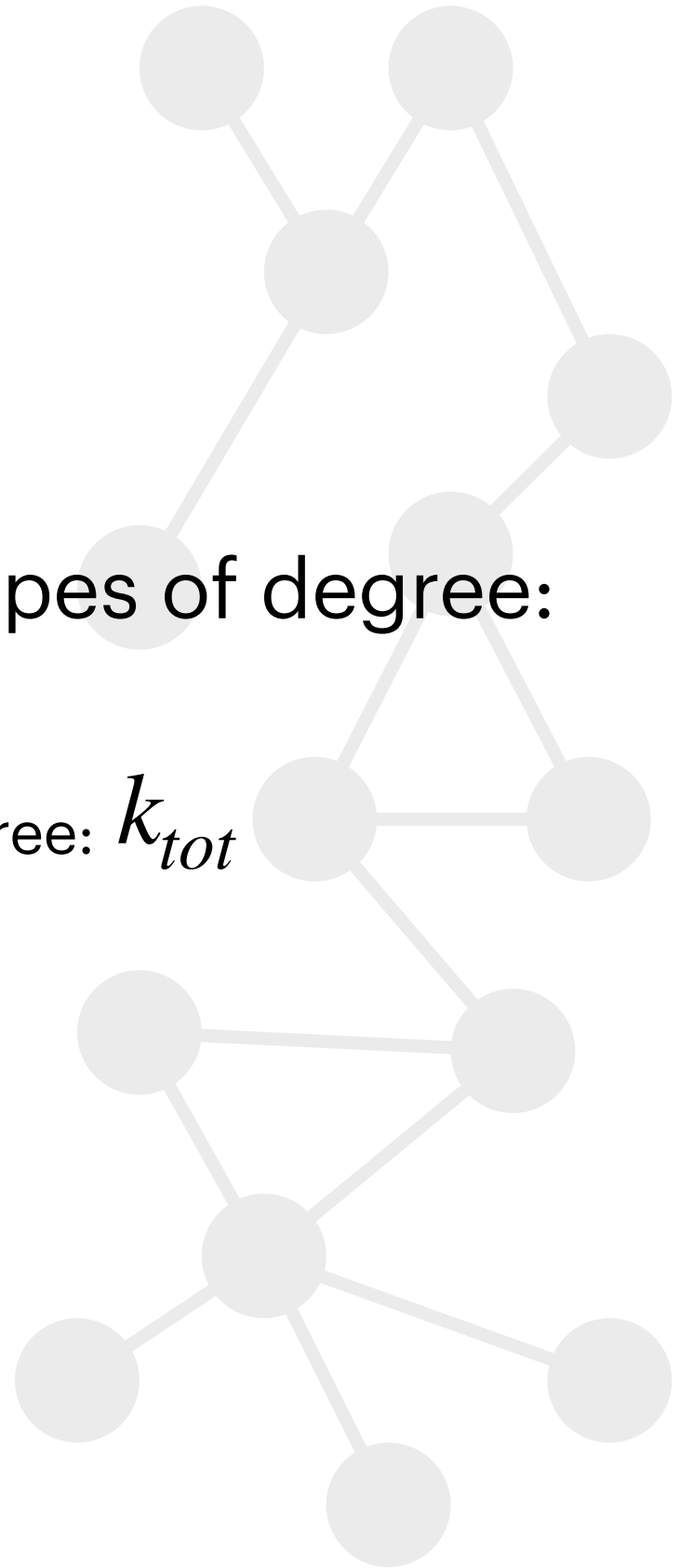
# DEGREE

In a directed network, there are three types of degree:

In-degree:  $k_{in}$

Out-degree:  $k_{out}$

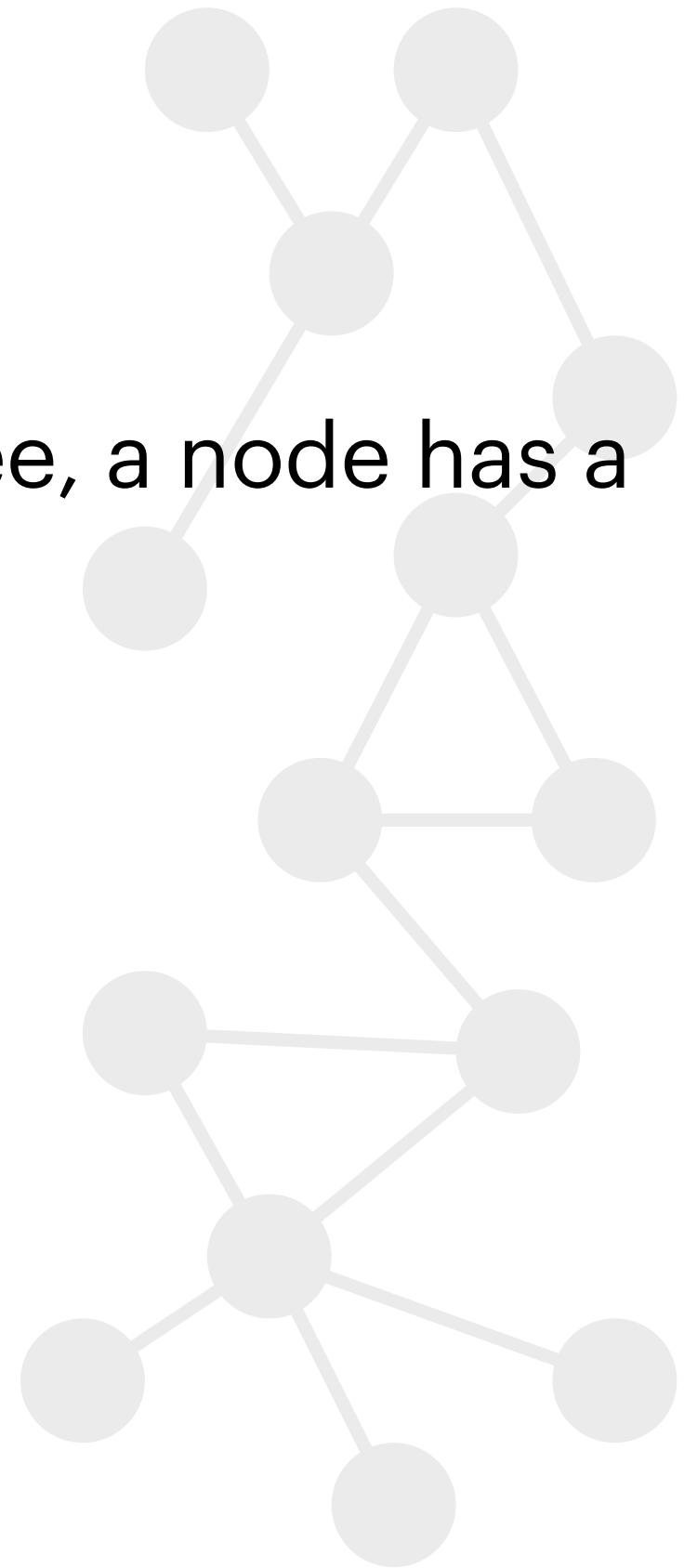
Total-degree:  $k_{tot}$



# DEGREE

In a weighted network, instead of a degree, a node has a **strength**:

