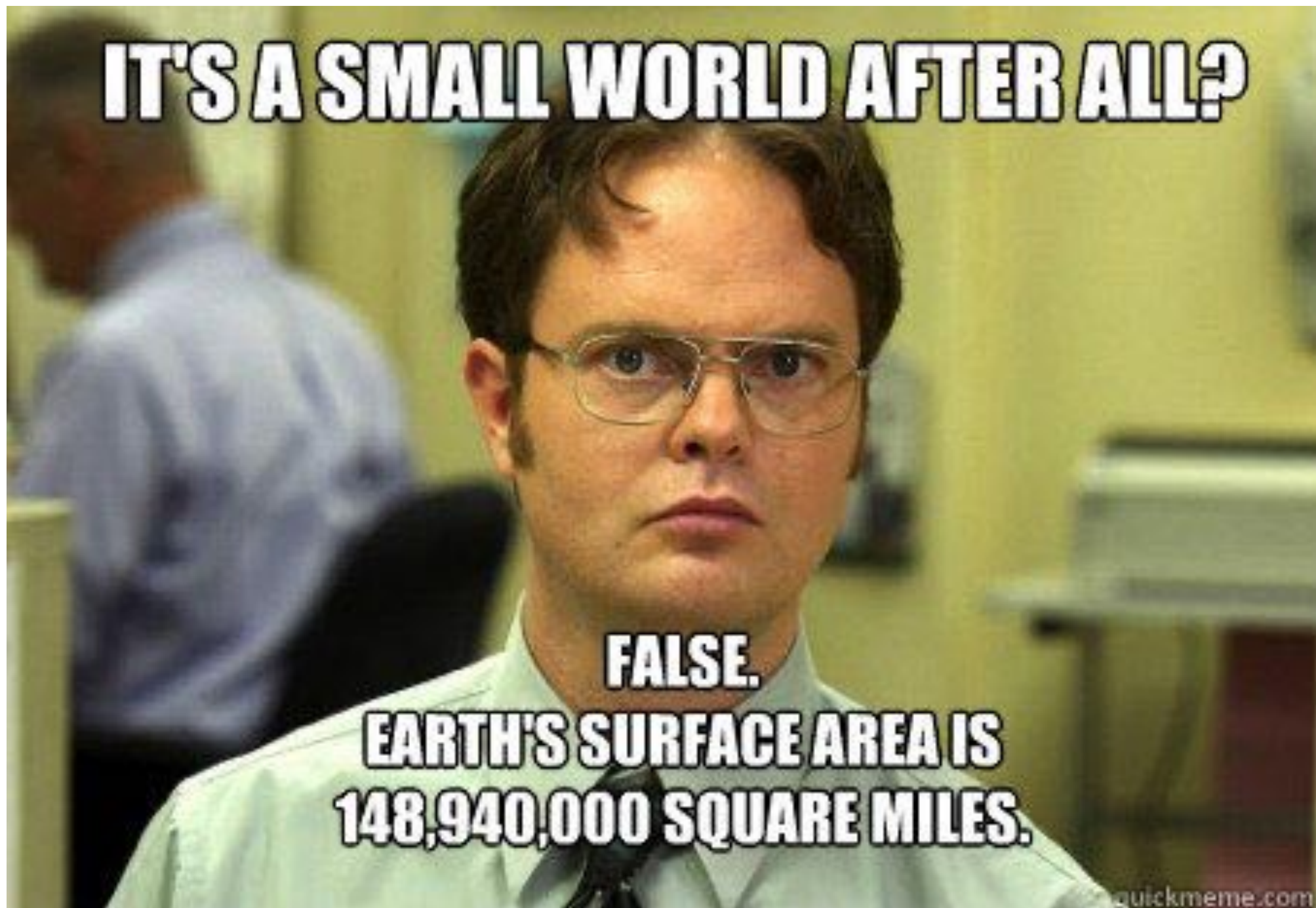


SMALL WORLDS

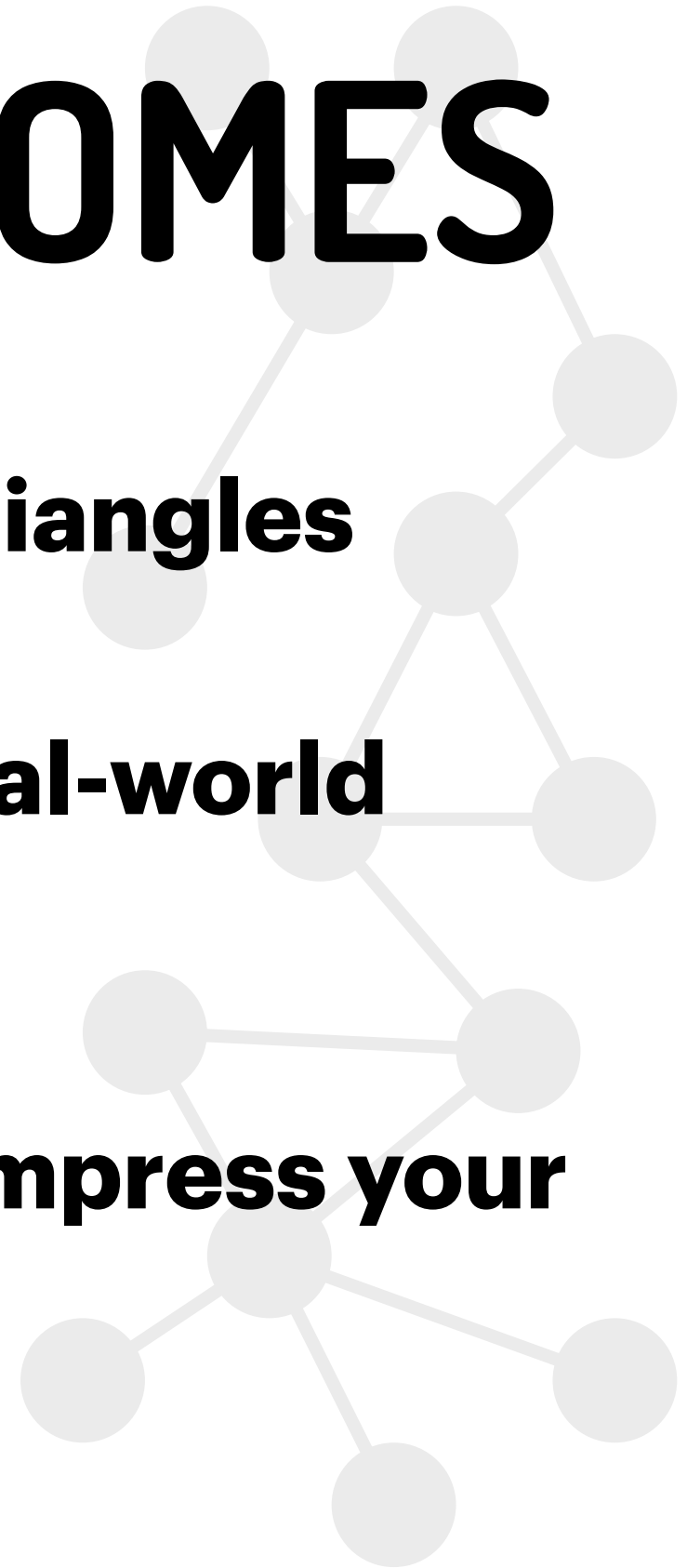


LEARNING OUTCOMES

Learn about **similarity, paths, and triangles**

See how these concepts apply to **real-world networks**

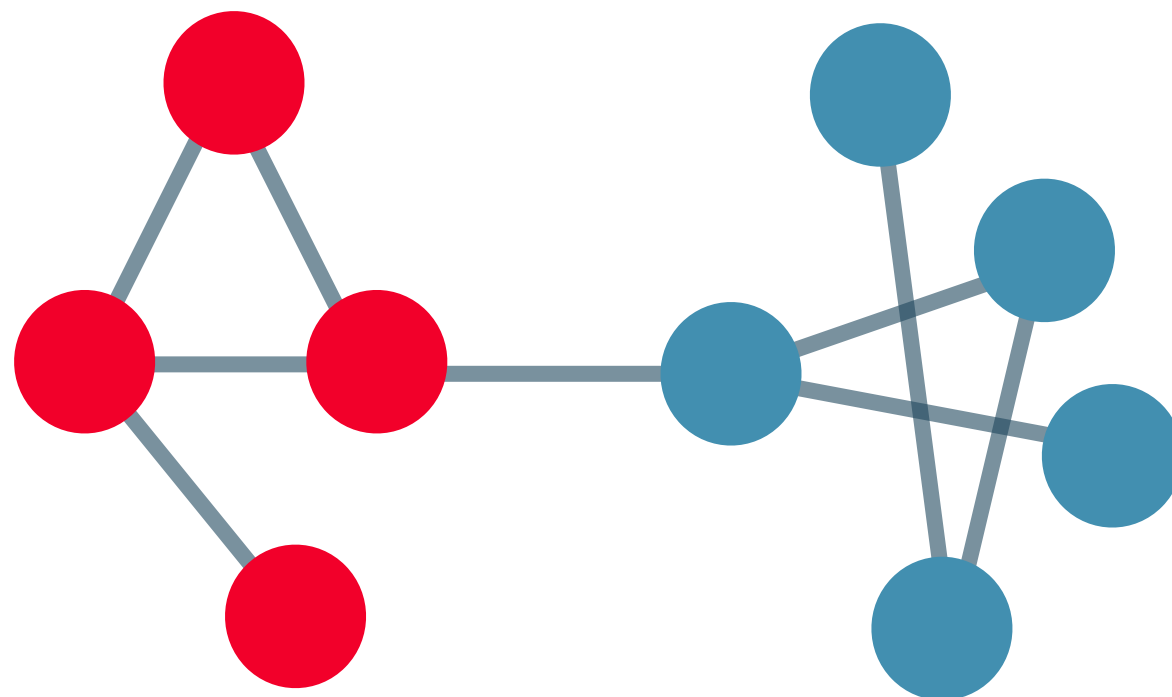
Learn **fun-facts** about networks to **impress your friends**



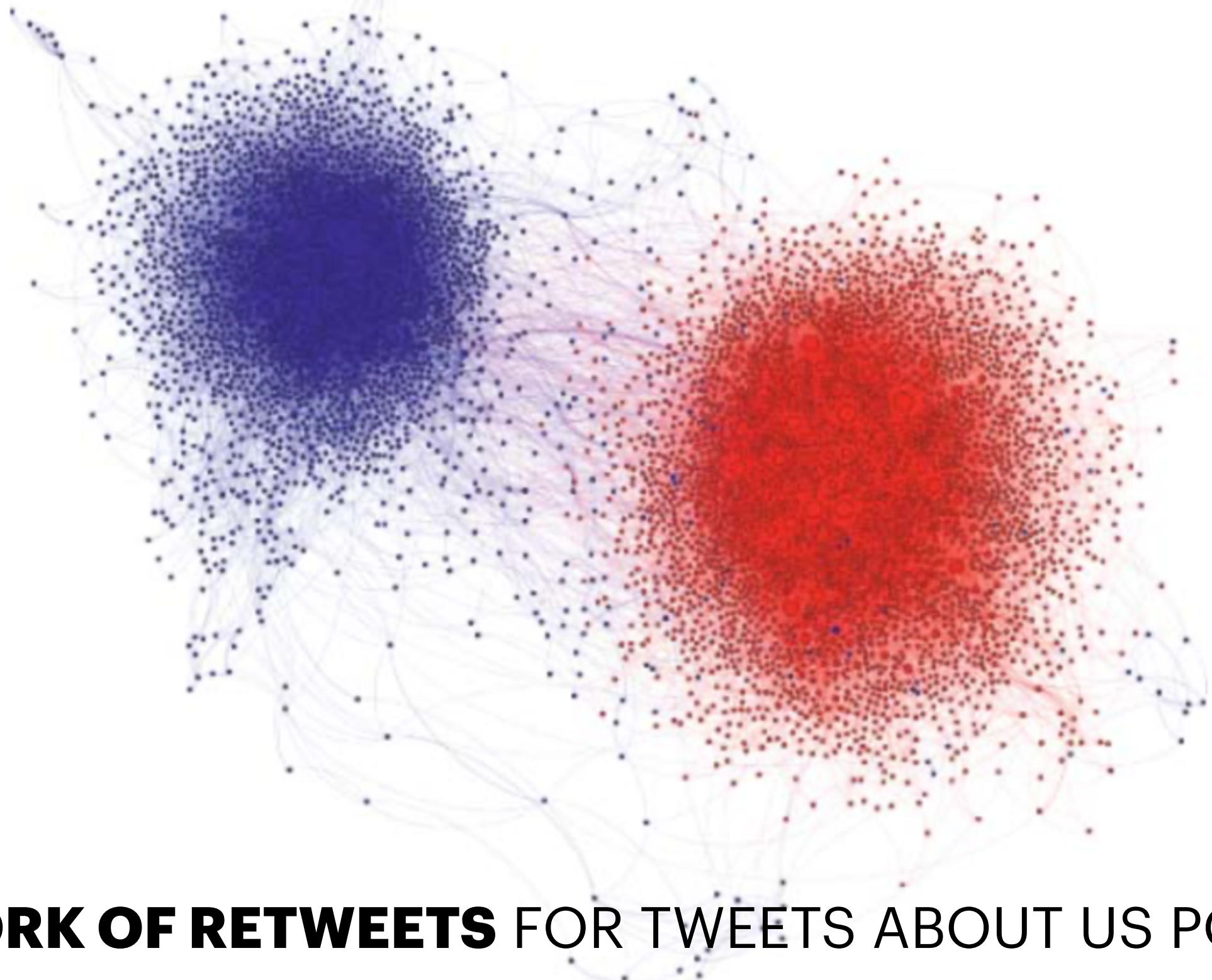
ASSORTATIVITY



ASSORTATIVITY



ASSORTATIVITY

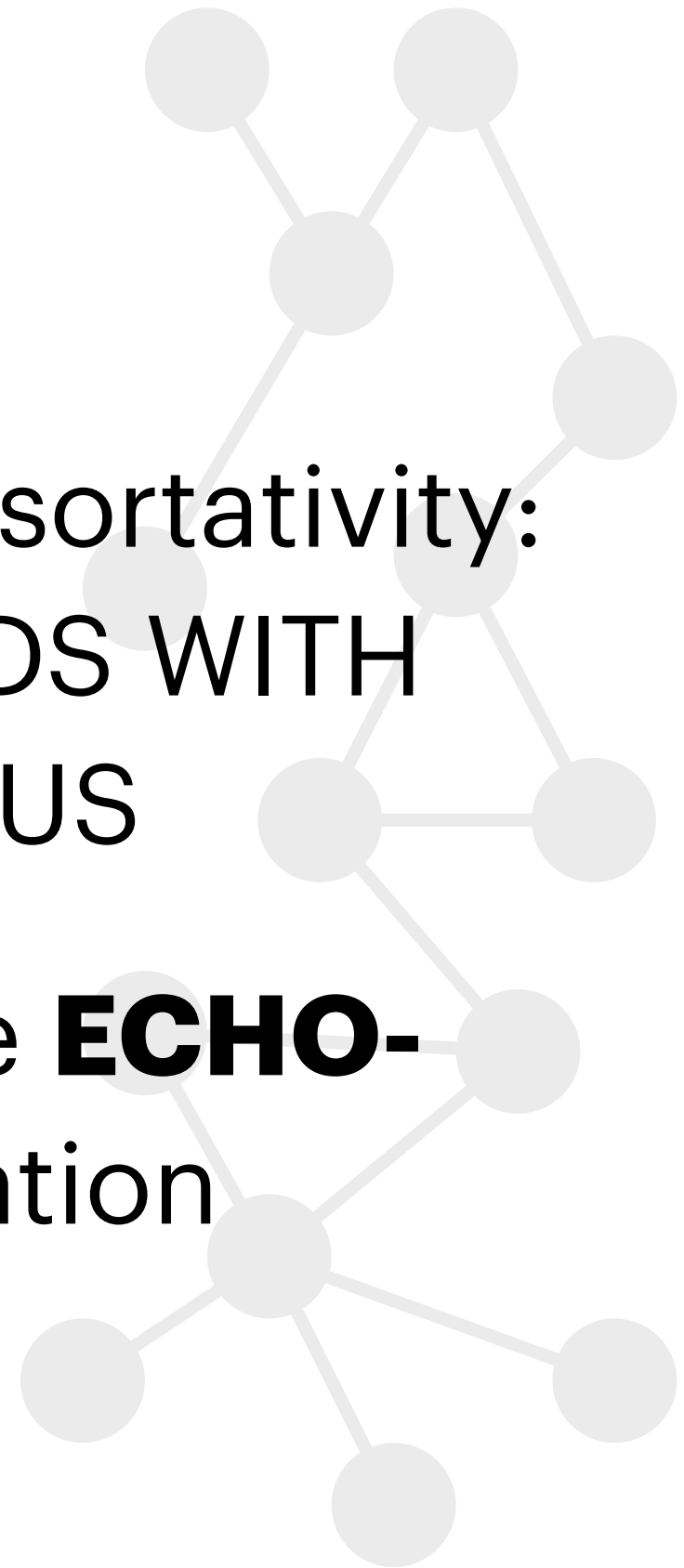


NETWORK OF RETWEETS FOR TWEETS ABOUT US POLITICS
REDS ARE CONSERVATIVES AND BLUES ARE DEMOCRATS

ASSORTATIVITY

HOMOPHILY is what causes assortativity:
WE TEND TO BECOME FRIENDS WITH
PEOPLE WHO ARE LIKE US

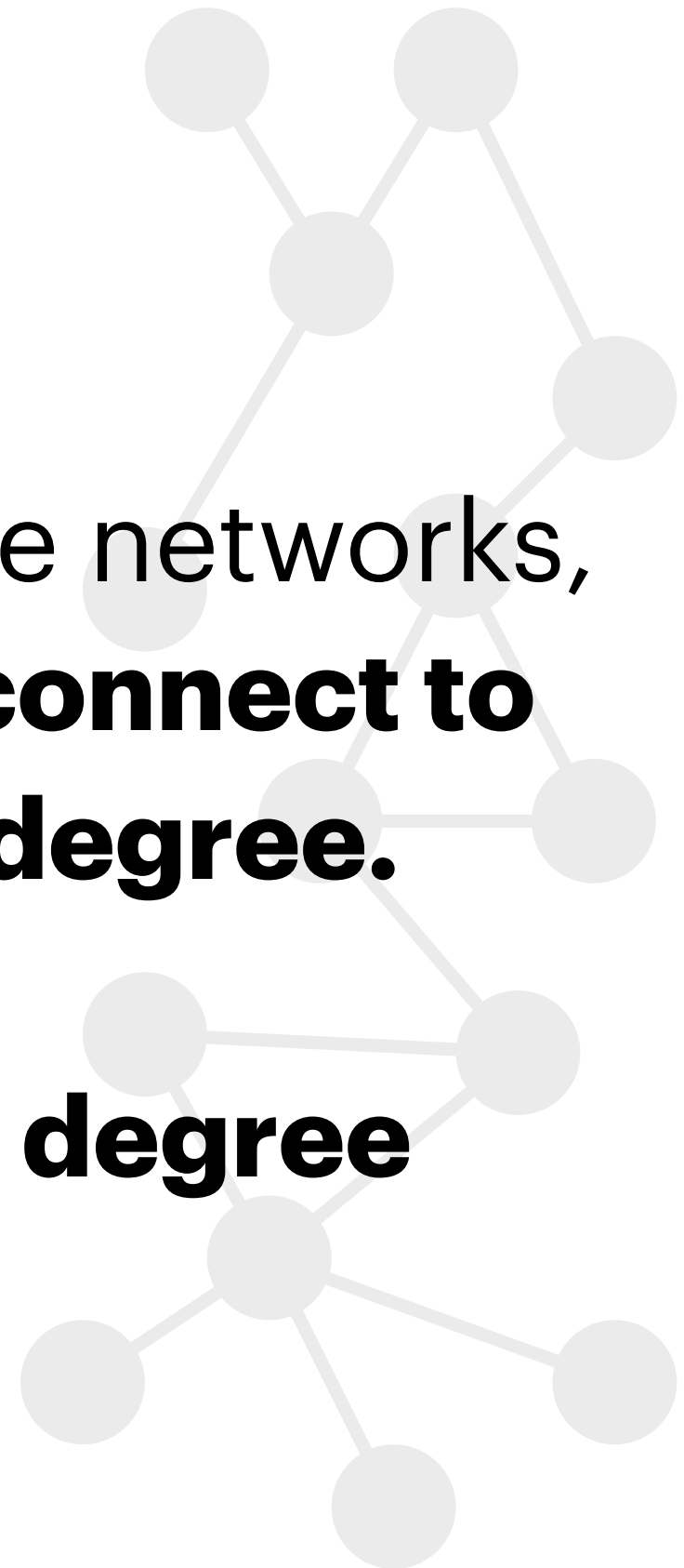
This, however, tends to cause **ECHO-CHAMBERS** and segregation