$$s_i = \sum_j w_{ij}$$



# DEGREE

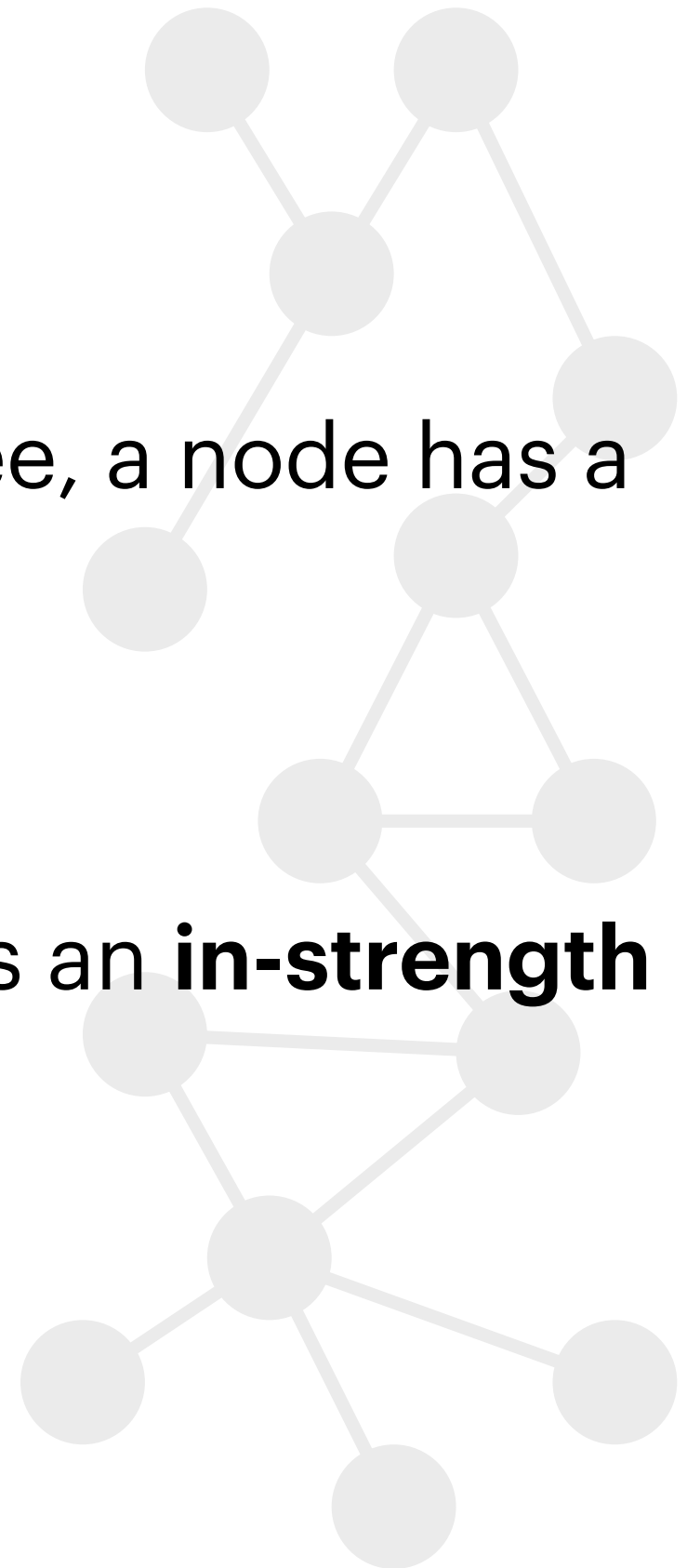
In a weighted network, instead of a degree, a node has a **strength**:

$$s_i = \sum_j w_{ij}$$

In a directed weighted network, a node has an **in-strength** and an **out-strength**:

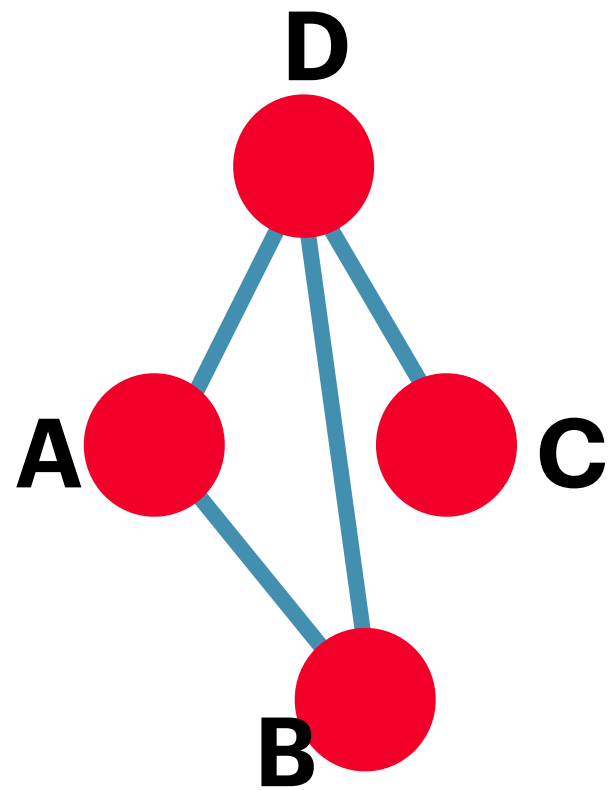
$$s_i^{in} = \sum_j w_{ji}$$

$$s_i^{out} = \sum_j w_{ij}$$



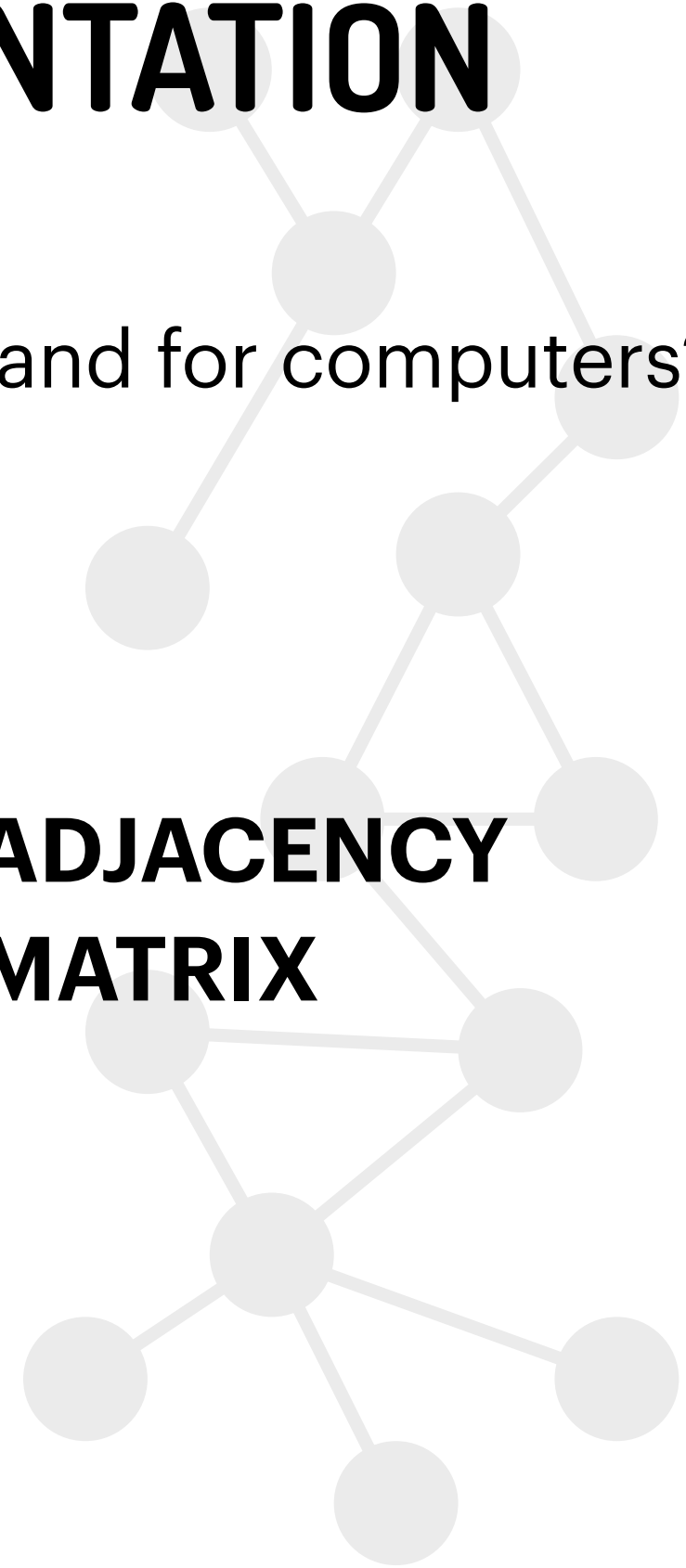
# MATHEMATICAL REPRESENTATION

How do we represent networks mathematically and for computers?