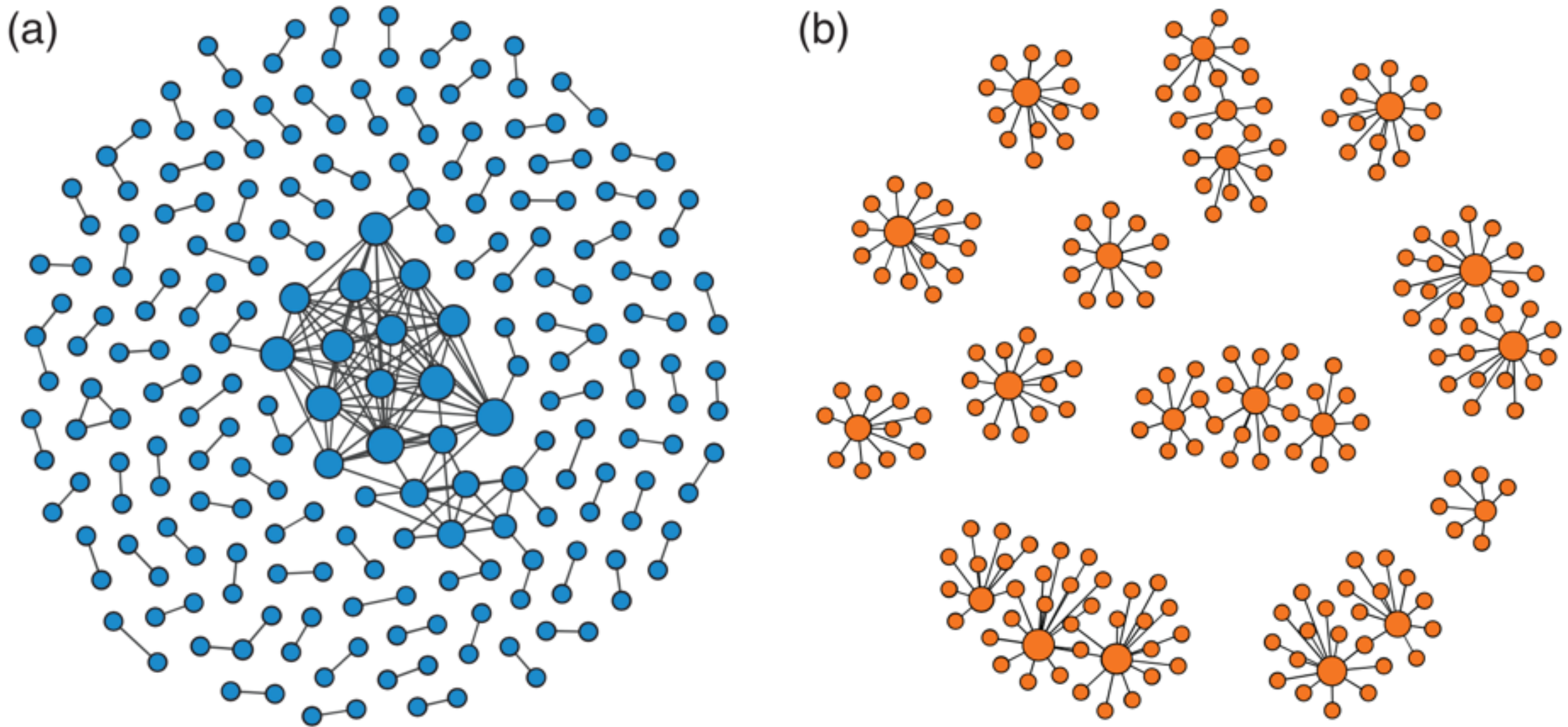
ASSORTATIVITY

Not only social networks: in some networks, nodes with **high** (low) **degree connect to** other nodes with **high** (low) **degree.**

This effect takes the name of **degree correlation**



ASSORTATIVITY



Network degree assortativity illustrated by (a) an assortative network and (b) a disassortative network.

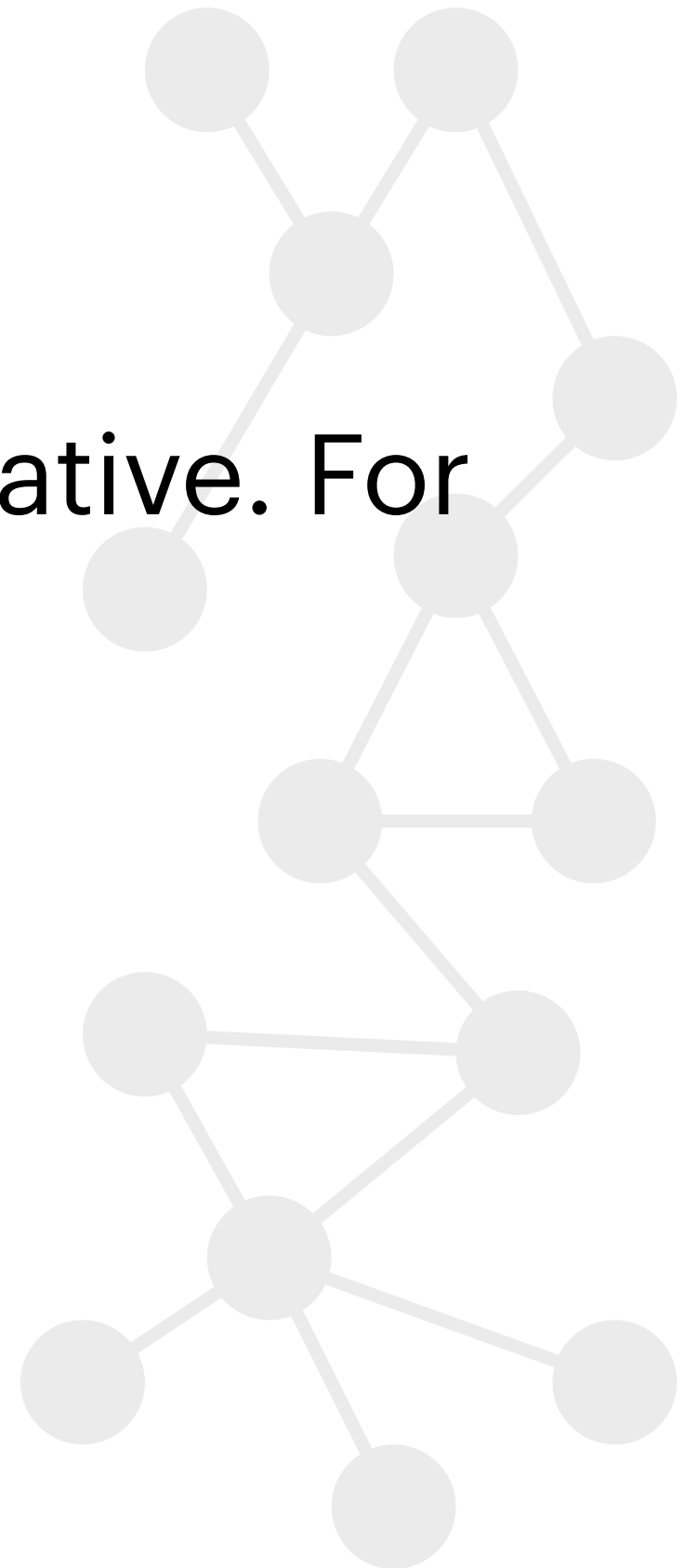
ASSORTATIVITY

Some networks are **not** assortative. For example:

World wide web

Ecological networks

Biological networks

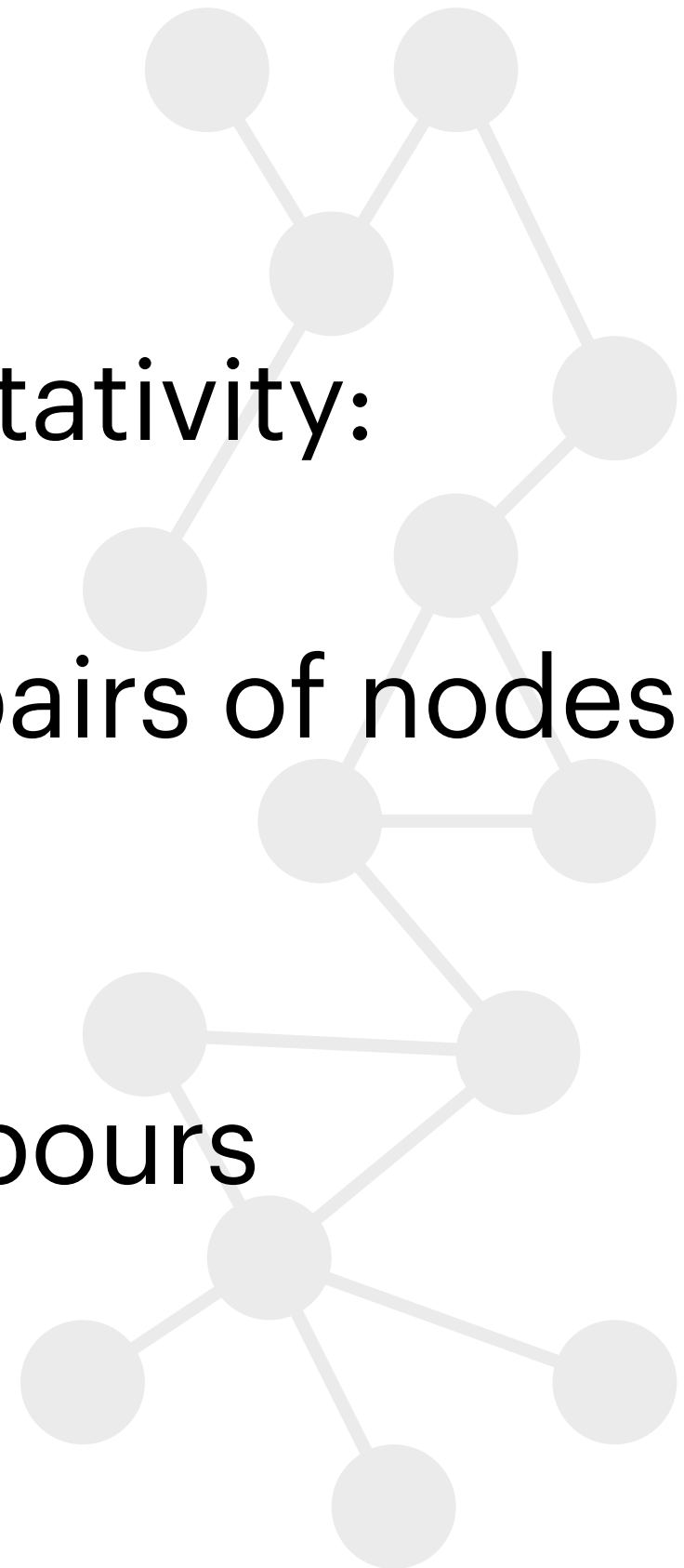


ASSORTATIVITY

Two ways to **compute** assortativity:

correlation between degrees of pairs of nodes
(usually Pearson)

average degree of neighbours



ASSORTATIVITY

K-nearest neighbours

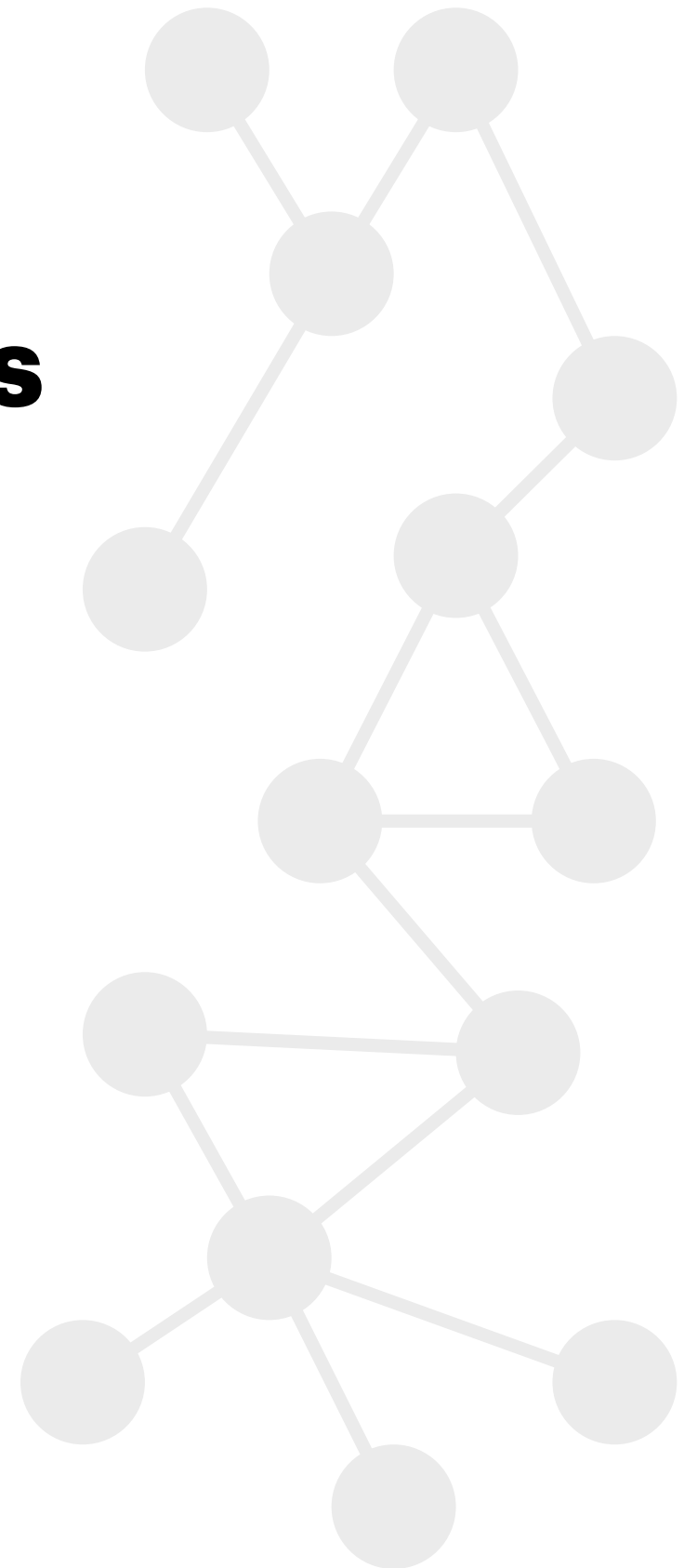
$$k_{nn}(i) = \frac{1}{k_i} \sum_j a_{ij} k_j$$

ASSORTATIVITY

K-nearest neighbours

Knn of node i

$$k_{nn}(i) = \frac{1}{k_i} \sum_j a_{ij} k_j$$

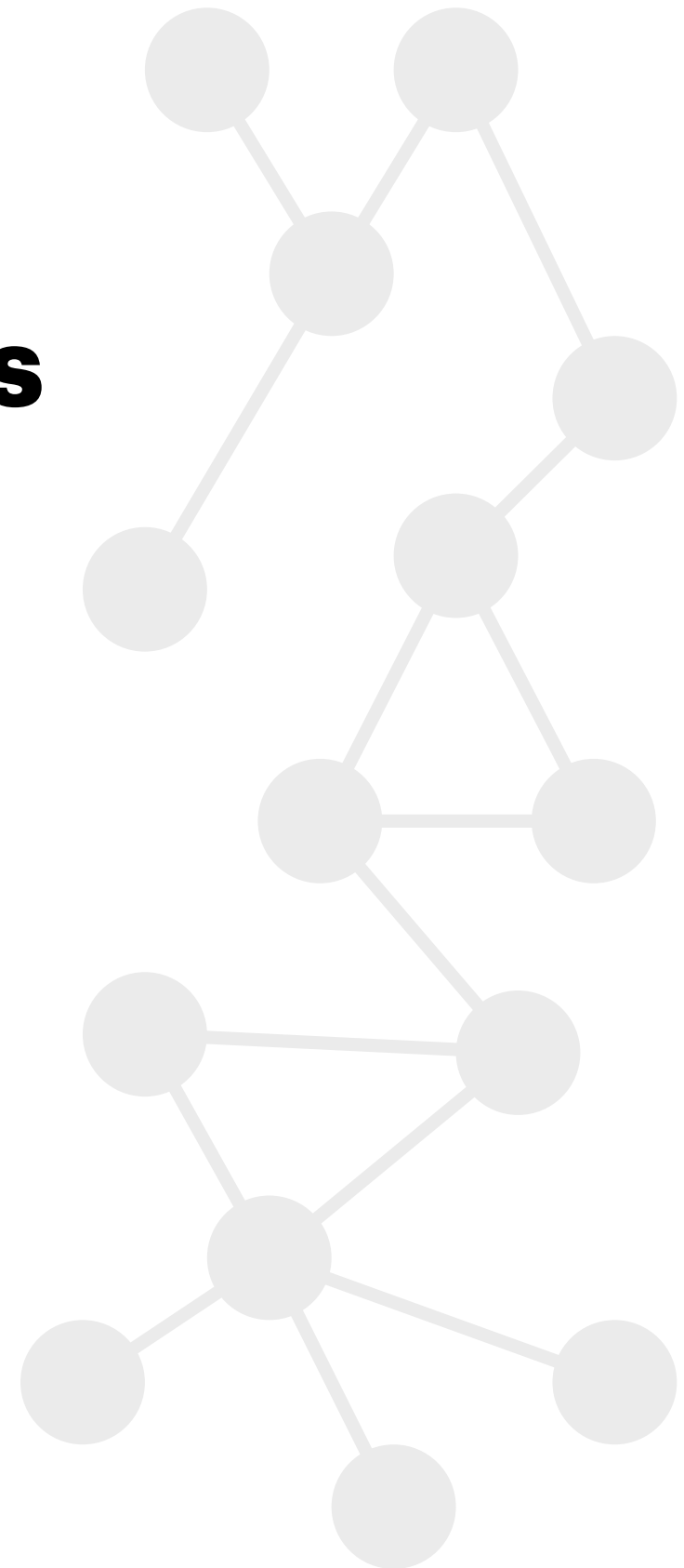


ASSORTATIVITY

K-nearest neighbours

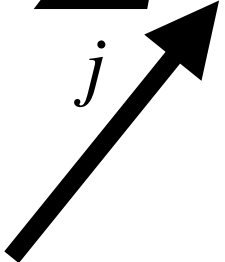
$$k_{nn}(i) = \frac{1}{k_i} \sum_j a_{ij} k_j$$