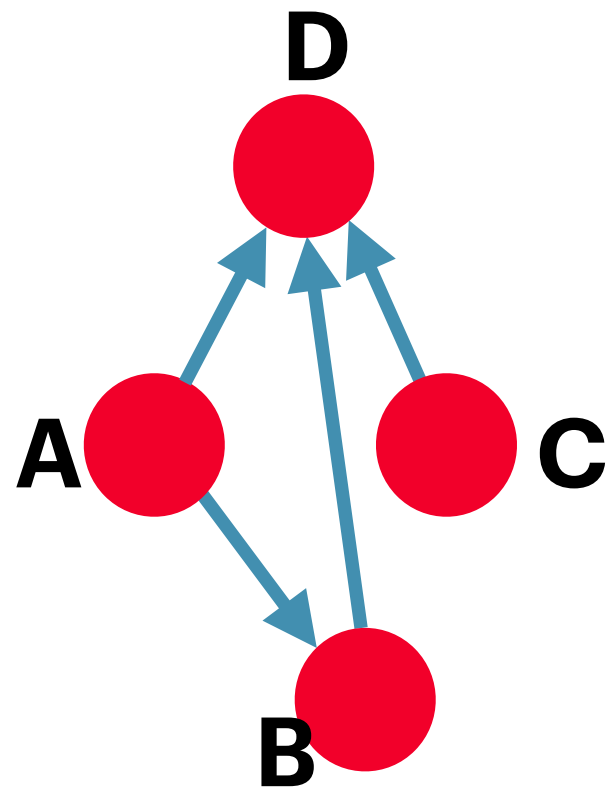
|   | A | B | C | D |
|---|---|---|---|---|
| A | 0 | 1 | 0 | 1 |
| B | 1 | 0 | 0 | 1 |
| C | 0 | 0 | 0 | 1 |
| D | 1 | 1 | 1 | 0 |

**ADJACENCY  
MATRIX**



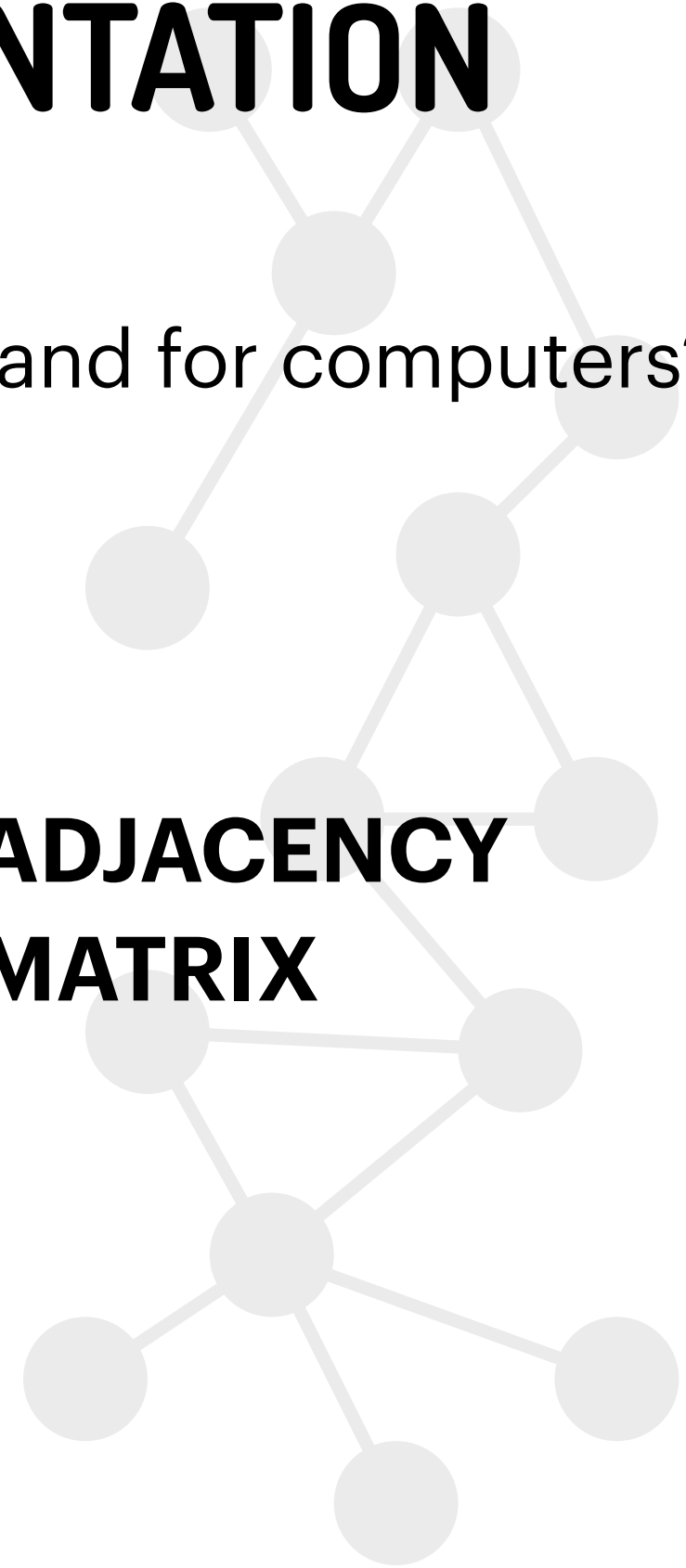
# MATHEMATICAL REPRESENTATION

How do we represent networks mathematically and for computers?



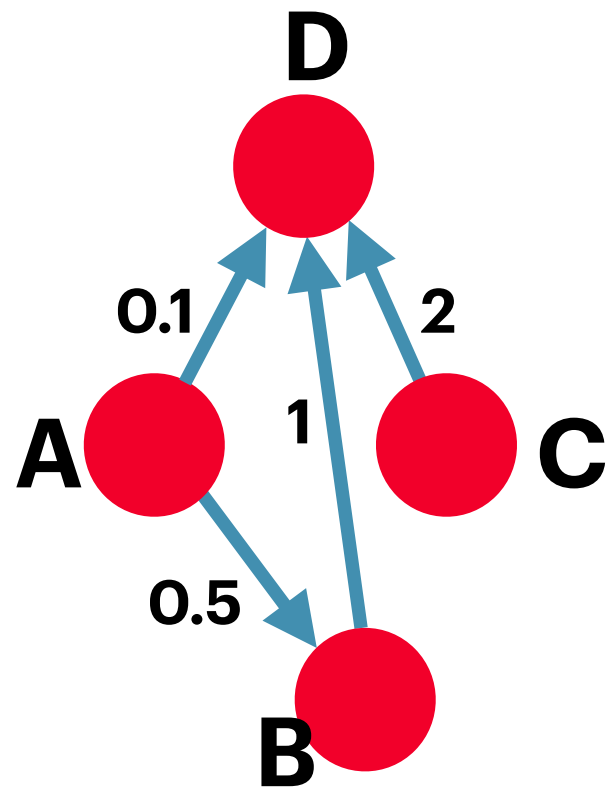
|   | A | B | C | D |
|---|---|---|---|---|
| A | 0 | 1 | 0 | 1 |
| B | 0 | 0 | 0 | 1 |
| C | 0 | 0 | 0 | 1 |
| D | 0 | 0 | 0 | 0 |