Number of neighbours of i

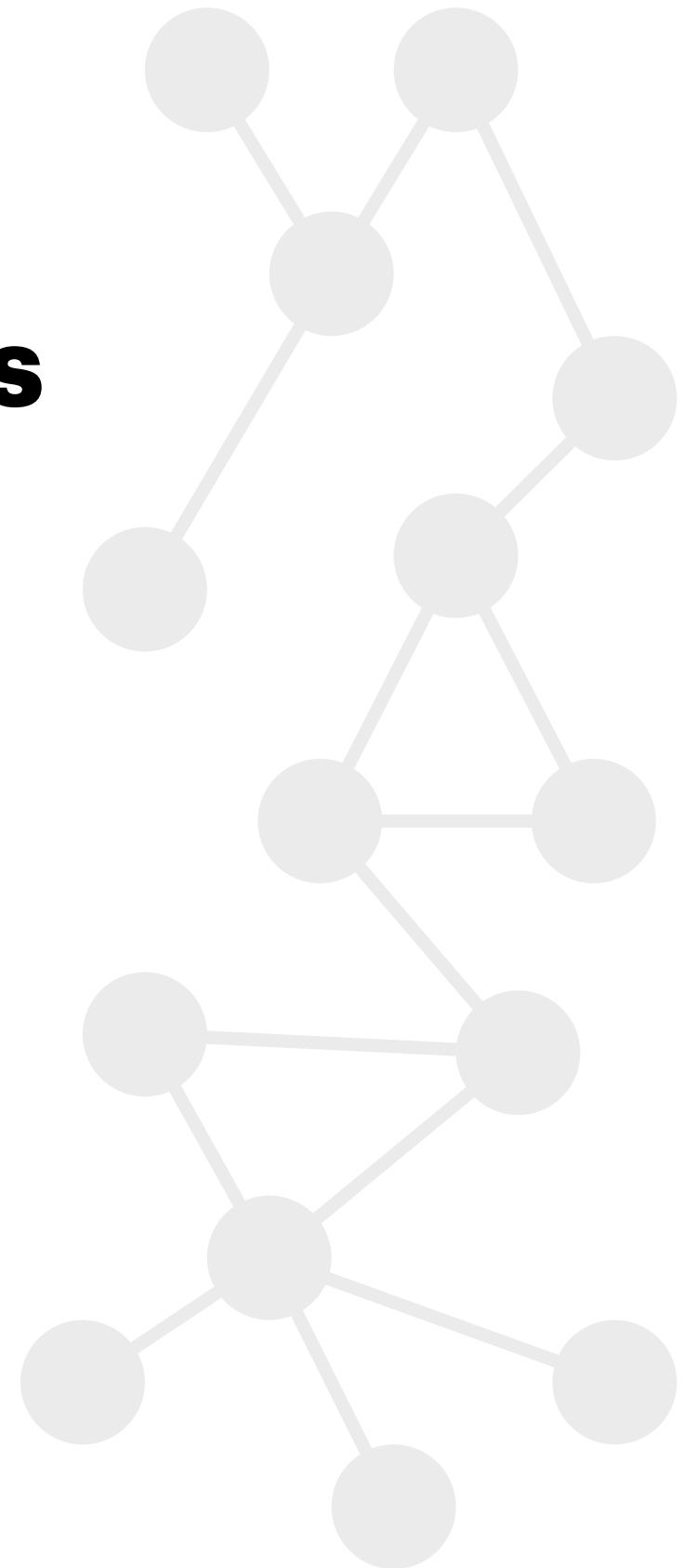


ASSORTATIVITY

K-nearest neighbours

$$k_{nn}(i) = \frac{1}{k_i} \sum_j a_{ij} k_j$$


1 if i and j are neighbours,
0 otherwise



ASSORTATIVITY

K-nearest neighbours

$$k_{nn}(i) = \frac{1}{k_i} \sum_j a_{ij} k_j$$

Degree of node j

ASSORTATIVITY

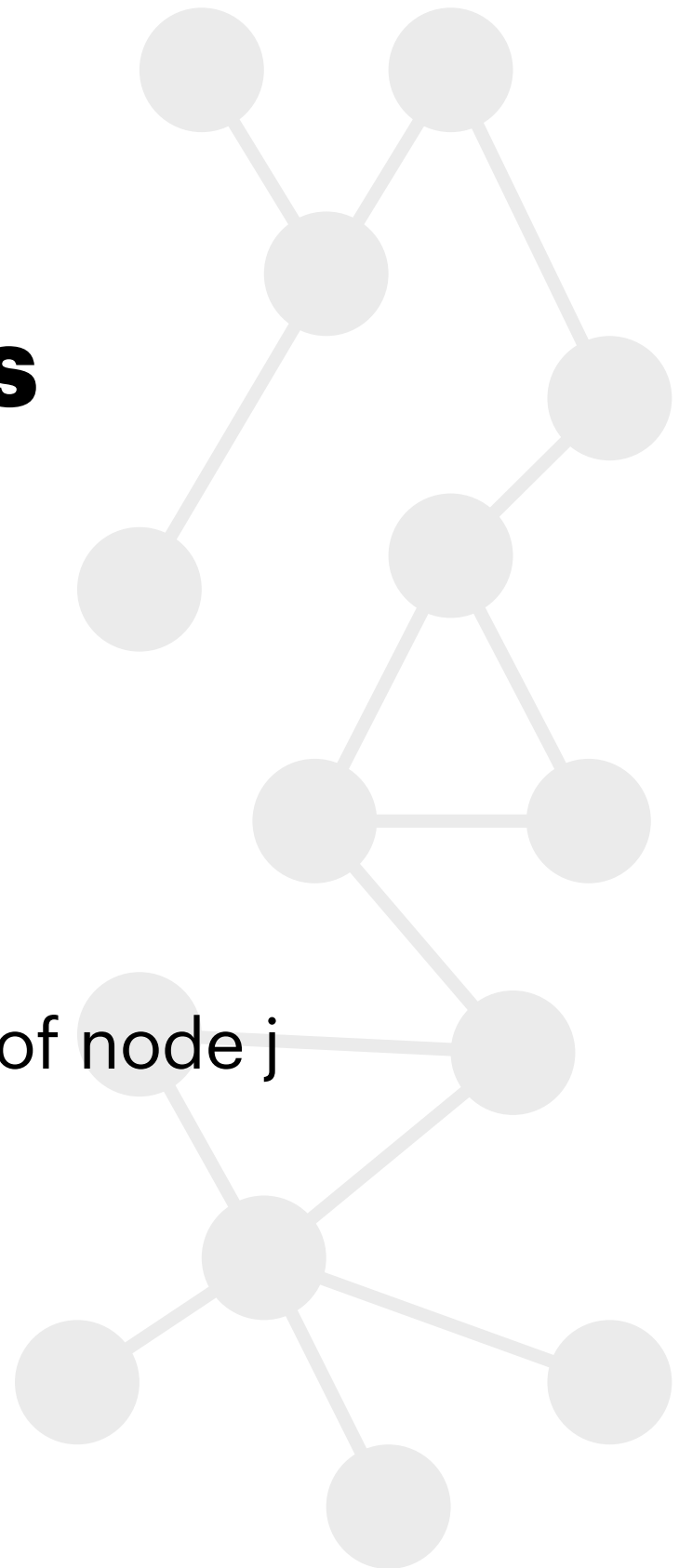
K-nearest neighbours

$$k_{nn}(i) = \frac{1}{k_i} \sum_j a_{ij} k_j$$

Number of neighbours of i

1 if i and j are neighbours,
0 otherwise

Degree of node j

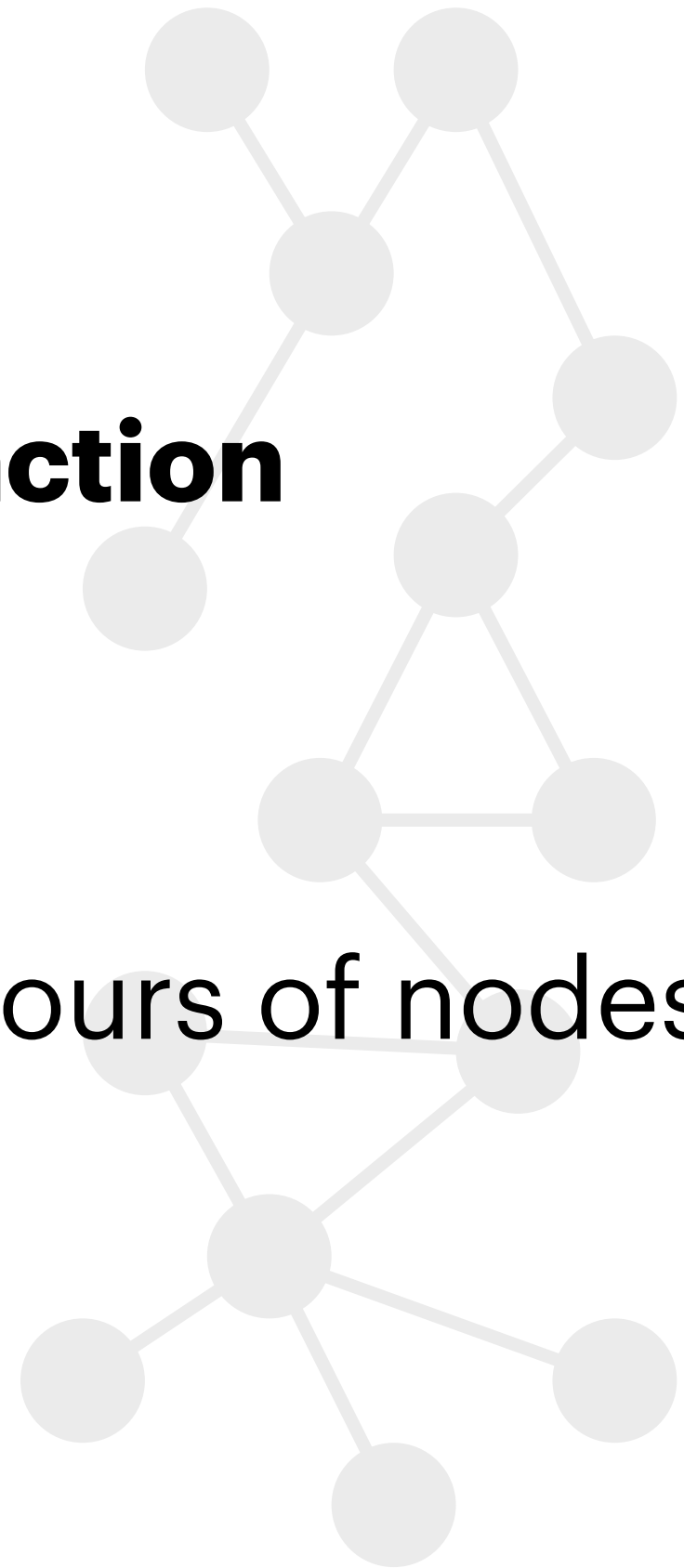


ASSORTATIVITY

K-nearest neighbours function

$$\langle k_{nn}(k) \rangle$$

The average degree of the neighbours of nodes
of degree k

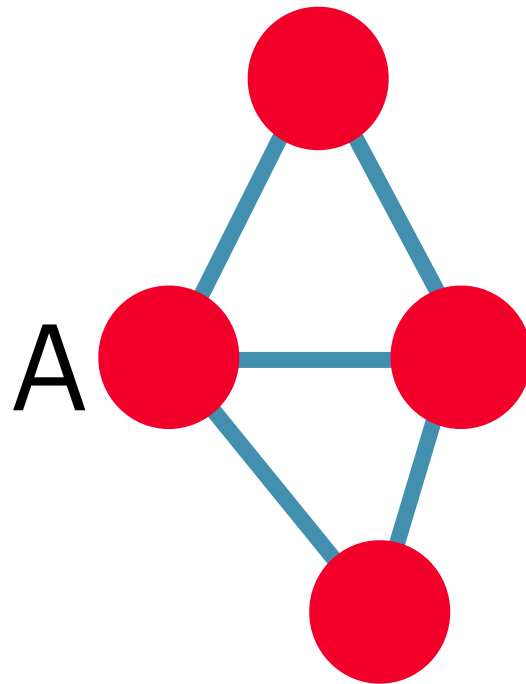


EXERCISE:

GIVEN THE FOLLOWING NETWORK COMPUTE

$$k_{nn}(A)$$

$$\langle k_{nn}(2) \rangle$$

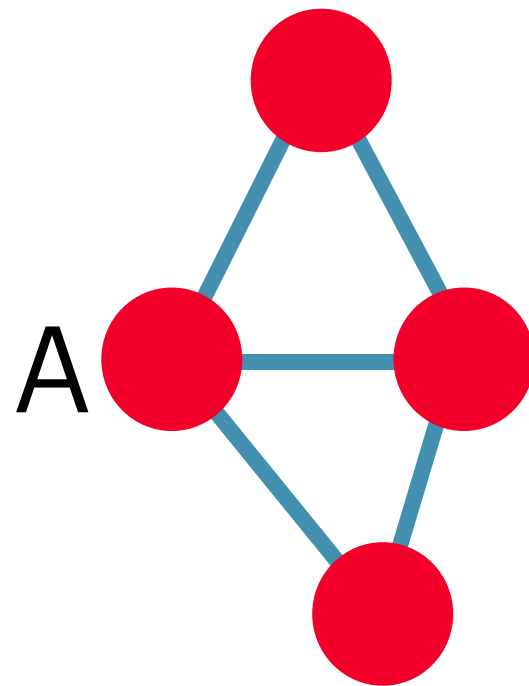


EXERCISE:

GIVEN THE FOLLOWING NETWORK COMPUTE

$$k_{nn}(A)$$

$$\langle k_{nn}(2) \rangle$$



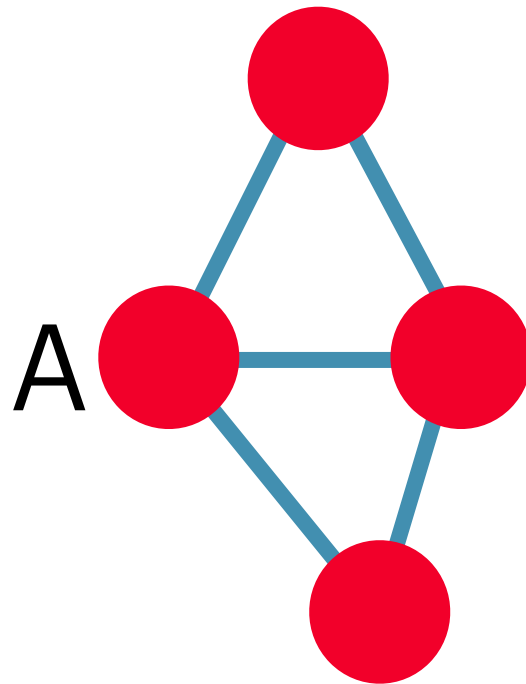
$$k_{nn}(A) = \frac{1}{k_A} \sum_j a_{ij} k_j$$

EXERCISE:

GIVEN THE FOLLOWING NETWORK COMPUTE

$$k_{nn}(A)$$

$$\langle k_{nn}(2) \rangle$$



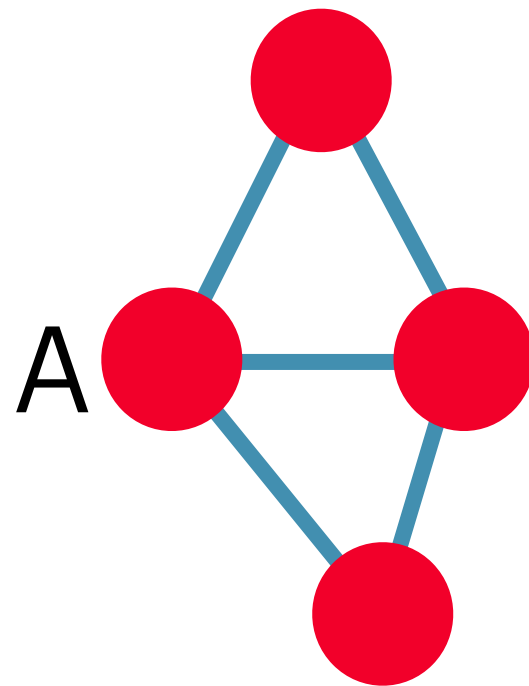
$$k_{nn}(A) = \frac{1}{3} \sum_j a_{ij} k_j$$

EXERCISE:

GIVEN THE FOLLOWING NETWORK COMPUTE

$$k_{nn}(A)$$

$$\langle k_{nn}(2) \rangle$$



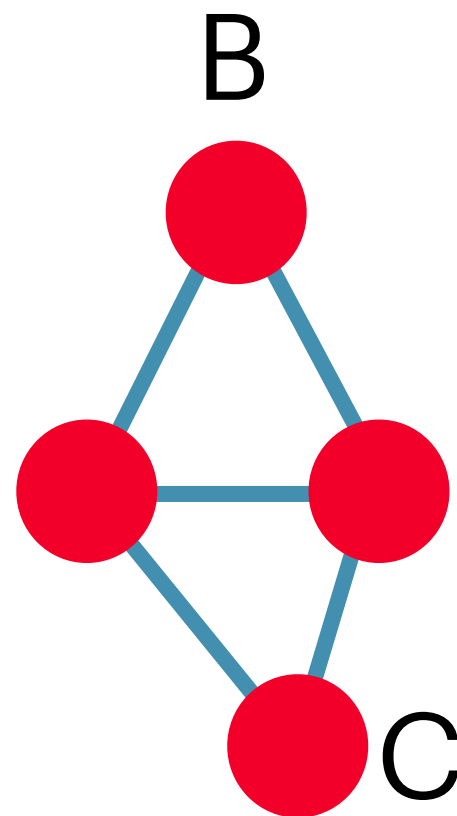
$$k_{nn}(A) = \frac{1}{3}(1 * 2 + 1 * 3 + 1 * 2) = \frac{7}{3}$$

EXERCISE:

GIVEN THE FOLLOWING NETWORK COMPUTE

$$k_{nn}(A)$$

$$\langle k_{nn}(2) \rangle$$



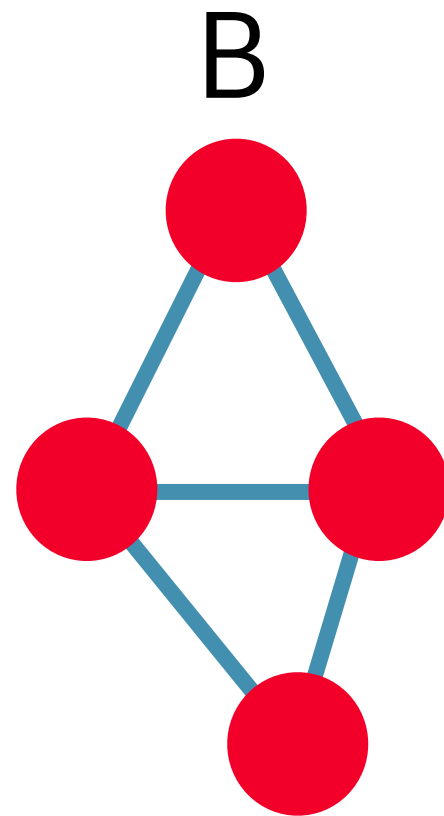
Which nodes have degree of 2?

EXERCISE:

GIVEN THE FOLLOWING NETWORK COMPUTE

$$k_{nn}(A)$$

$$\langle k_{nn}(2) \rangle$$



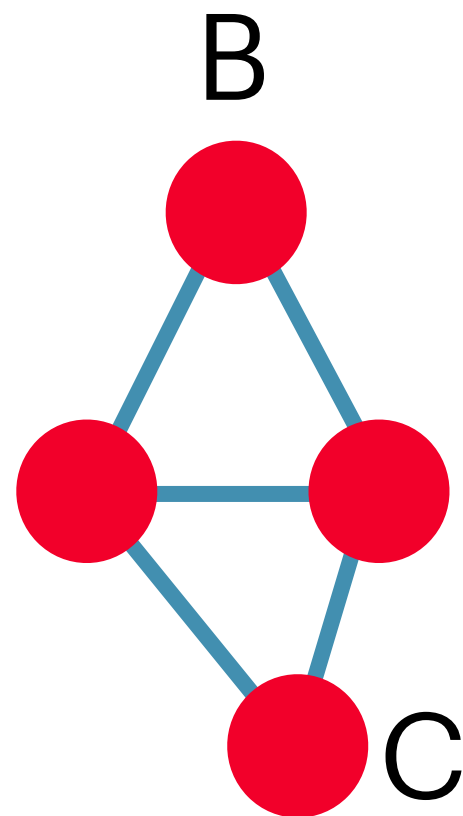
$$k_{nn}(B) = ?$$

EXERCISE:

GIVEN THE FOLLOWING NETWORK COMPUTE

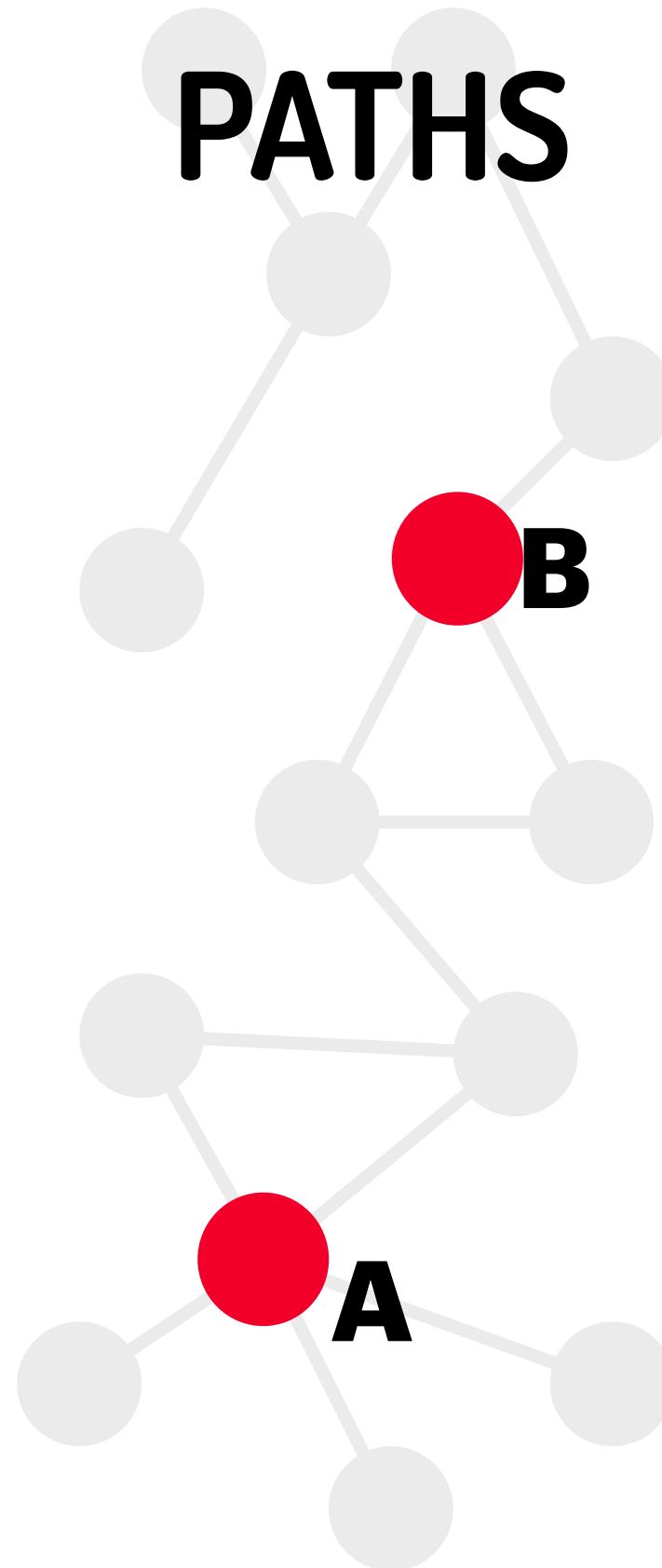
$$k_{nn}(A)$$

$$\langle k_{nn}(2) \rangle$$

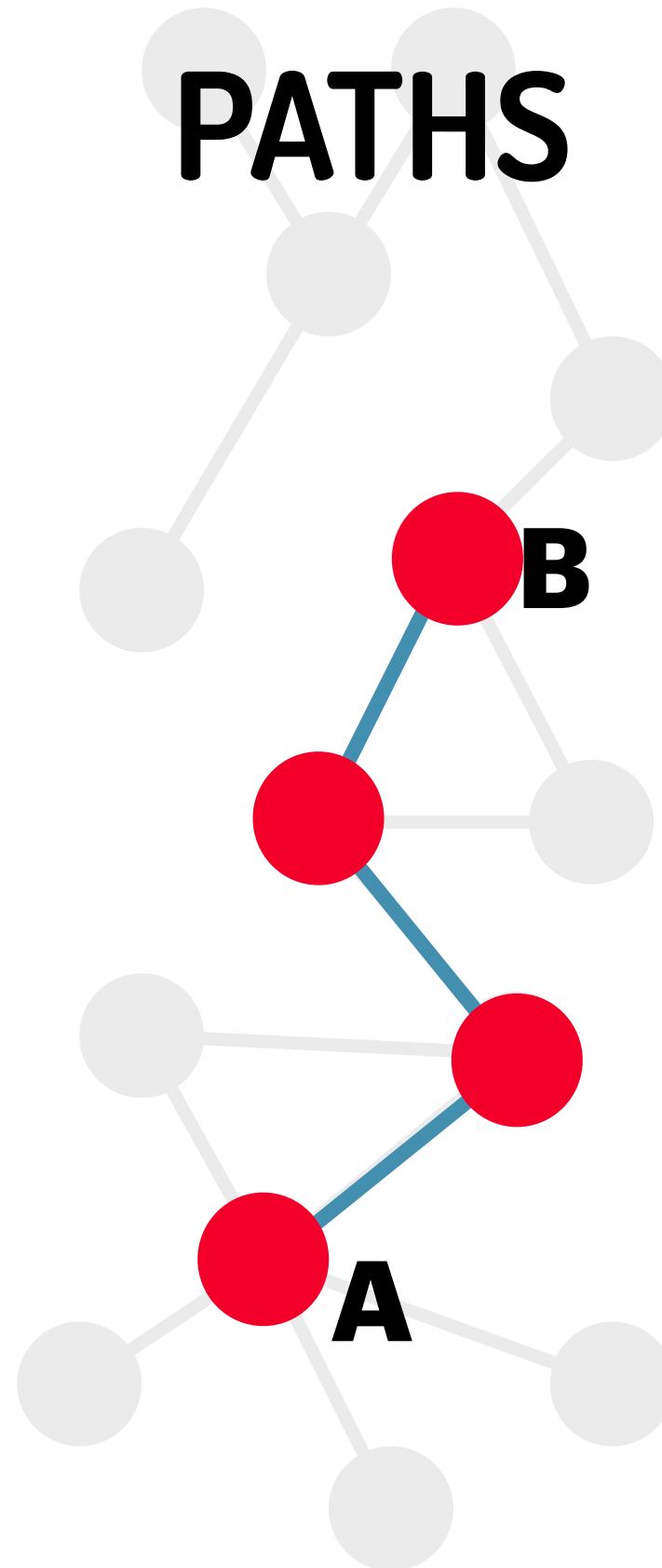


Pro tip: network is symmetric for these nodes

PATHS



PATHS



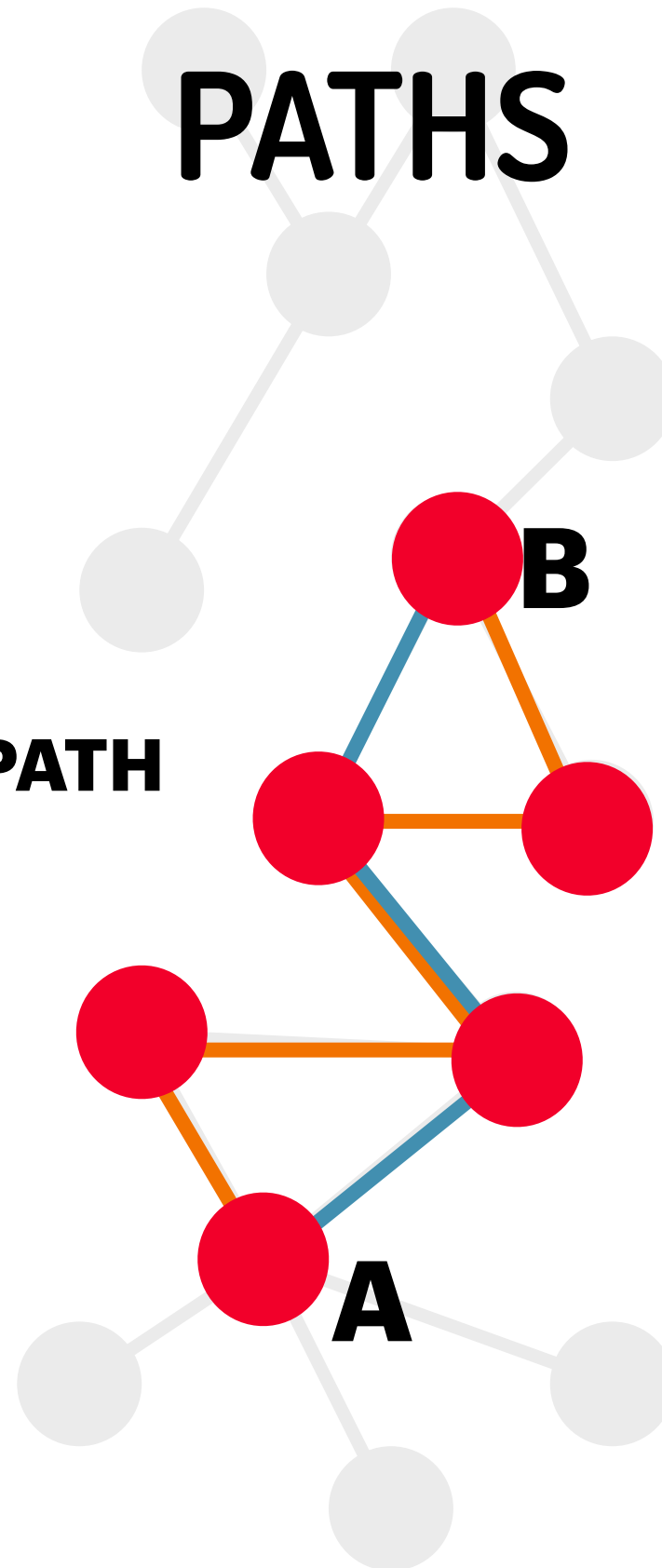
PATHS

The diagram shows a graph with red nodes and edges. Two paths are highlighted: Path A (blue) and Path B (orange). The graph is set against a background of light gray nodes and edges.

```
graph TD; A((A)) --- B((B)); A --- C(( )); C --- D(( )); D --- E(( )); E --- F(( )); F --- G(( )); G --- H(( )); H --- I(( )); I --- J(( )); J --- K(( )); K --- L(( )); L --- M(( )); M --- N(( )); N --- O(( )); O --- P(( )); P --- Q(( )); Q --- R(( )); R --- S(( )); S --- T(( )); T --- U(( )); U --- V(( )); V --- W(( )); W --- X(( )); X --- Y(( )); Y --- Z(( ));
```

PATHS

 **SHORTEST PATH**
 ℓ_{AB}

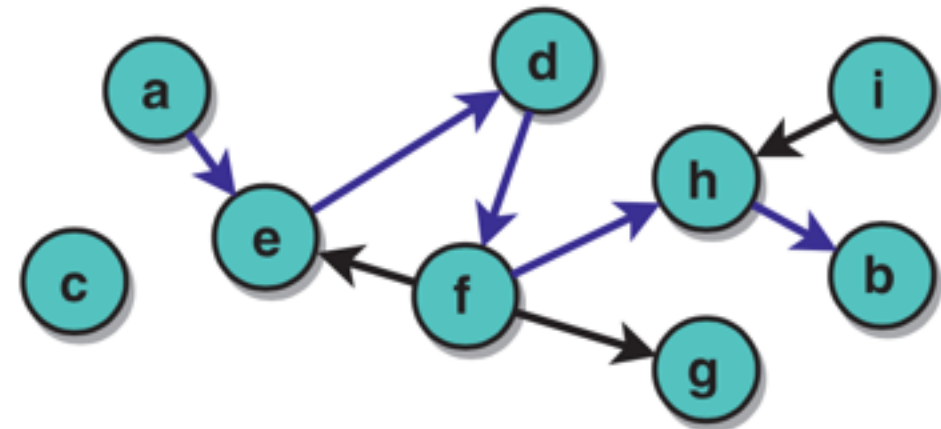
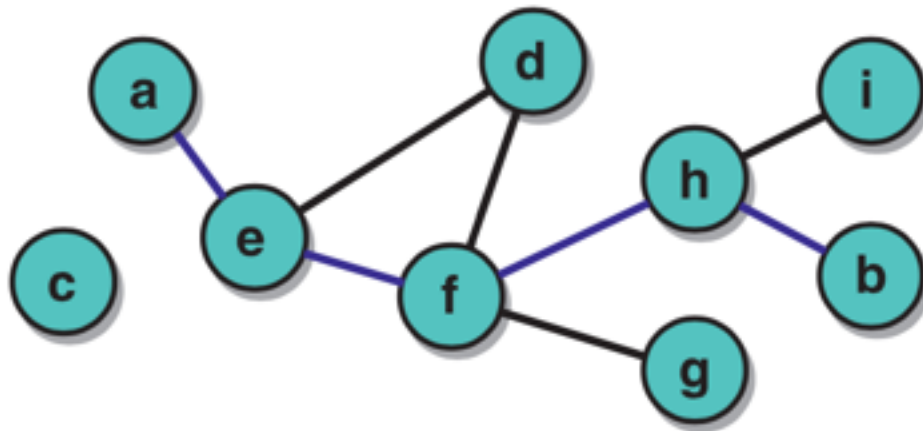


PATHS

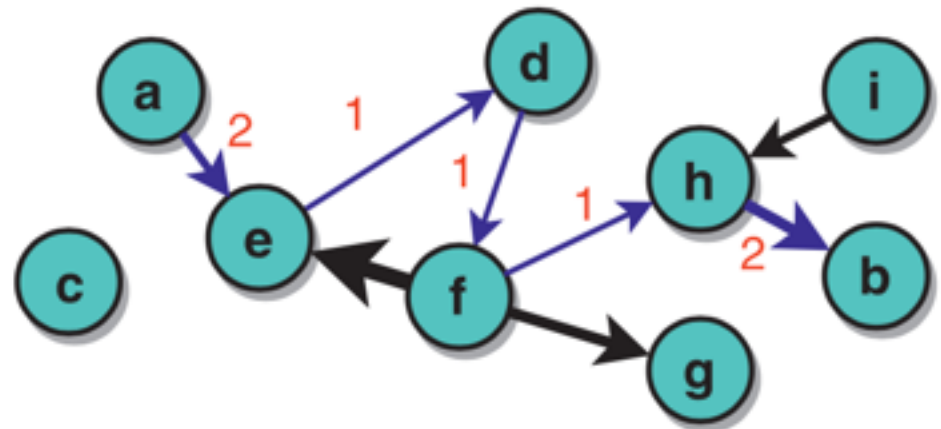
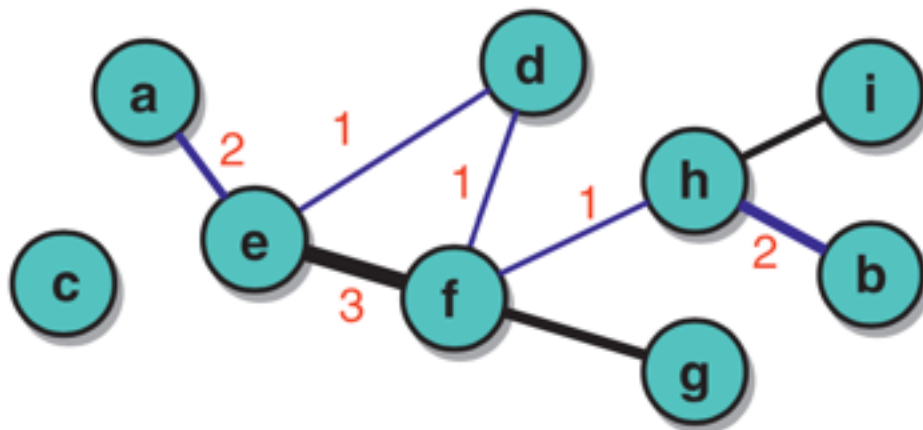
Undirected

Directed

Unweighted



Weighted

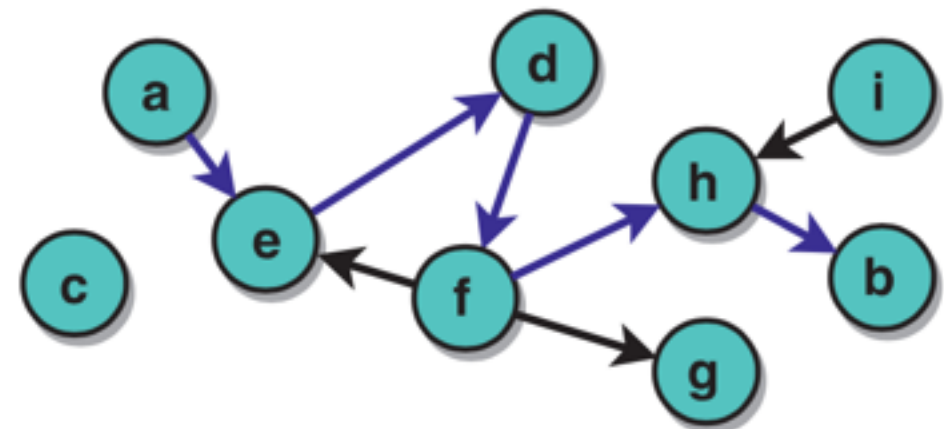
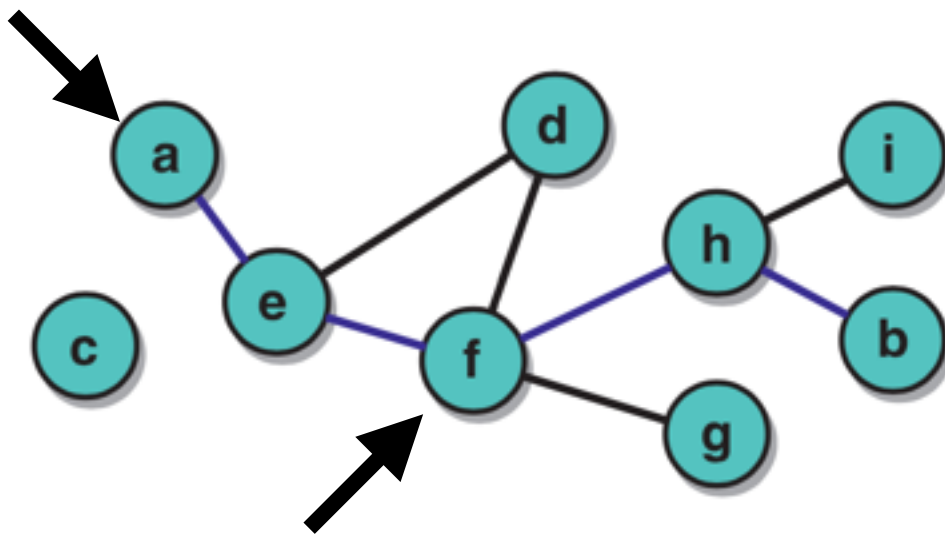