**ADJACENCY  
MATRIX**



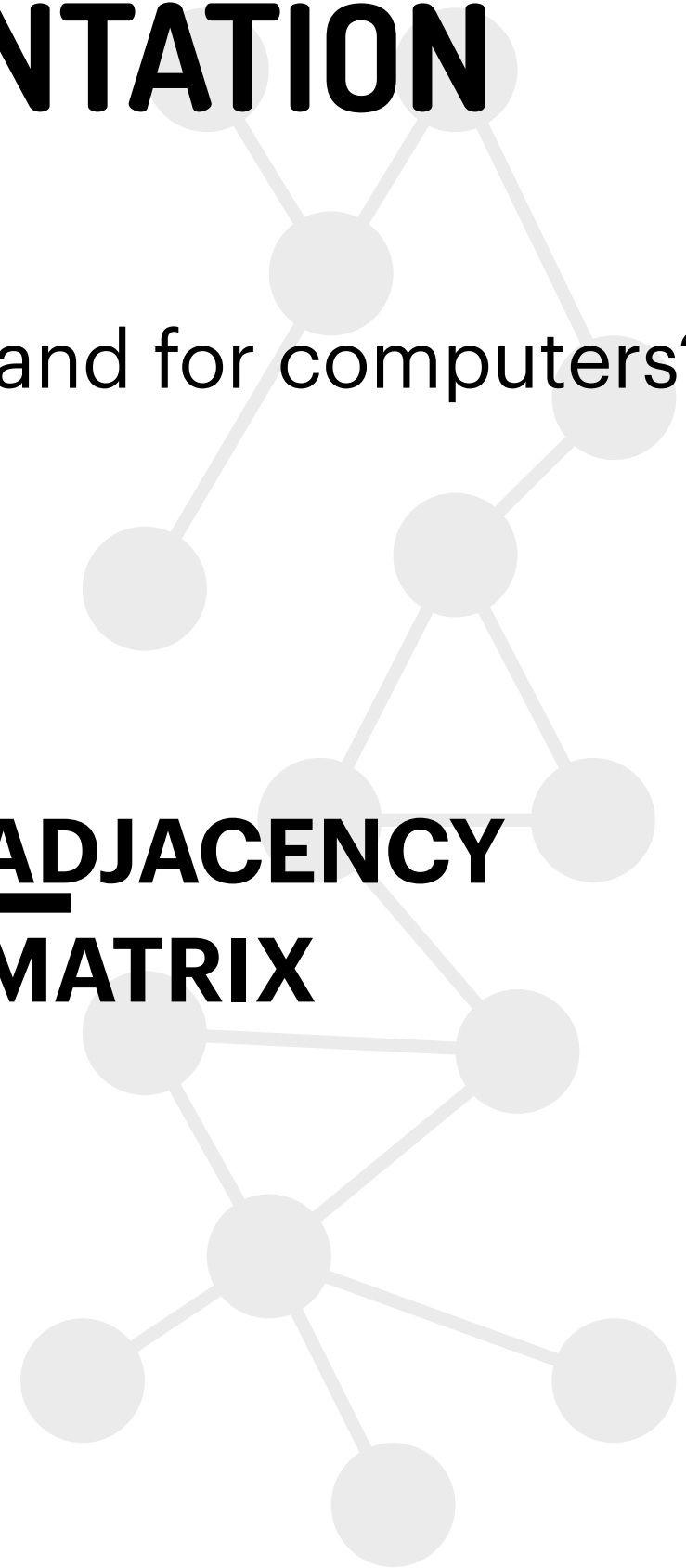
# MATHEMATICAL REPRESENTATION

How do we represent networks mathematically and for computers?



|   | A | B   | C | D   |
|---|---|-----|---|-----|
| A | 0 | 0.5 | 0 | 0.1 |
| B | 0 | 0   | 0 | 1   |
| C | 0 | 0   | 0 | 2   |
| D | 0 | 0   | 0 | 0   |