PATHS

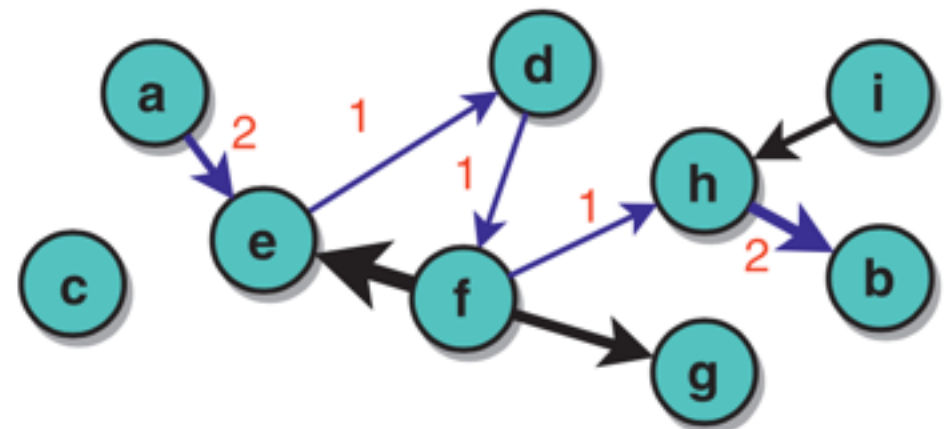
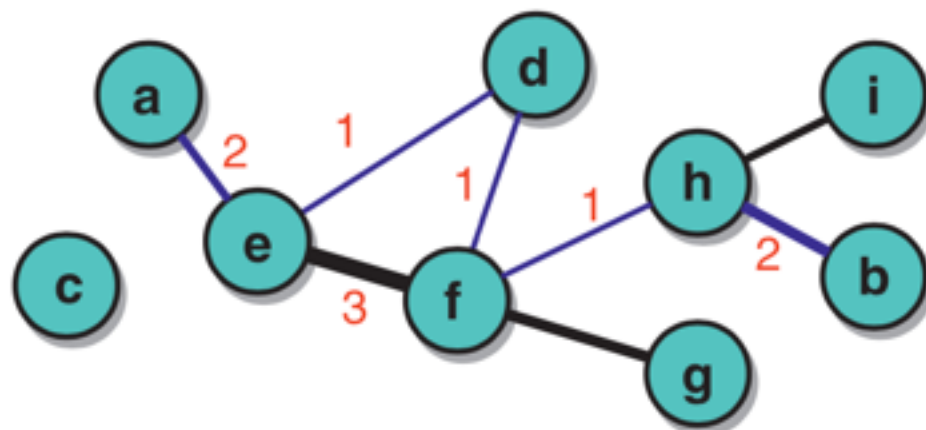
Undirected

Directed

Unweighted



Weighted

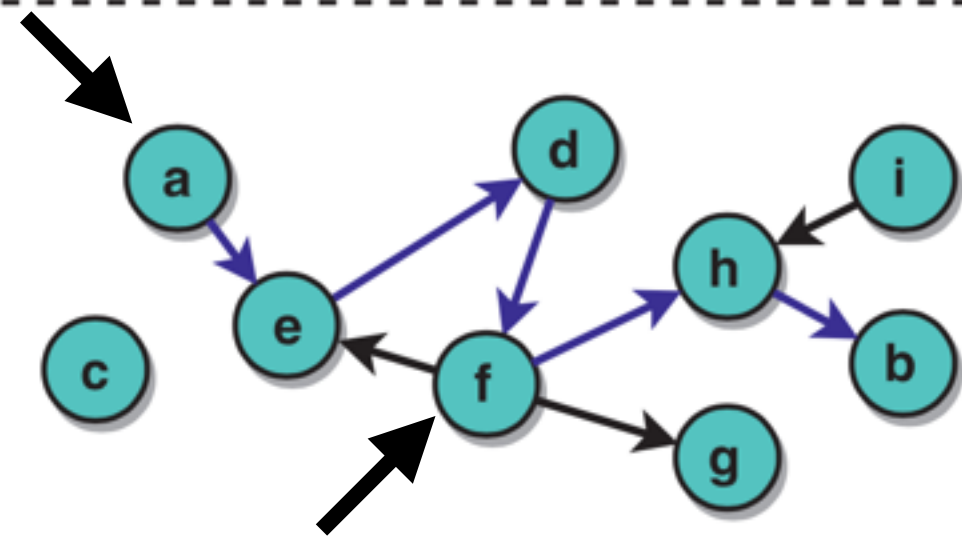
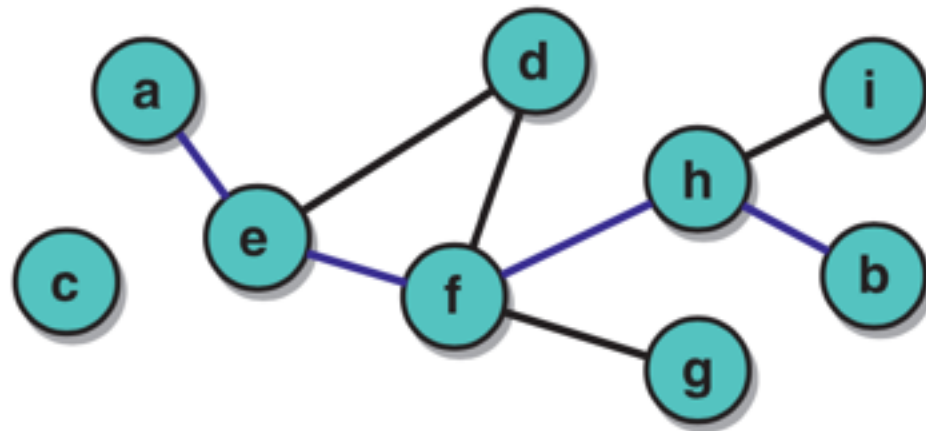


PATHS

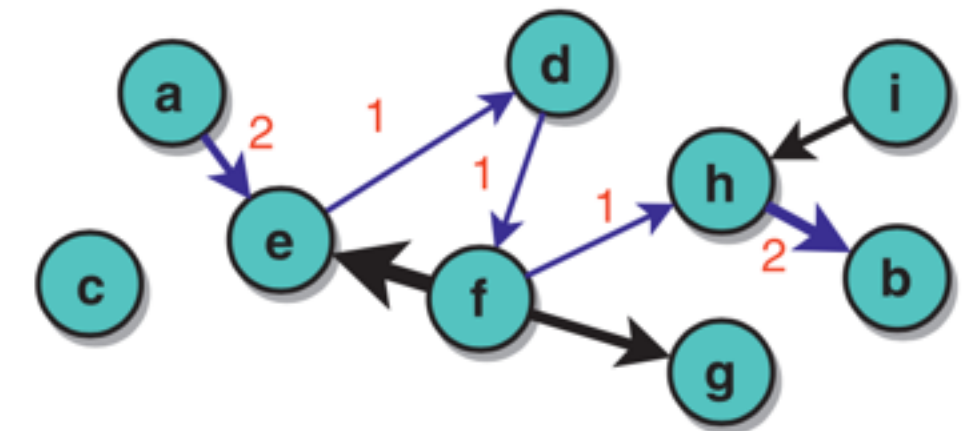
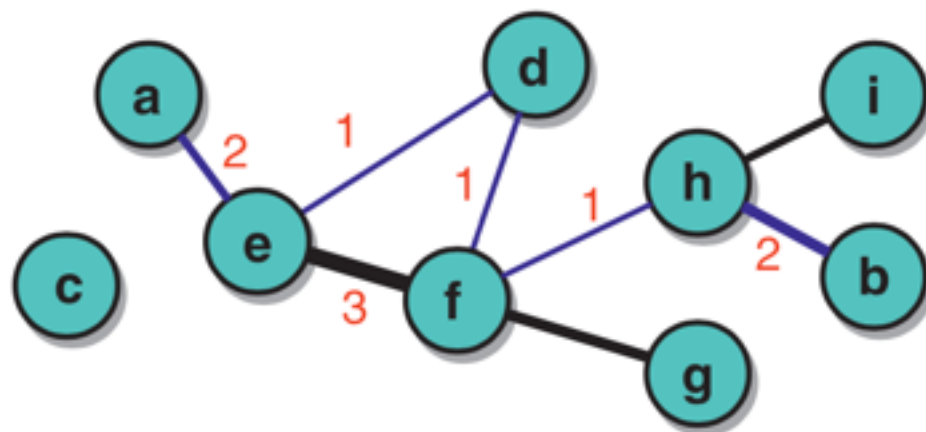
Undirected

Directed

Unweighted



Weighted

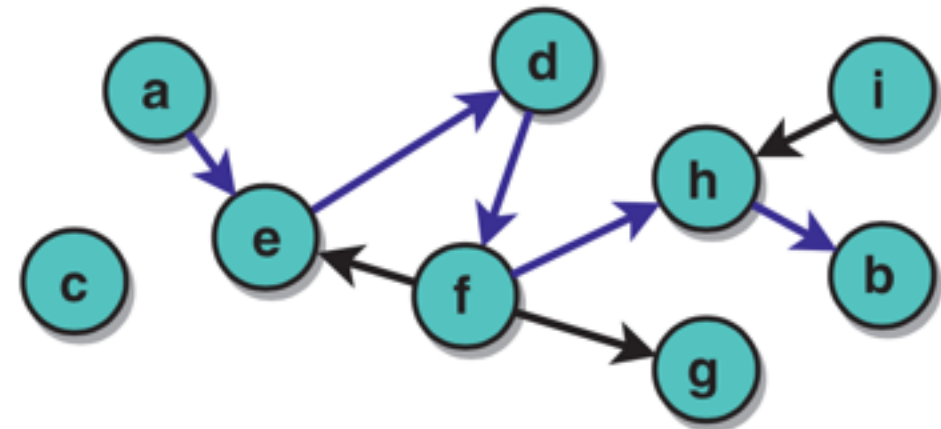
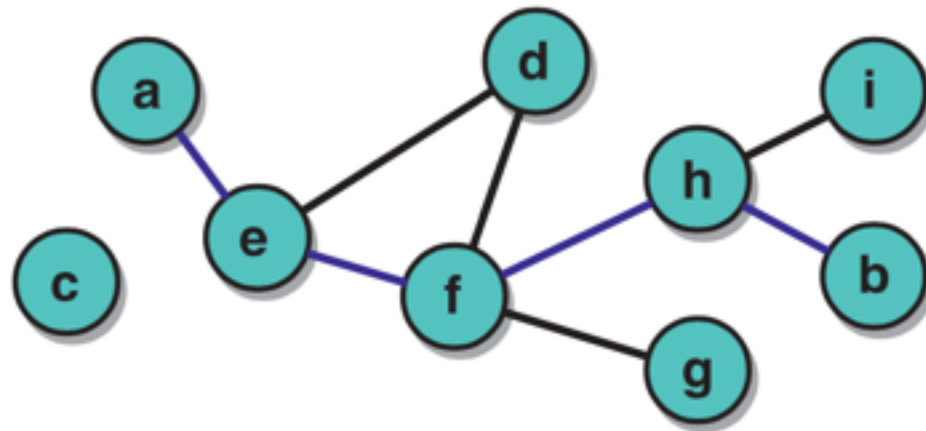


PATHS

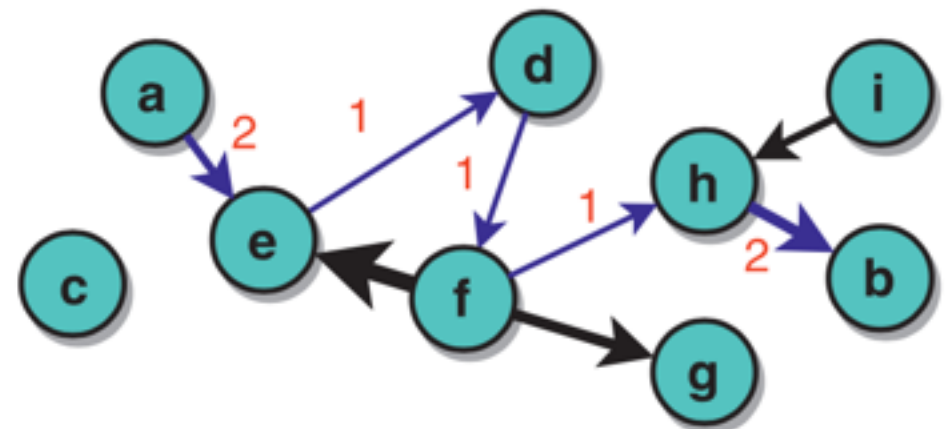
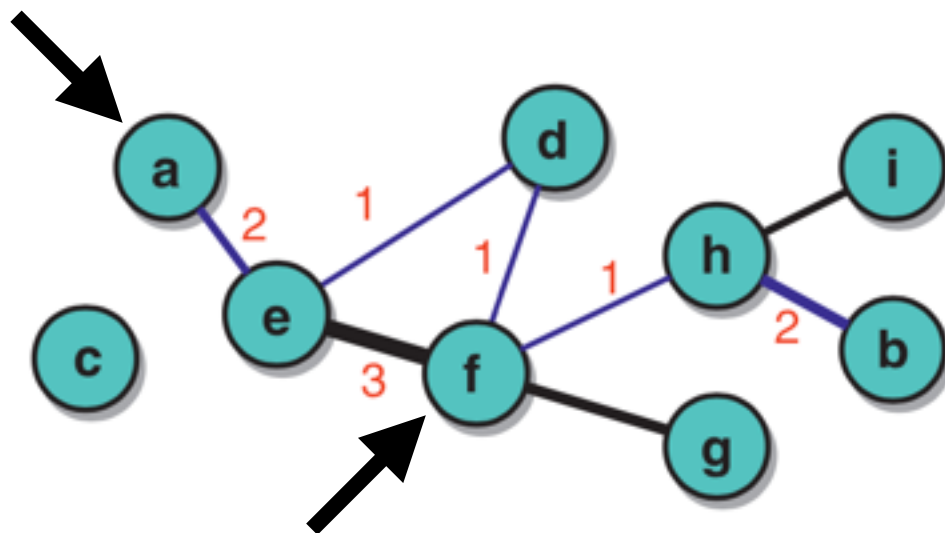
Undirected

Directed

Unweighted



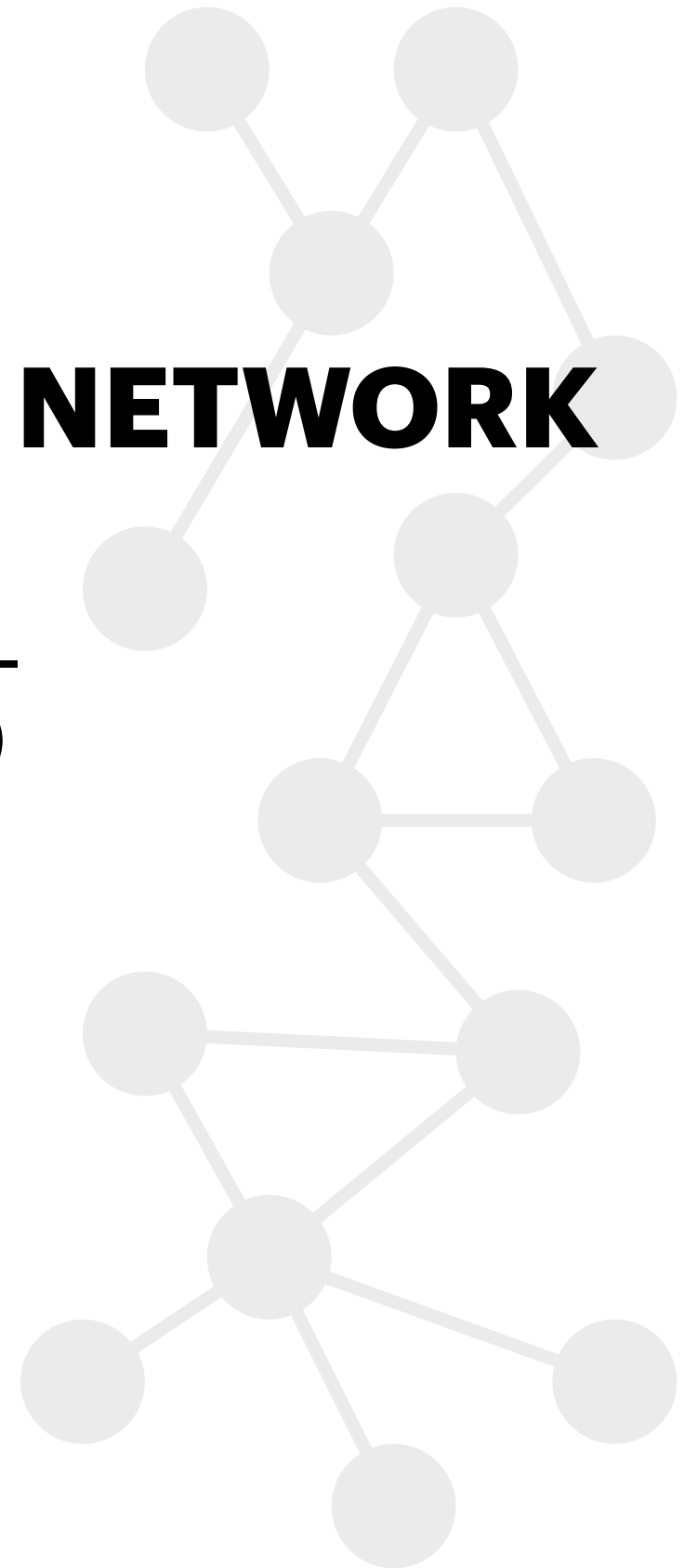
Weighted



PATHS

AVERAGE PATH - UNDIRECTED NETWORK

$$\langle \ell \rangle = \frac{\sum_{i,j} \ell_{ij}}{\binom{N}{2}} = 2 \frac{\sum_{i,j} \ell_{ij}}{N(N-1)}$$



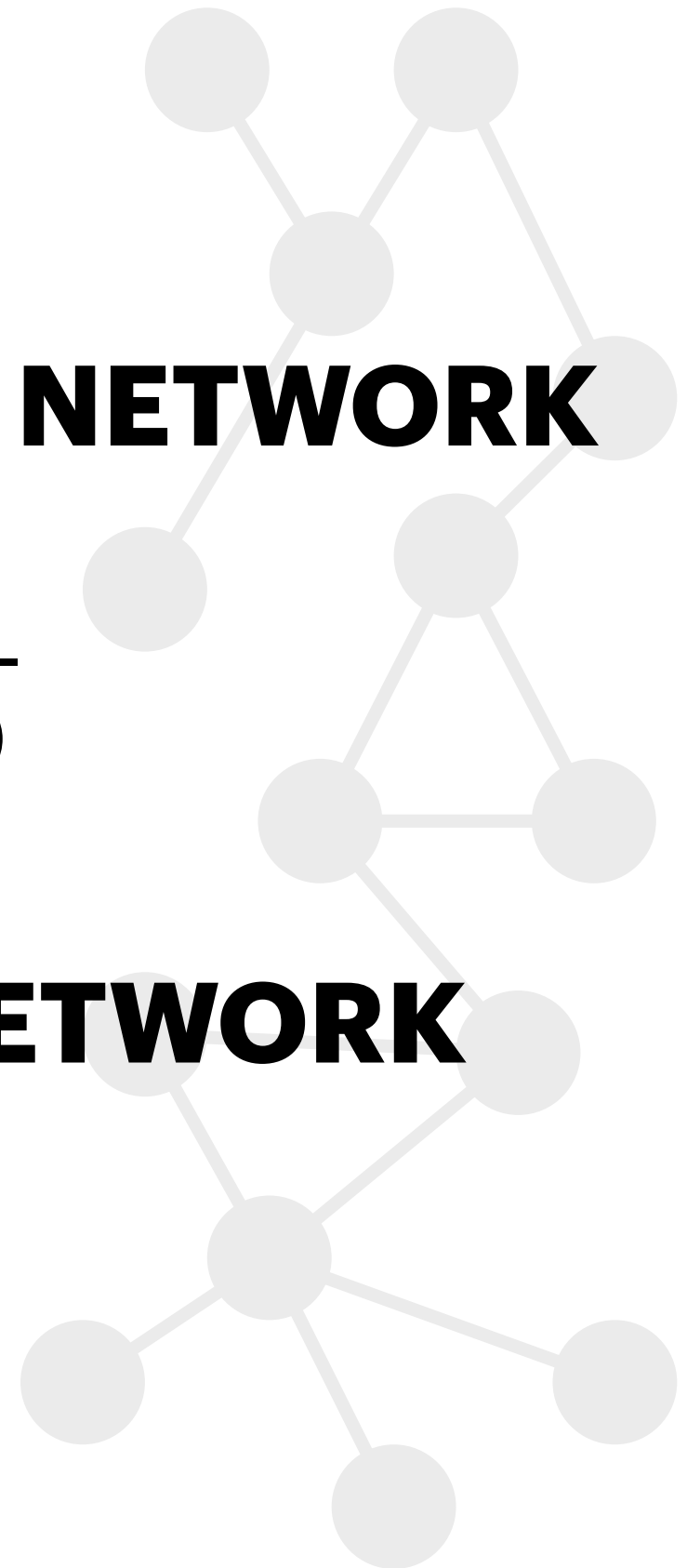
PATHS

AVERAGE PATH - UNDIRECTED NETWORK

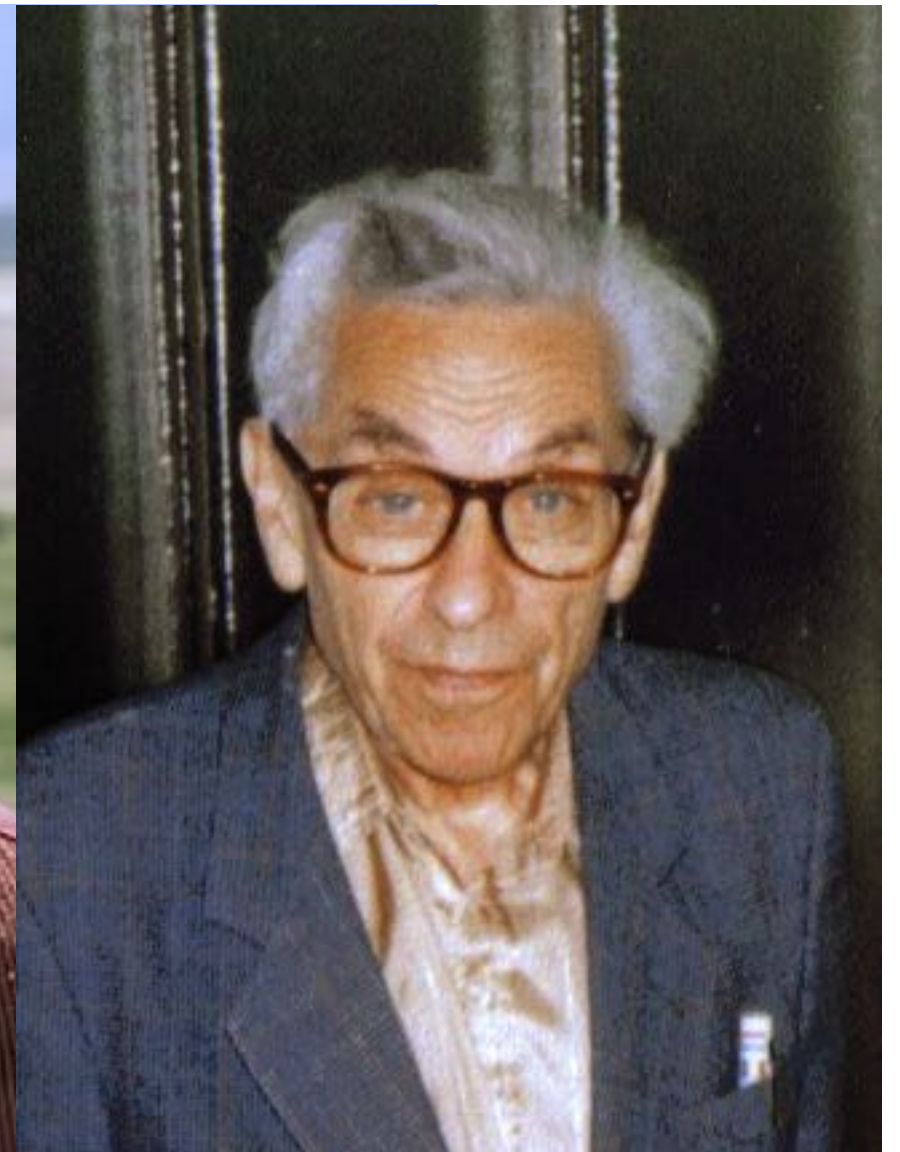
$$\langle \ell \rangle = \frac{\sum_{i,j} \ell_{ij}}{\binom{N}{2}} = 2 \frac{\sum_{i,j} \ell_{ij}}{N(N-1)}$$

DIAMETER - UNDIRECTED NETWORK

$$\ell_{max} = \max_{i,j} \ell_{ij}$$



SIX DEGREES OF SEPARATION

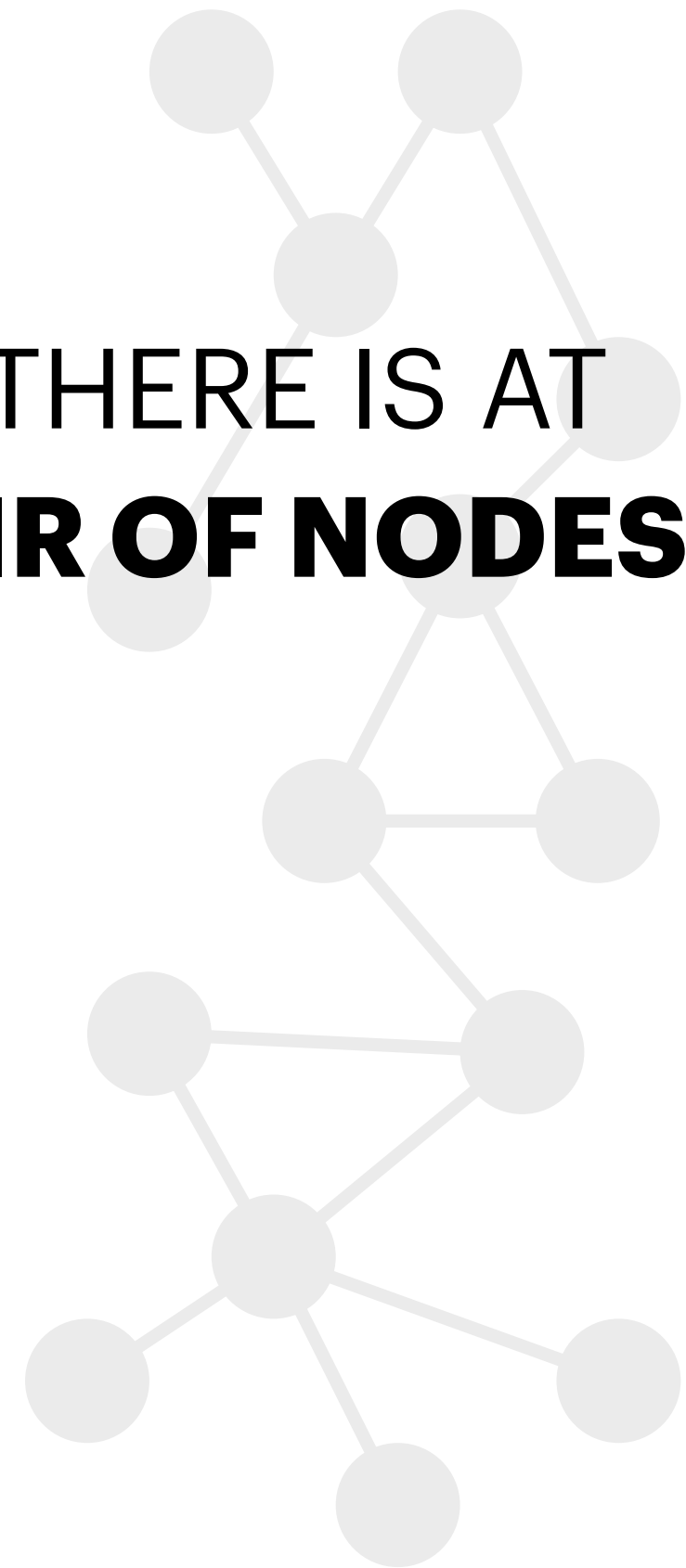


SIX DEGREES OF SEPARATION

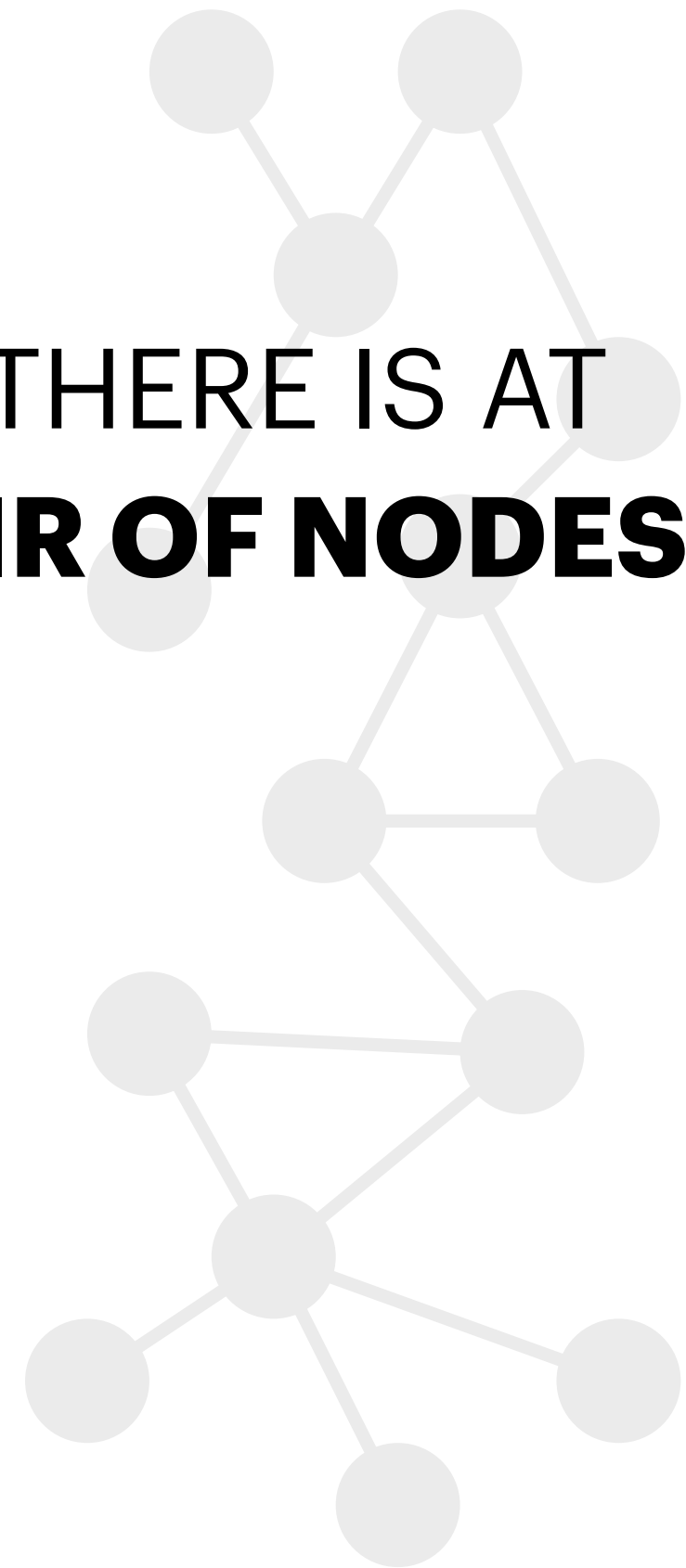
An average path length is said to be **short** if:

$$\langle \ell \rangle \approx \log(N)$$



A background network diagram consisting of several interconnected gray circular nodes and thin gray lines. The nodes are arranged in a way that suggests a complex, multi-layered structure, with some nodes acting as hubs connecting to multiple other nodes. The overall layout is centered and fills the background of the slide.

THERE IS AT R OF NODES

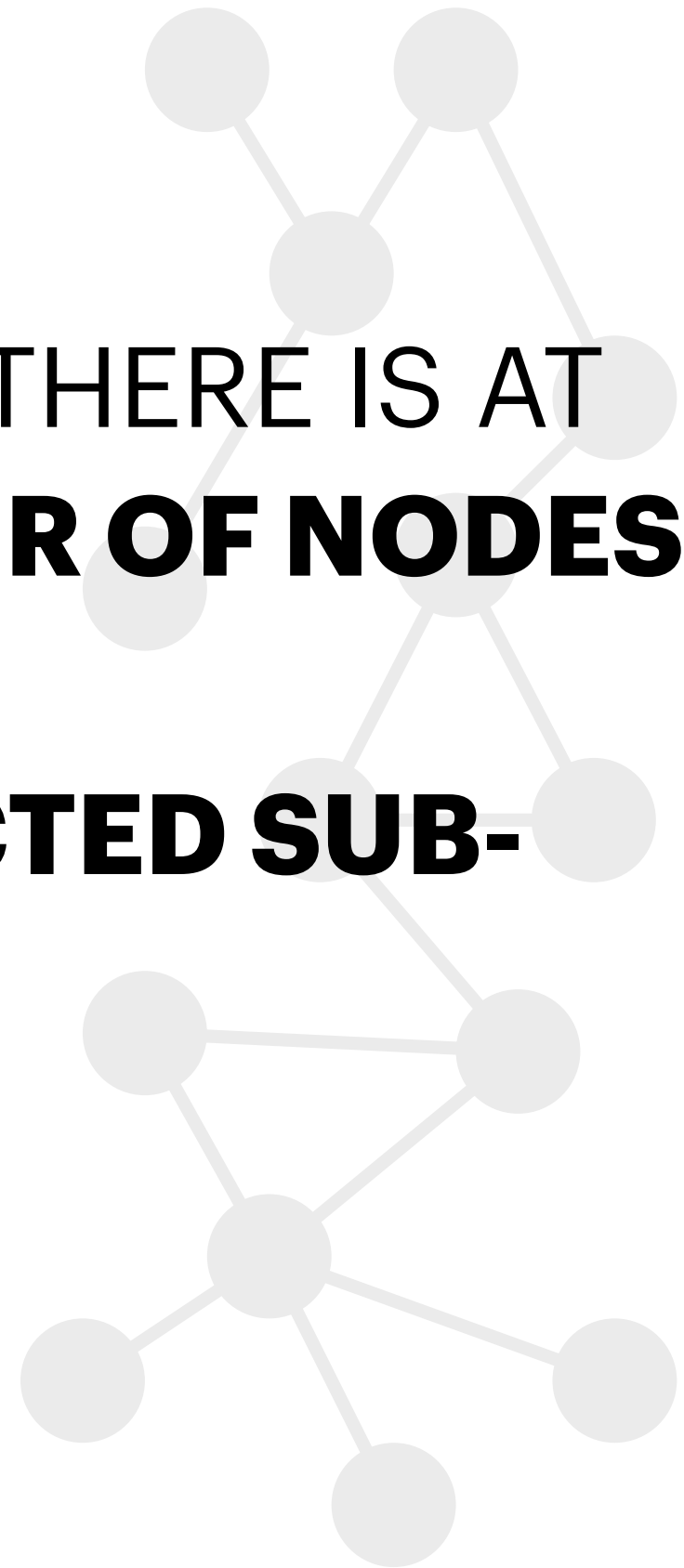
A background network diagram consisting of several interconnected gray circular nodes and thin gray lines. The nodes are arranged in a way that suggests a complex, multi-layered structure, with some nodes acting as hubs connecting to multiple other nodes. The overall layout is centered and fills the background of the slide.