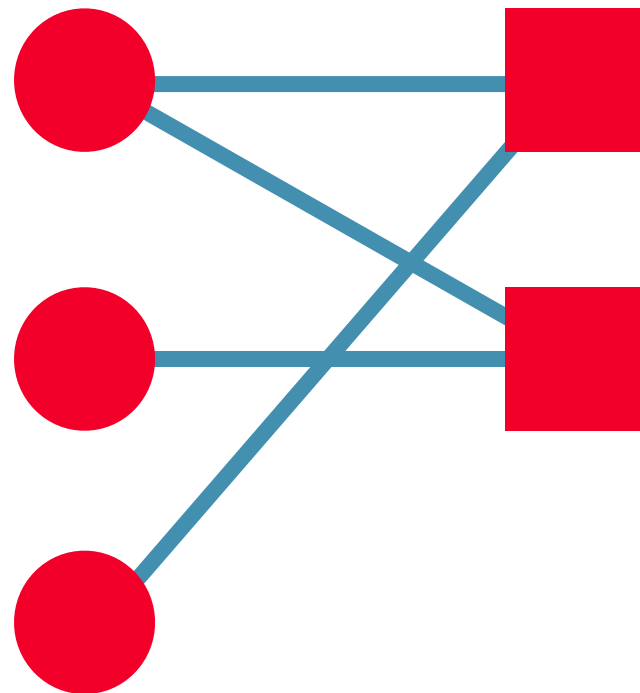
← **ADJACENCY  
MATRIX**



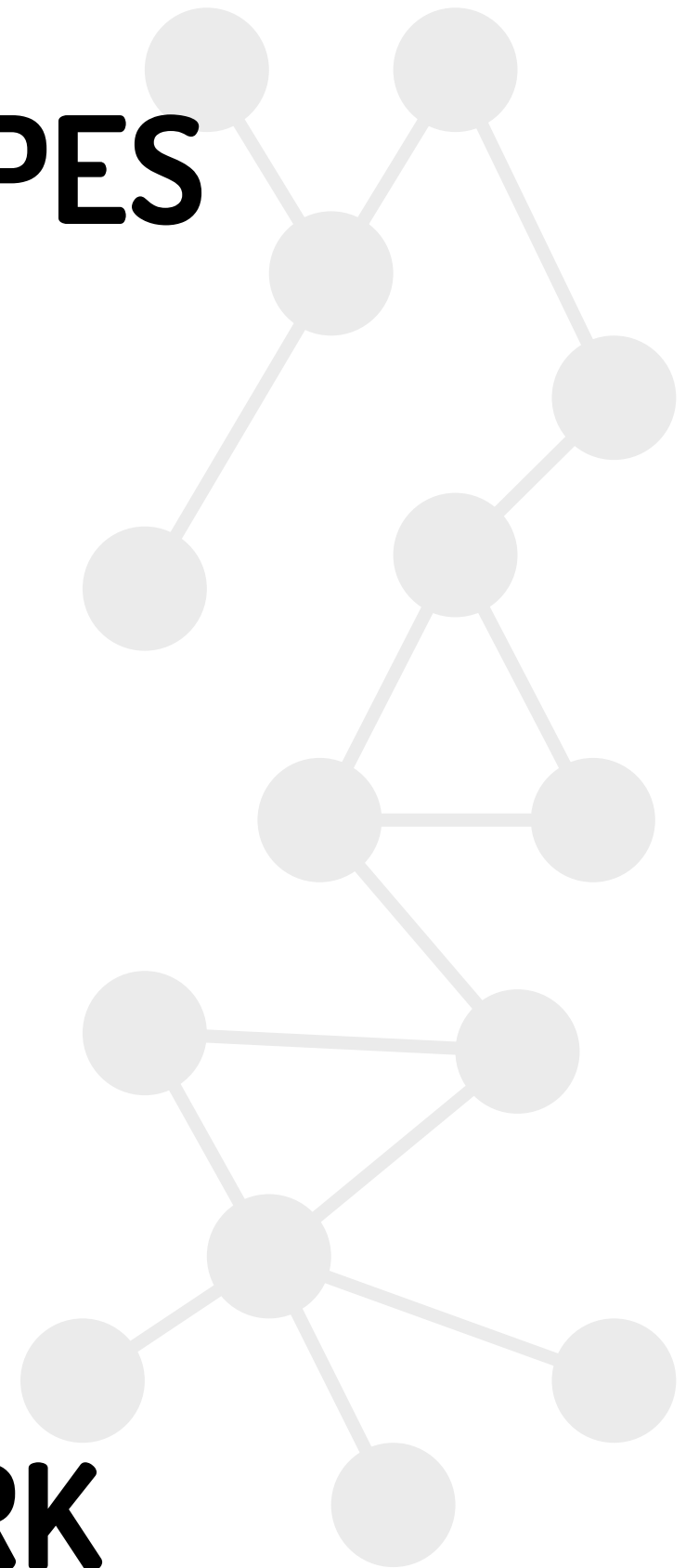
# OTHER NETWORK TYPES

**STUDENTS**

**COURSES**



**BIPARTITE NETWORK**



July-August, 2011

Asset type

Fund complex

NON-EUROZONE:

Certificate of Deposit

Other Repo

Financial Co. CP

GSE Repo

ABCP

Other Instrument

Treasury Repo

Other CP

VRDN

Insurance Co. Funding Agr.

Investment Co.

SIV Note

EUROZONE:

Certificate of Deposit

Financial Co. CP

GSE Repo

ABCP

Other Instrument

Other Repo

Other CP

Treasury Repo

A

B

C

D

E

F

G

H

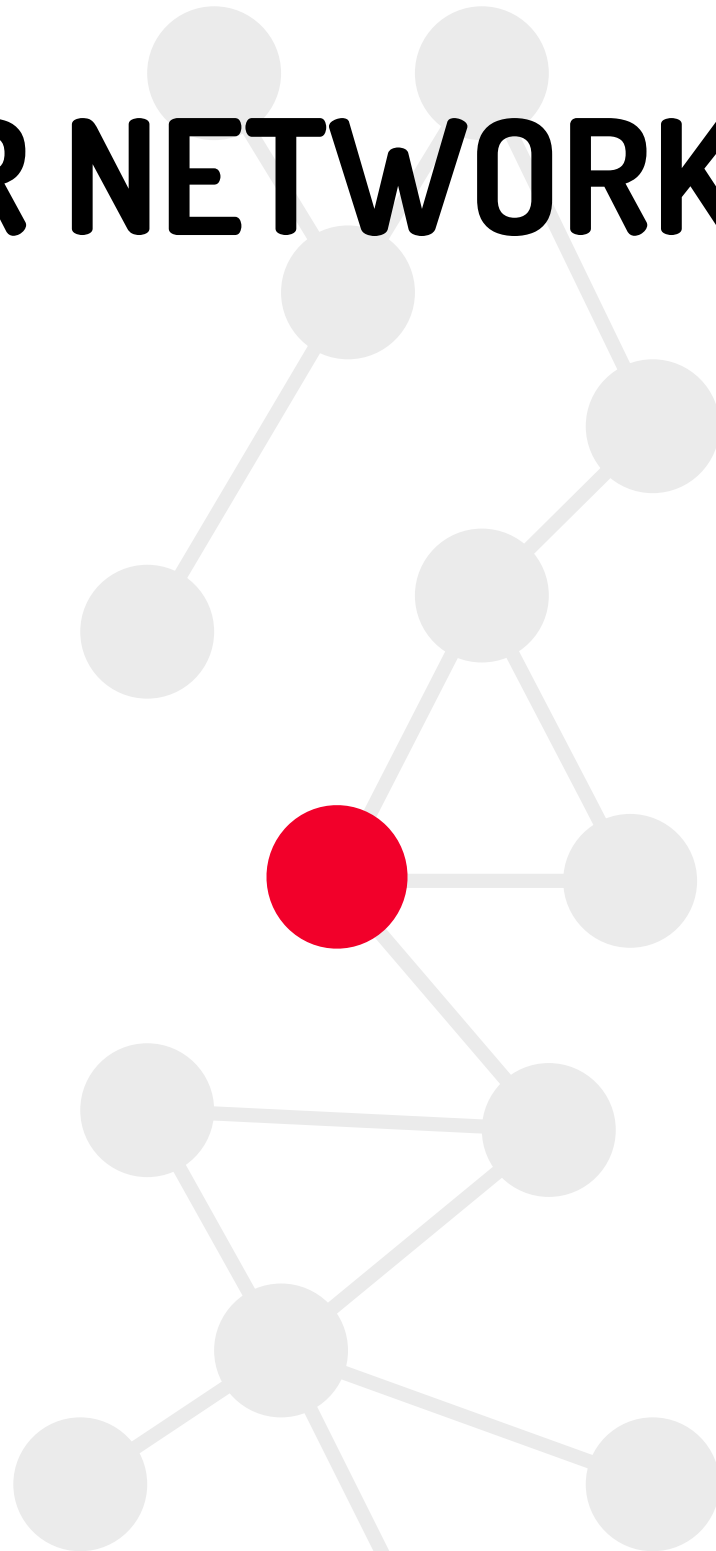
I

J

EXAMPLE: ASSET HOLDINGS

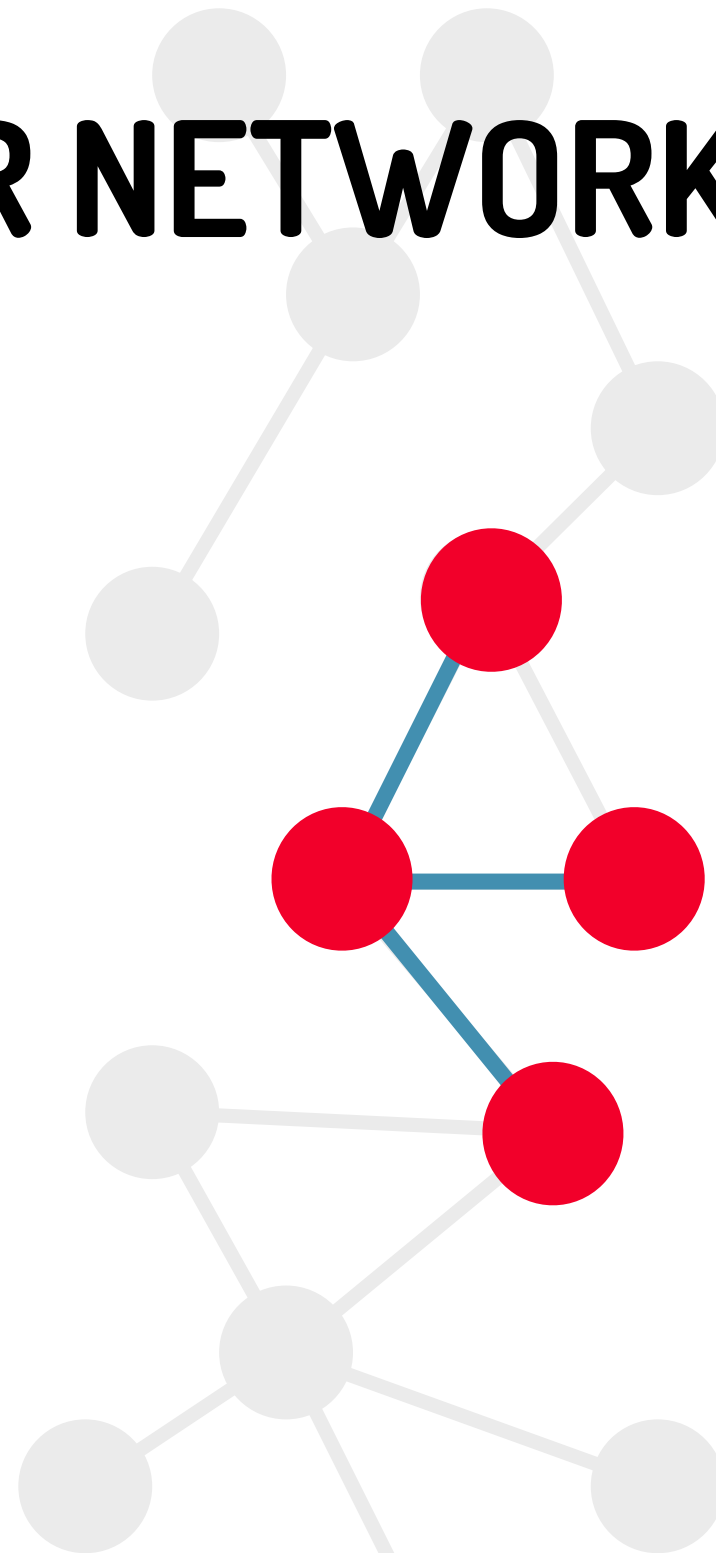
Image source : SEC N-MFP, OFR analysis