THERE IS AT R OF NODES

CONNECTEDNESS

A NETWORK IS **CONNECTED** IF THERE IS AT LEAST A **PATH** BETWEEN **ANY PAIR OF NODES**

A **COMPONENT** IS A **CONNECTED SUB-GRAPH**

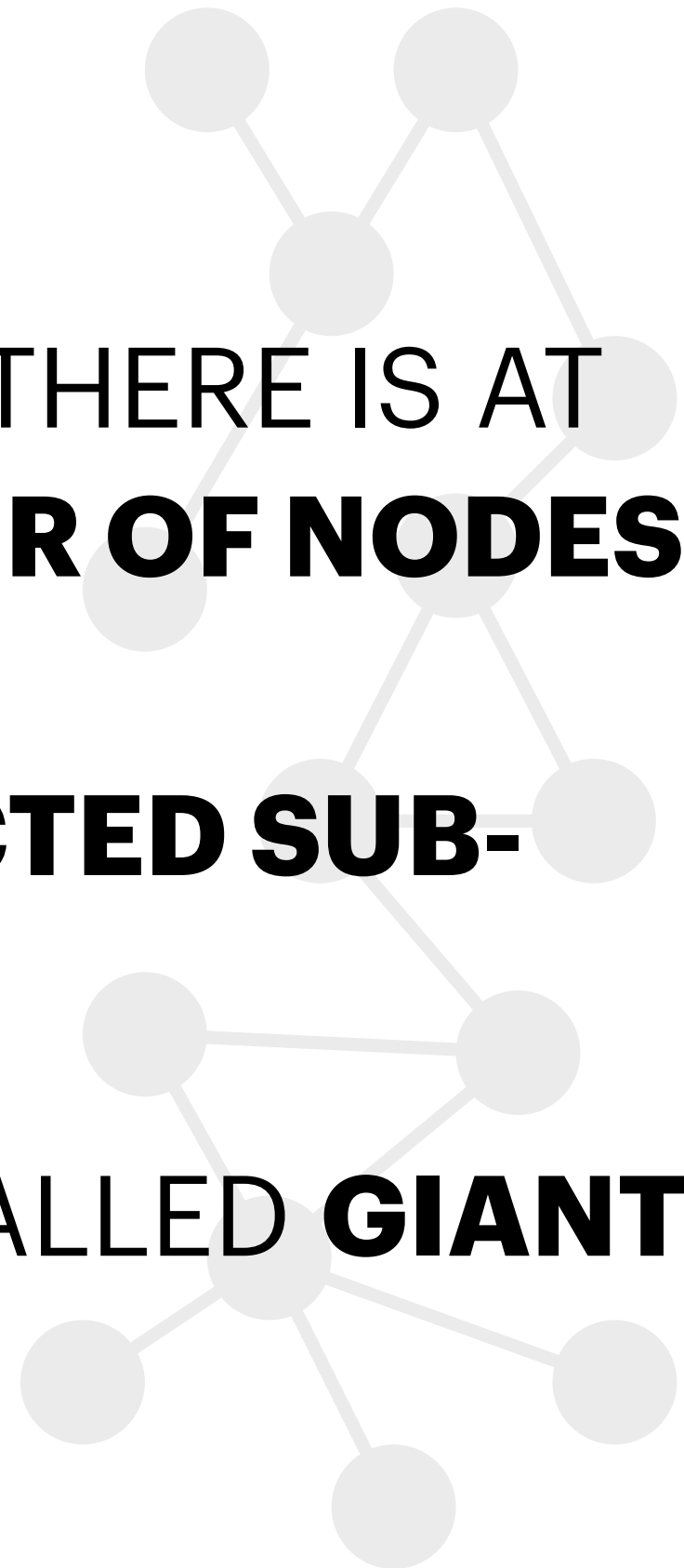


CONNECTEDNESS

A NETWORK IS **CONNECTED** IF THERE IS AT LEAST A **PATH** BETWEEN **ANY PAIR OF NODES**

A **COMPONENT** IS A **CONNECTED SUB-GRAPH**

THE **LARGEST** COMPONENT IS CALLED **GIANT COMPONENT**



CONNECTEDNESS

FOR DIRECTED NETWORKS COMPONENTS CAN
BE:

WEAKLY CONNECTED:

IF CONNECTED ONLY **DISREGARDING THE DIRECTION** OF LINKS



CONNECTEDNESS

FOR DIRECTED NETWORKS COMPONENTS CAN
BE:

WEAKLY CONNECTED:

IF CONNECTED ONLY **DISREGARDING THE DIRECTION** OF LINKS

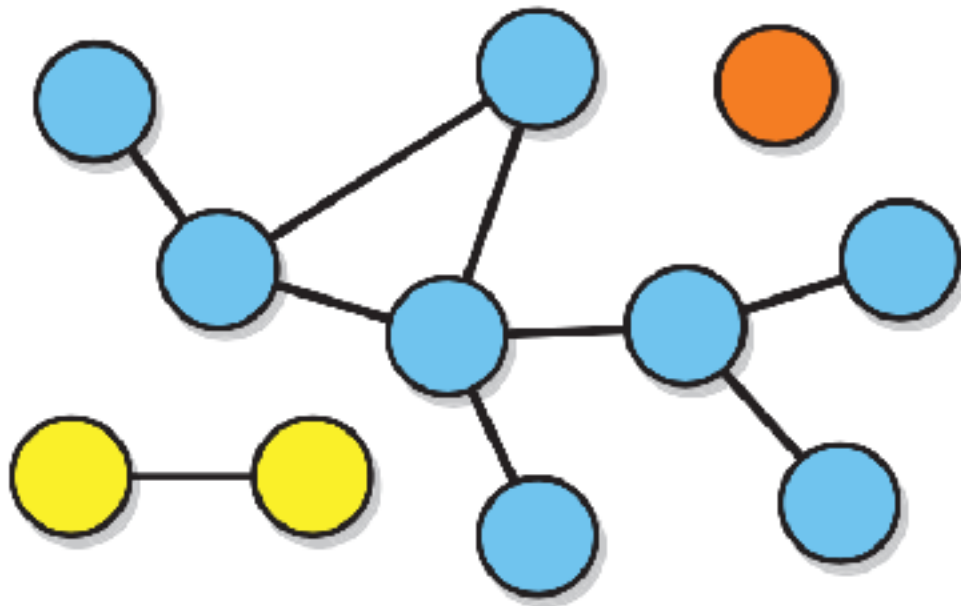
STRONGLY CONNECTED:

IF CONNECTED ALSO WHEN **CONSIDERING THE DIRECTION** OF LINKS



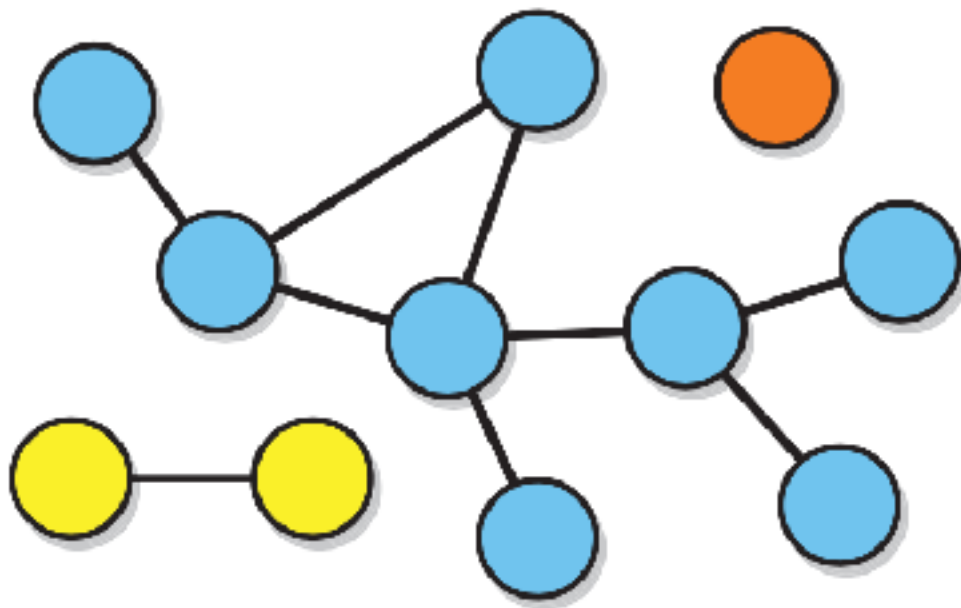
CONNECTEDNESS

Undirected

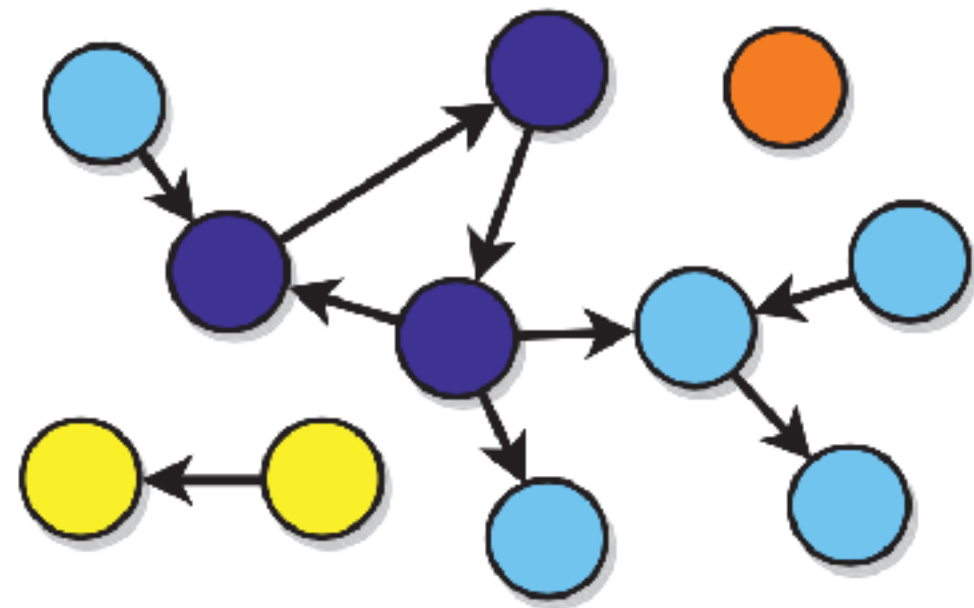


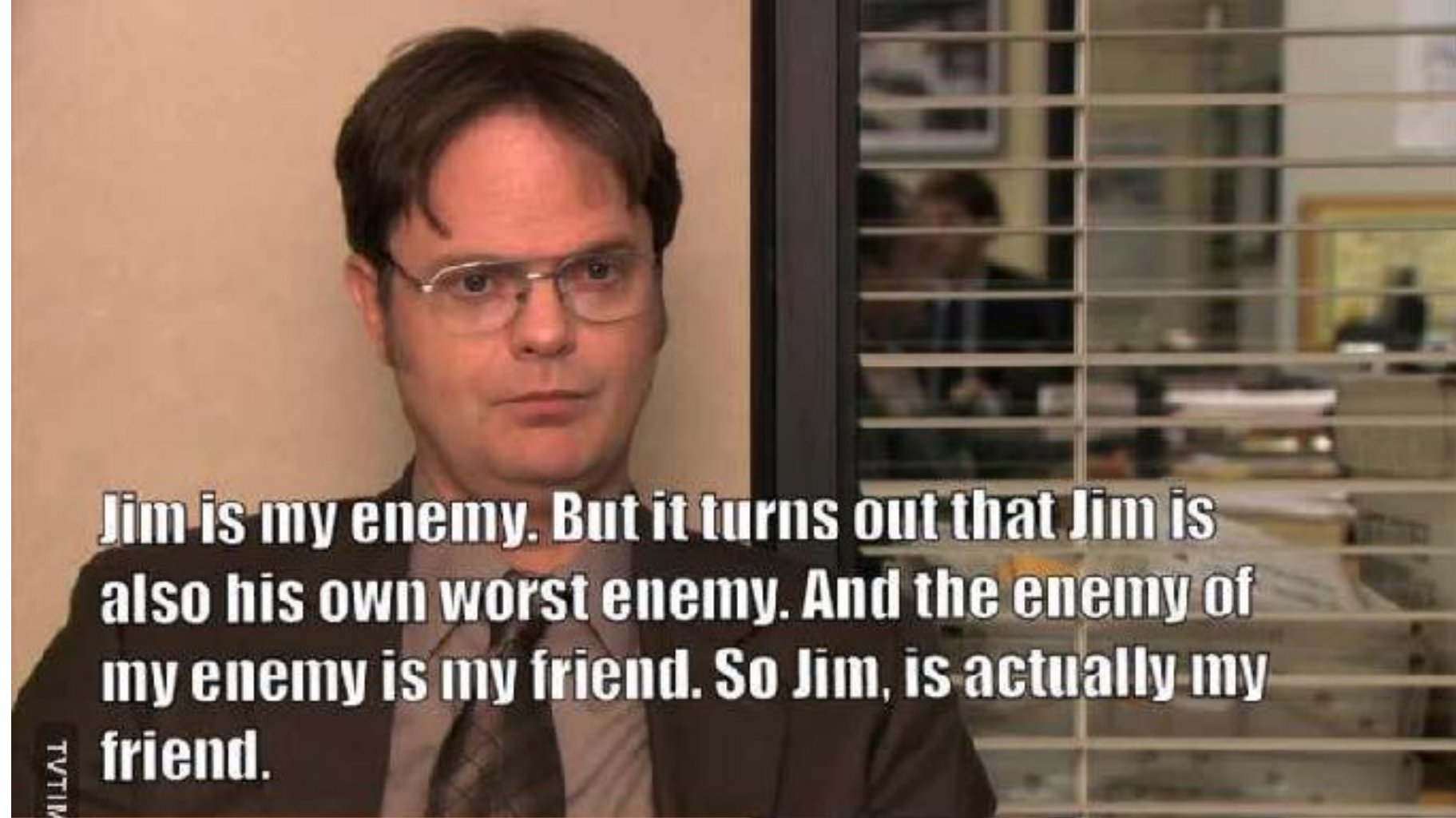
CONNECTEDNESS

Undirected



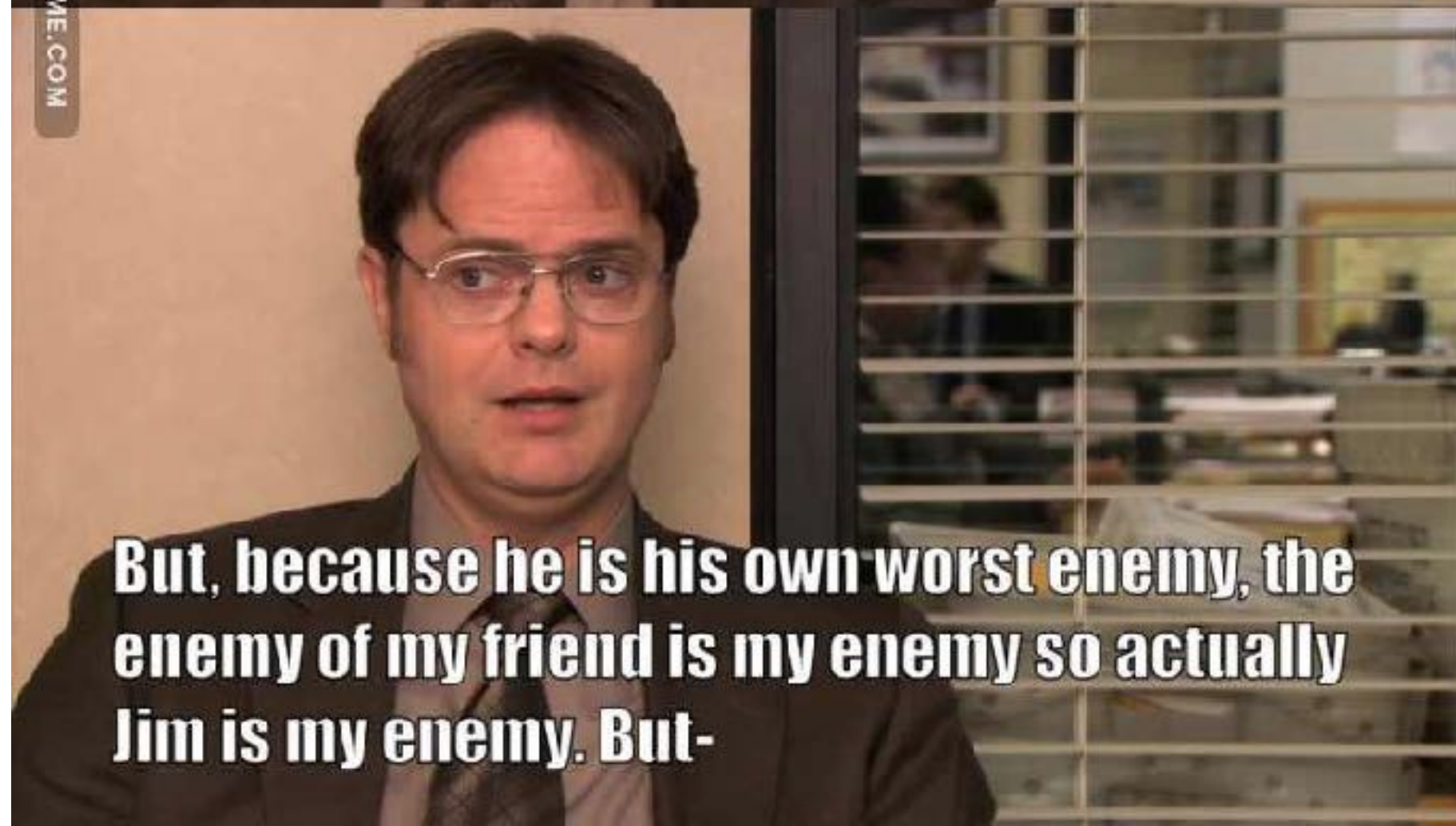
Directed





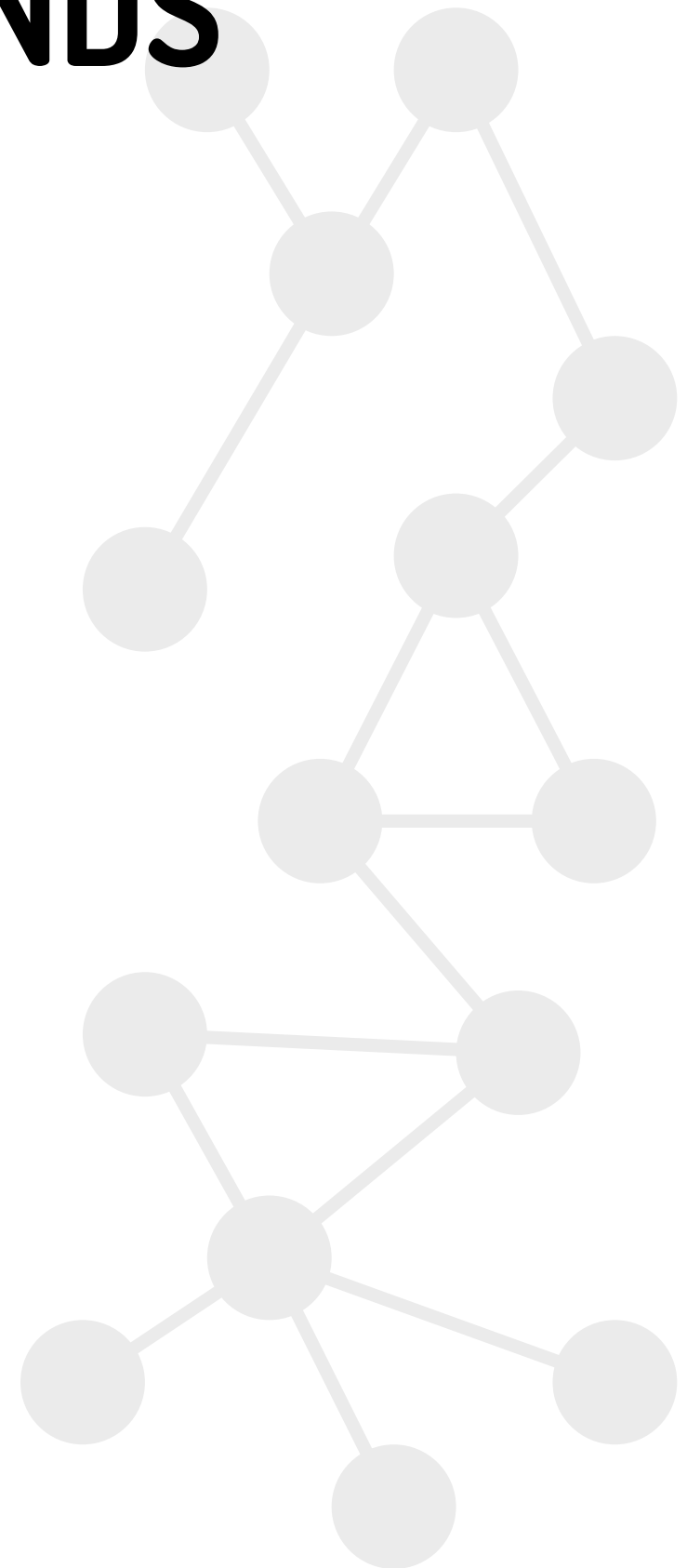
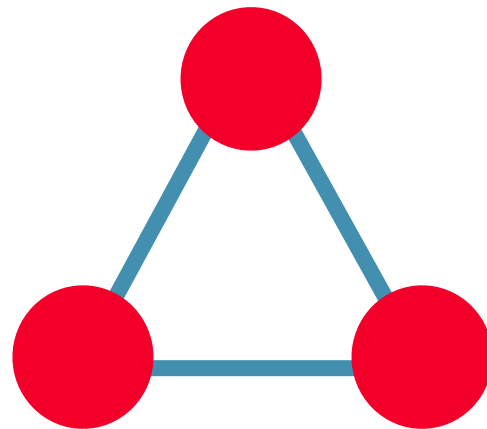
TVTIME.COM

Jim is my enemy. But it turns out that Jim is also his own worst enemy. And the enemy of my enemy is my friend. So Jim, is actually my friend.