# OTHER NETWORK TYPES



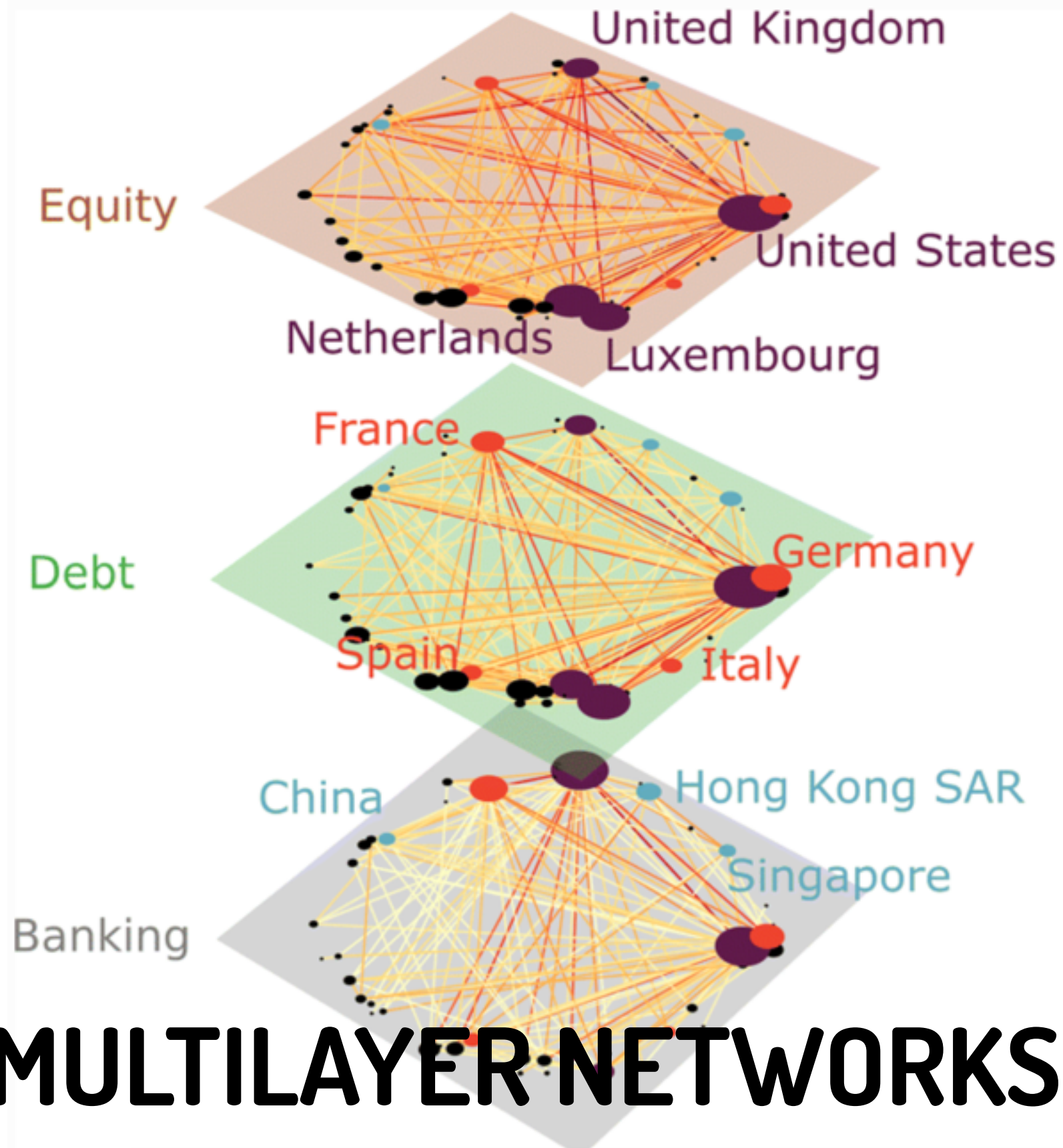
**SUB-NETWORKS AND EGO-NETWORKS**

# OTHER NETWORK TYPES

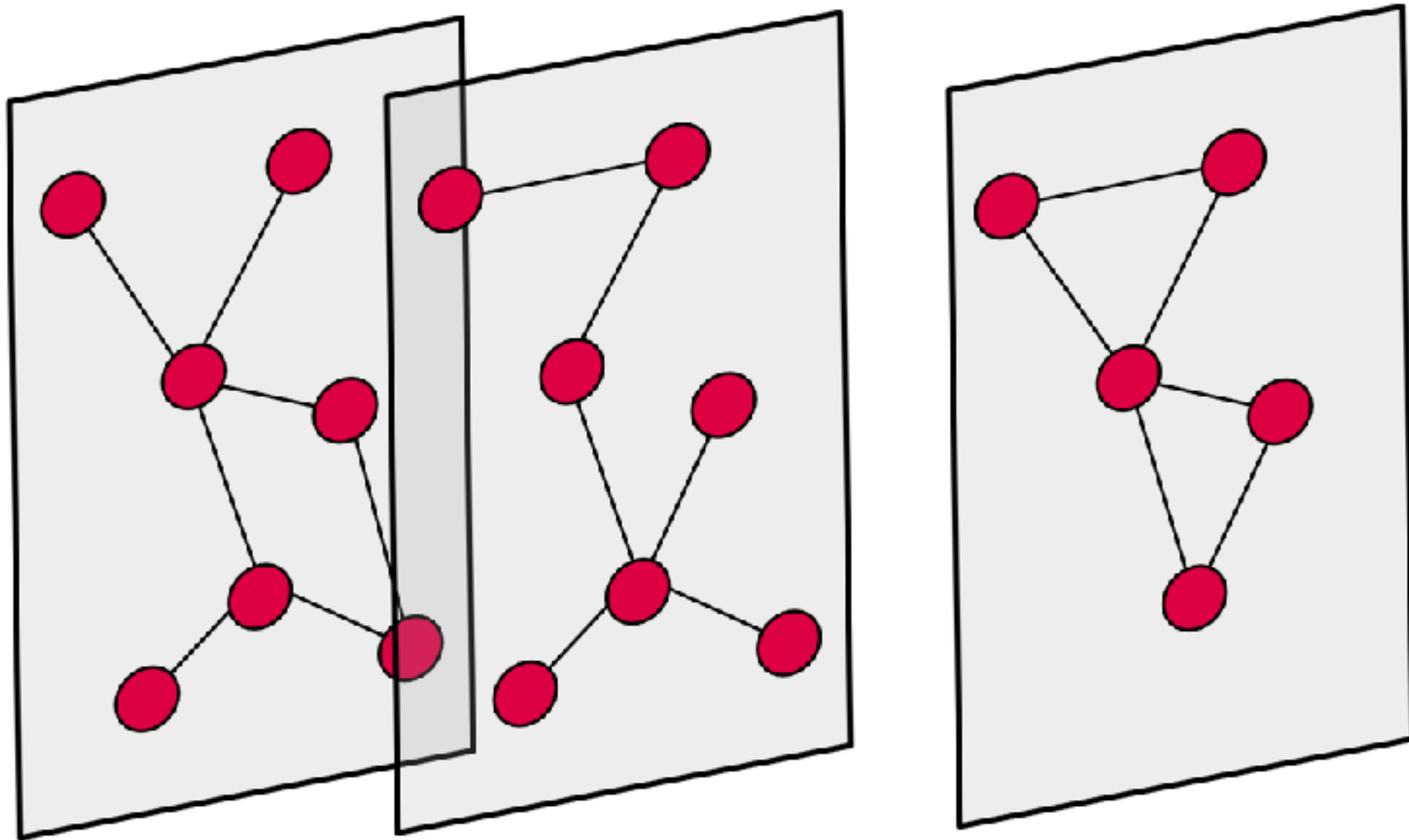


## SUB-NETWORKS AND EGO-NETWORKS

# OTHER NETWORK TYPES



# OTHER NETWORK TYPES



$t=1$

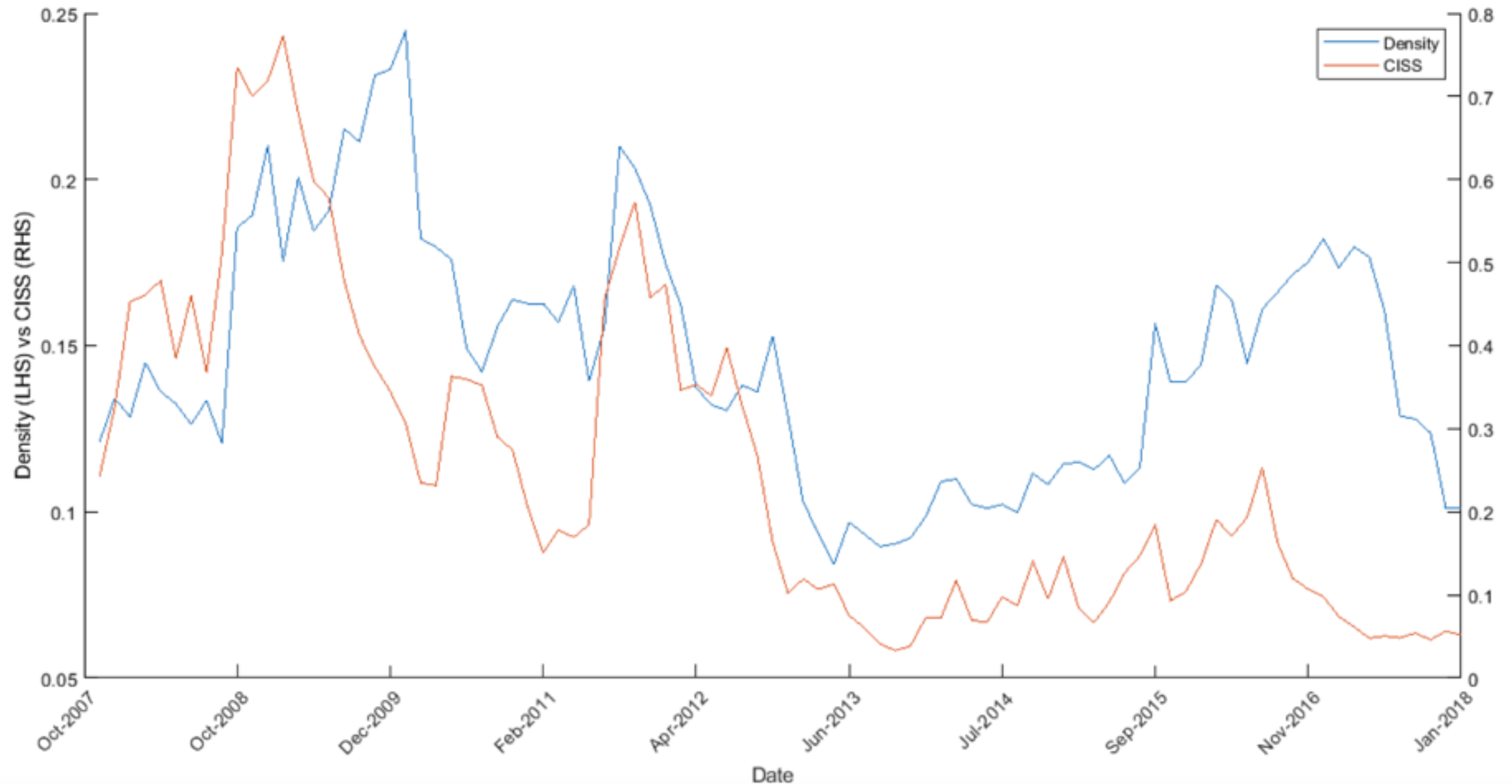
$t=2$

...

$t=T$

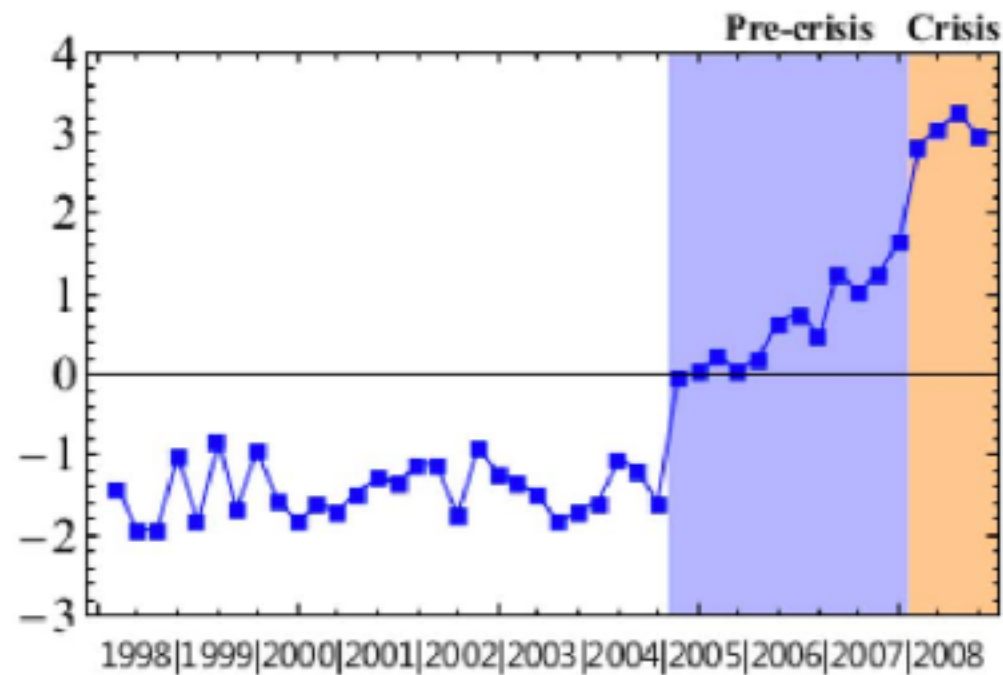
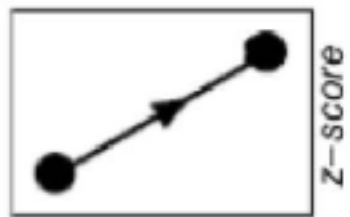
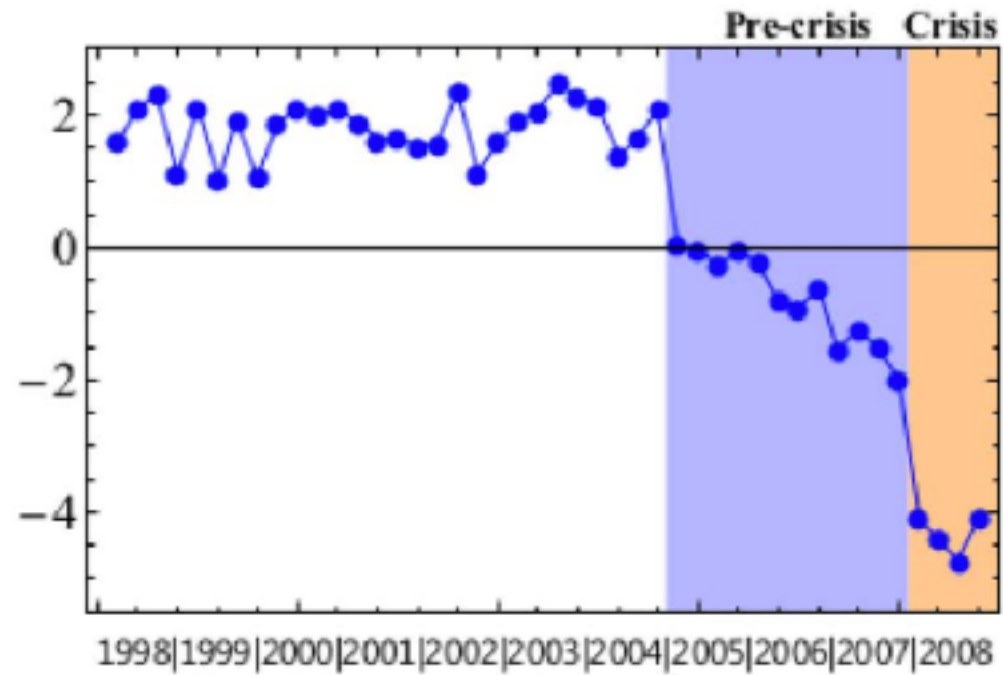
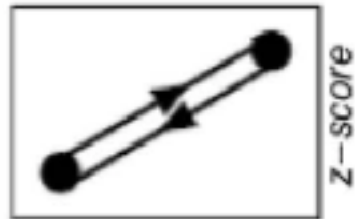
## TEMPORAL NETWORKS

# OTHER NETWORK TYPES



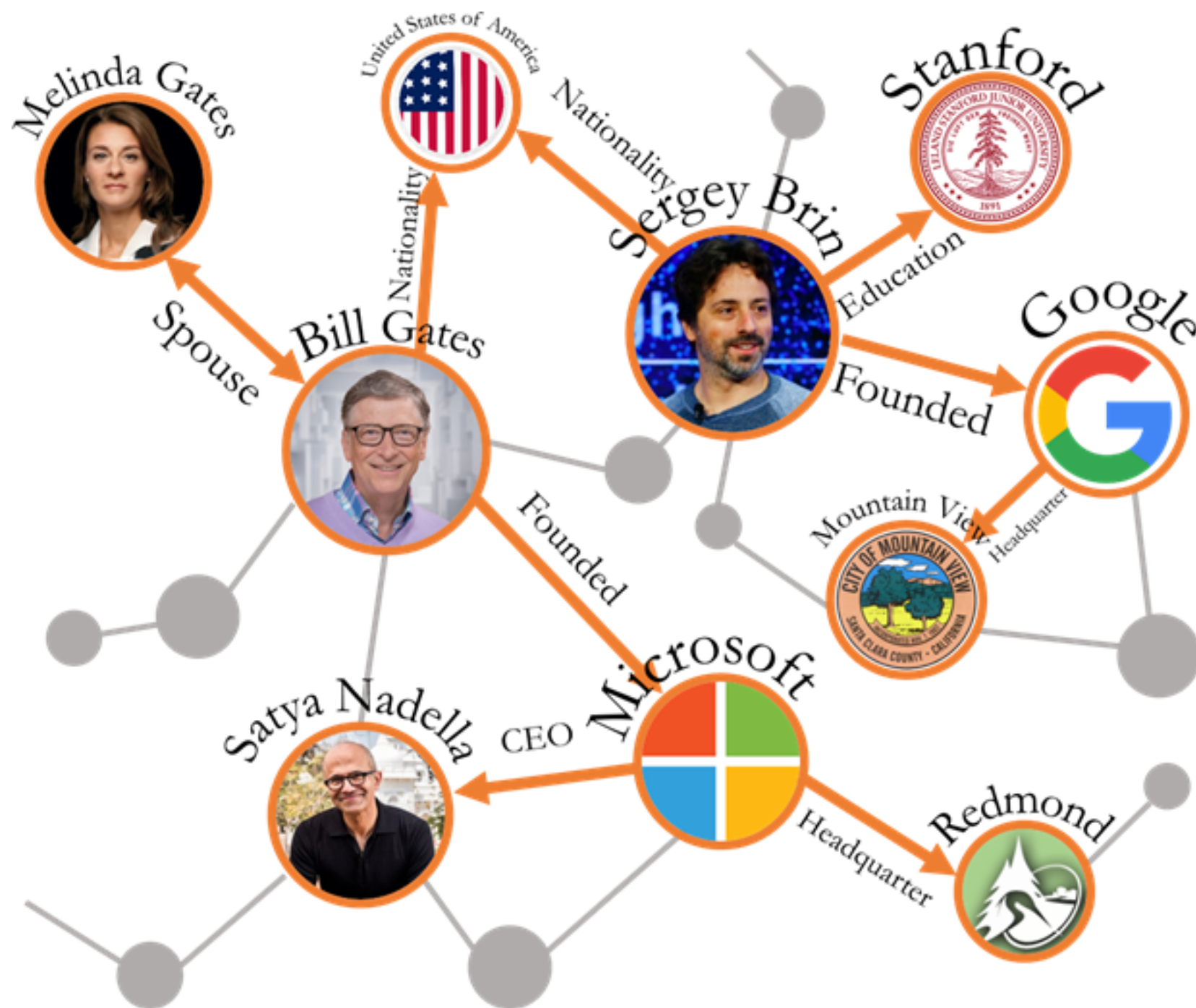
# TEMPORAL NETWORKS

# OTHER NETWORK TYPES



# TEMPORAL NETWORKS

# KNOWLEDGE GRAPHS



# SUMMARY

We learned how to **characterise network elements**

We learned how to **choose** the most appropriate **type of links**

We have **seen different types of networks**