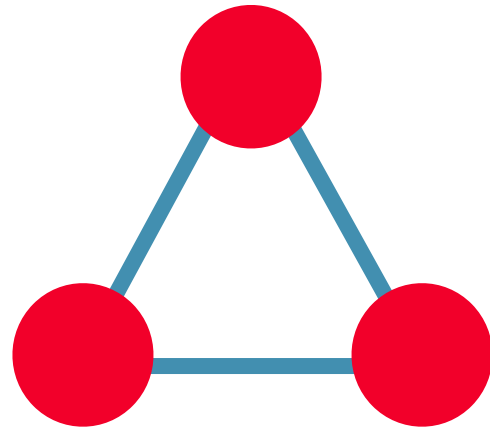


But, because he is his own worst enemy, the enemy of my friend is my enemy so actually Jim is my enemy. But-

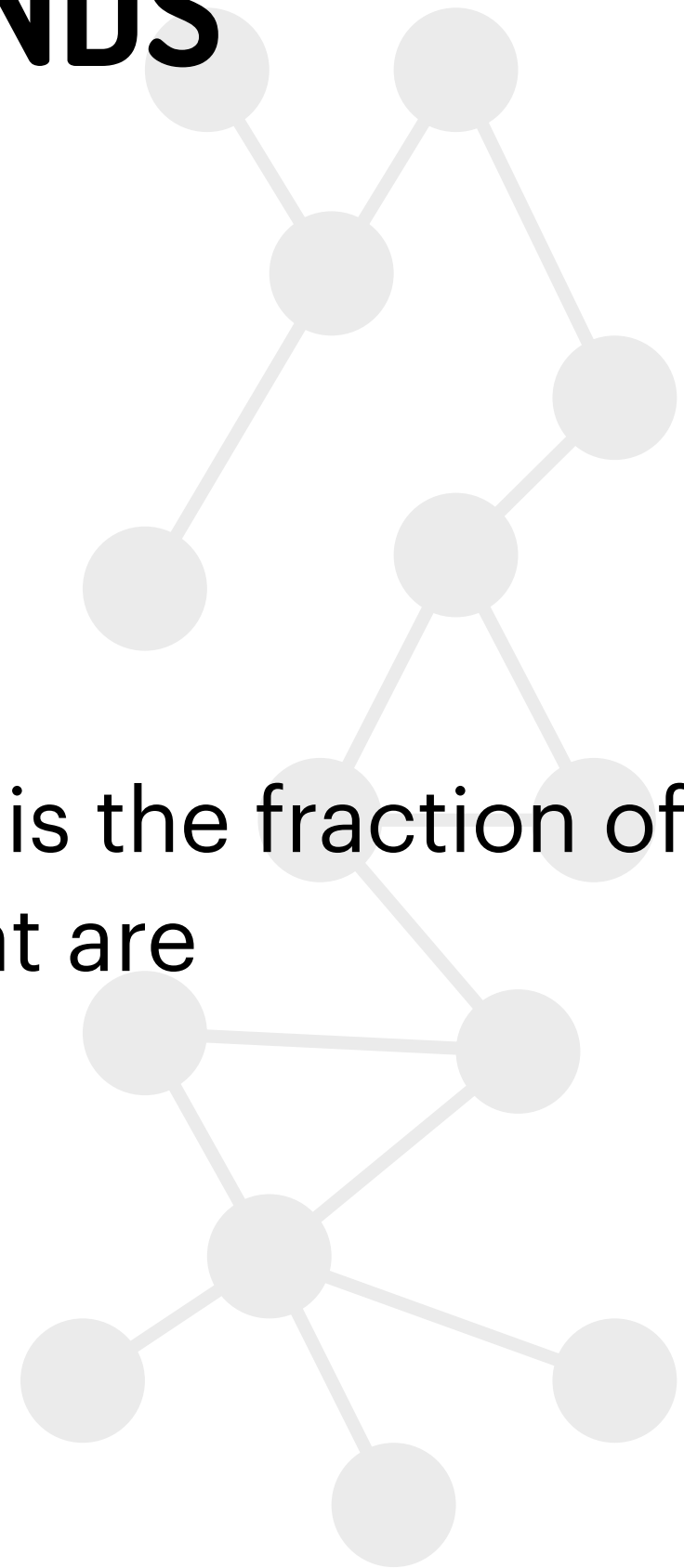
TRIANGLES AND FRIENDS



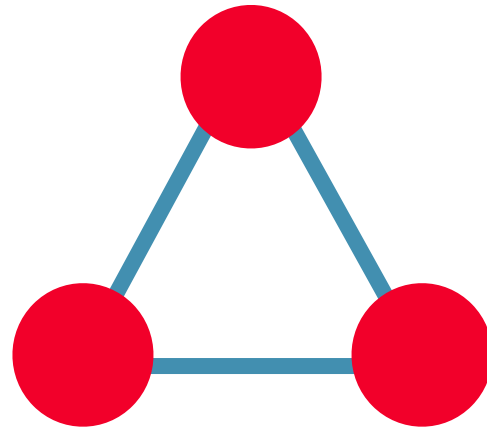
TRIANGLES AND FRIENDS



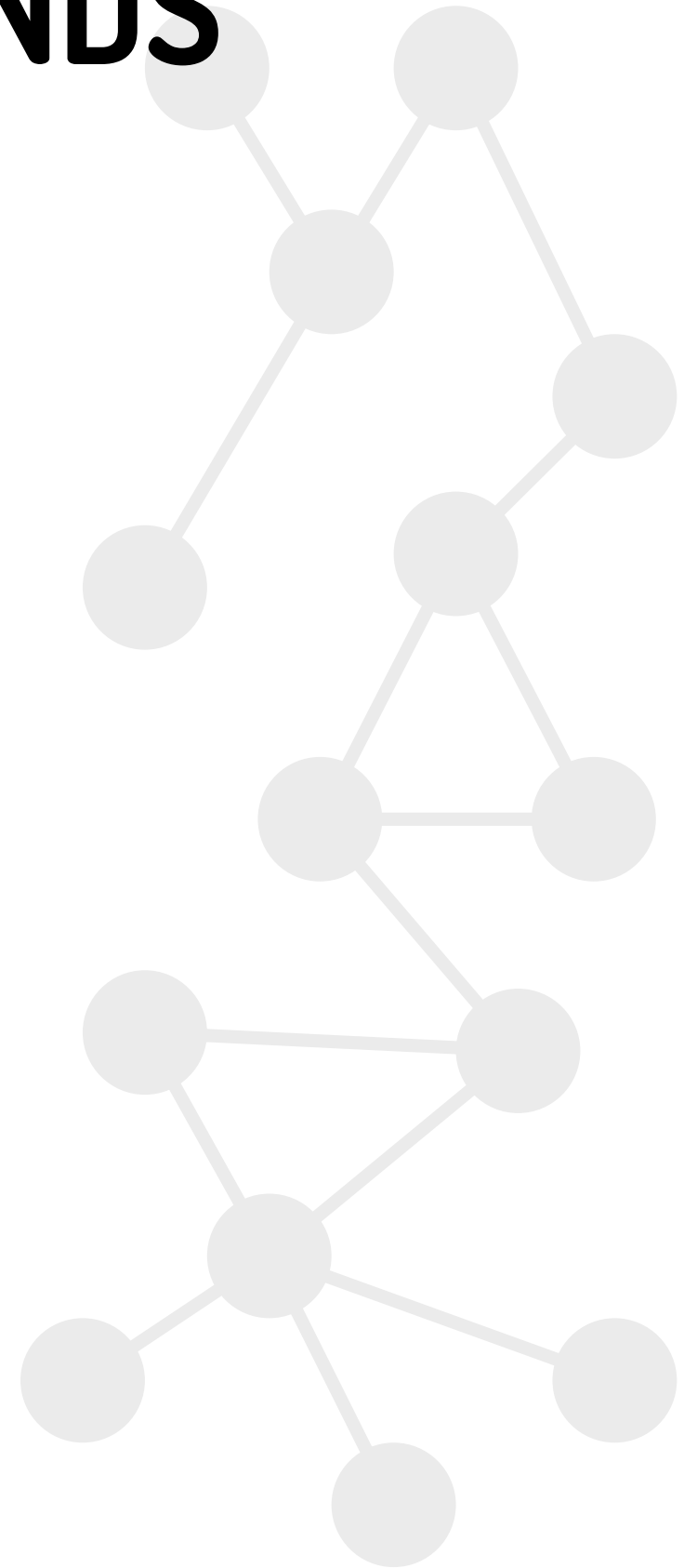
The **clustering coefficient** $C(i)$ of node i is the fraction of pairs of the neighbours of i that are **connected to each other**

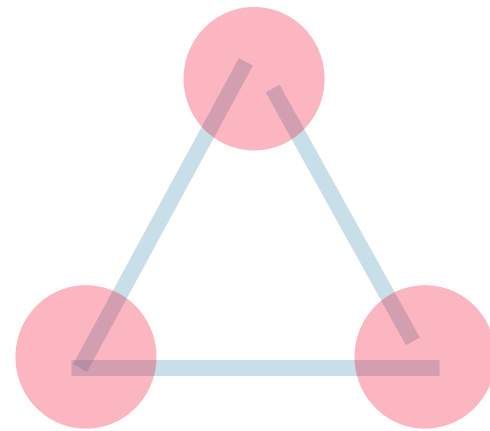


TRIANGLES AND FRIENDS

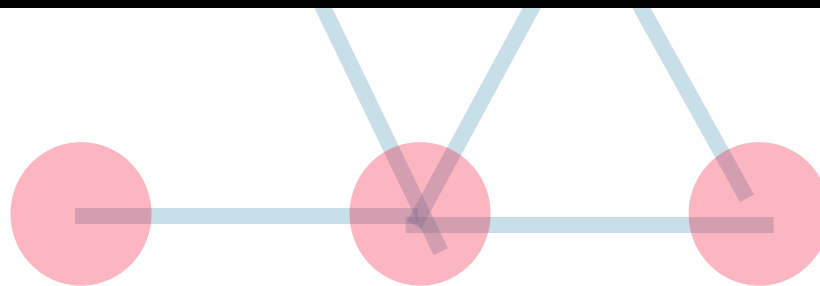


$$C(i) = \frac{\tau(i)}{\tau_{max}(i)} = \frac{2\tau(i)}{k_i(k_i - 1)}$$

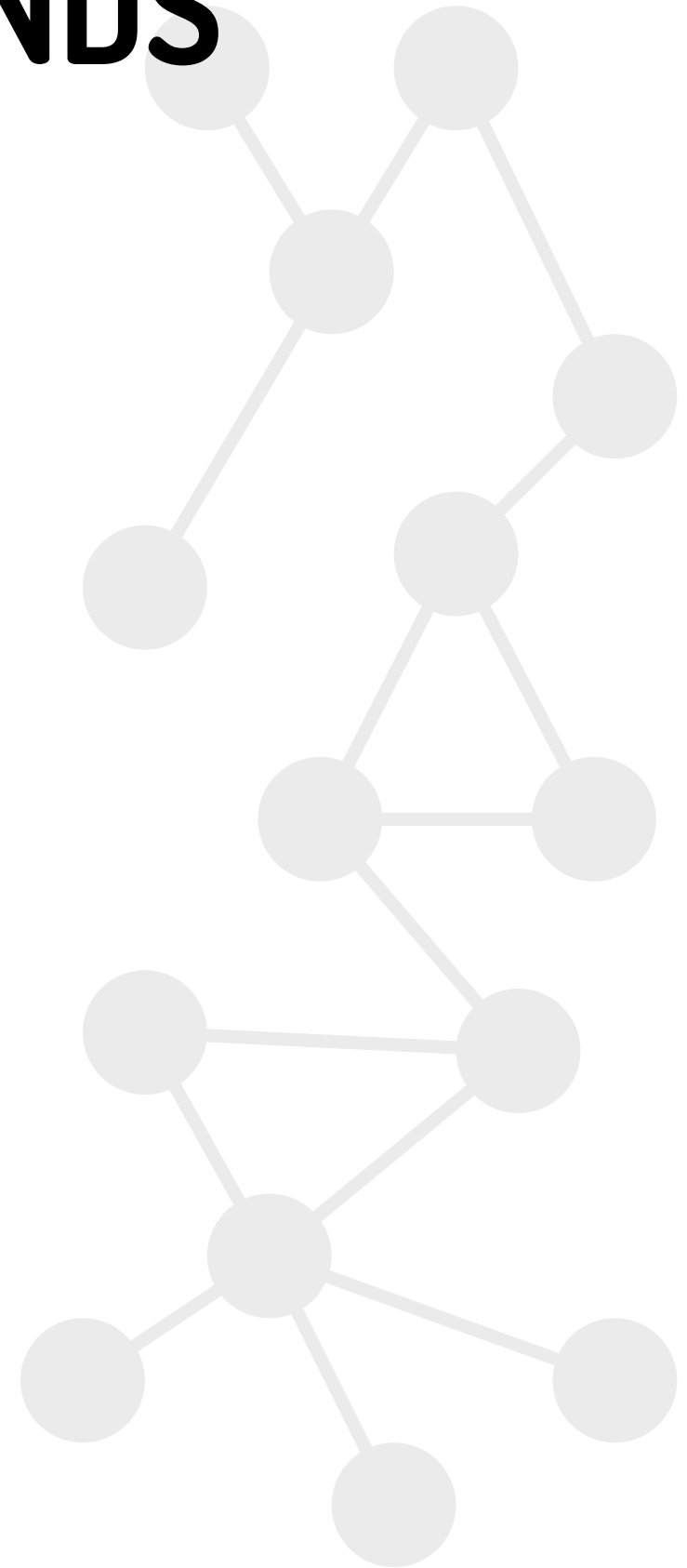
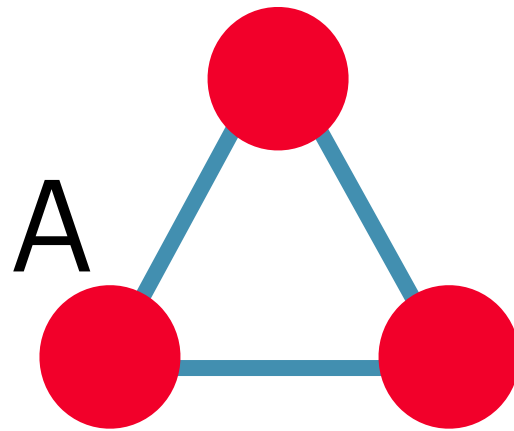




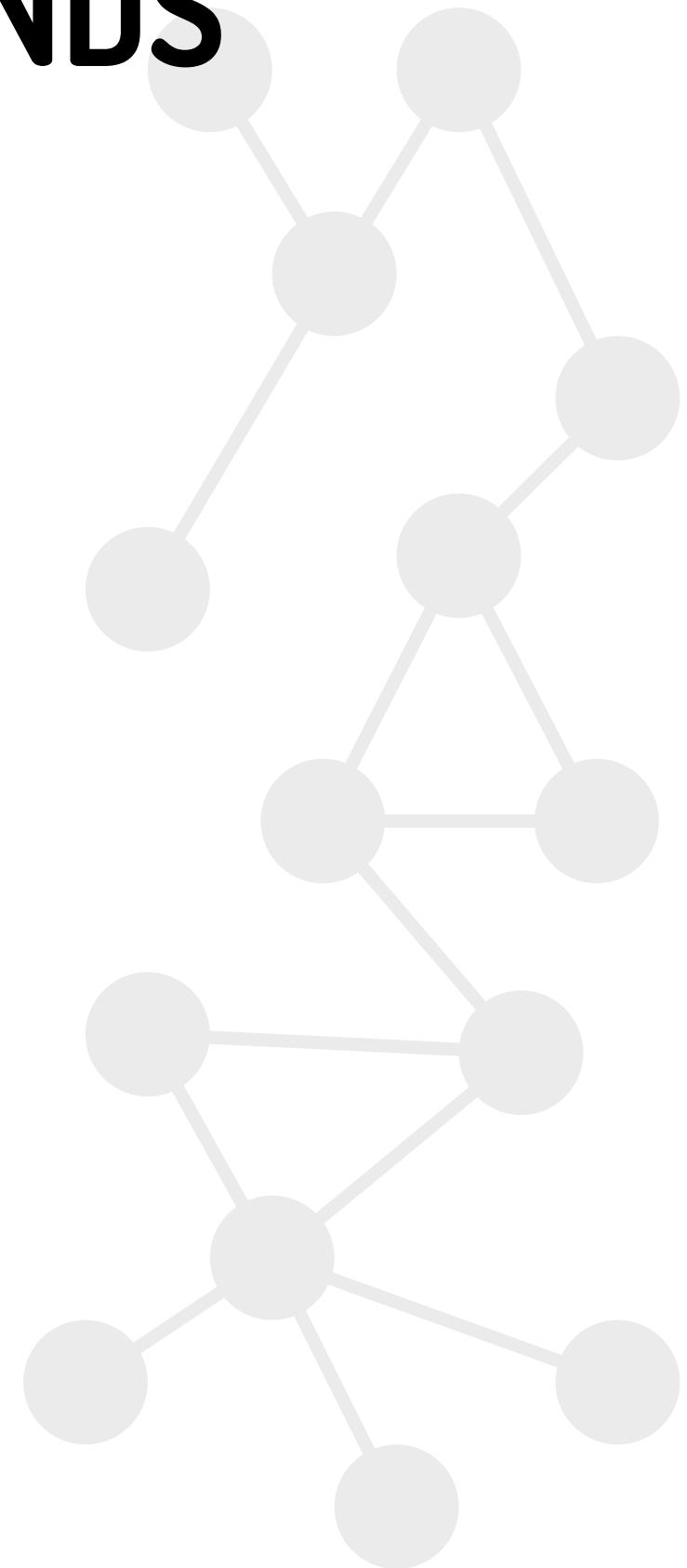
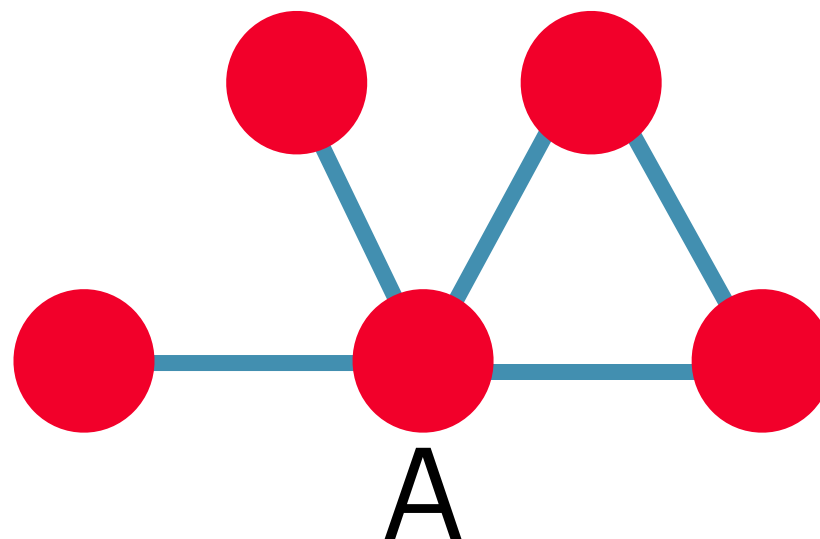
WHAT IS THE $C(A)$ IN THE FOLLOWING NETWORKS?



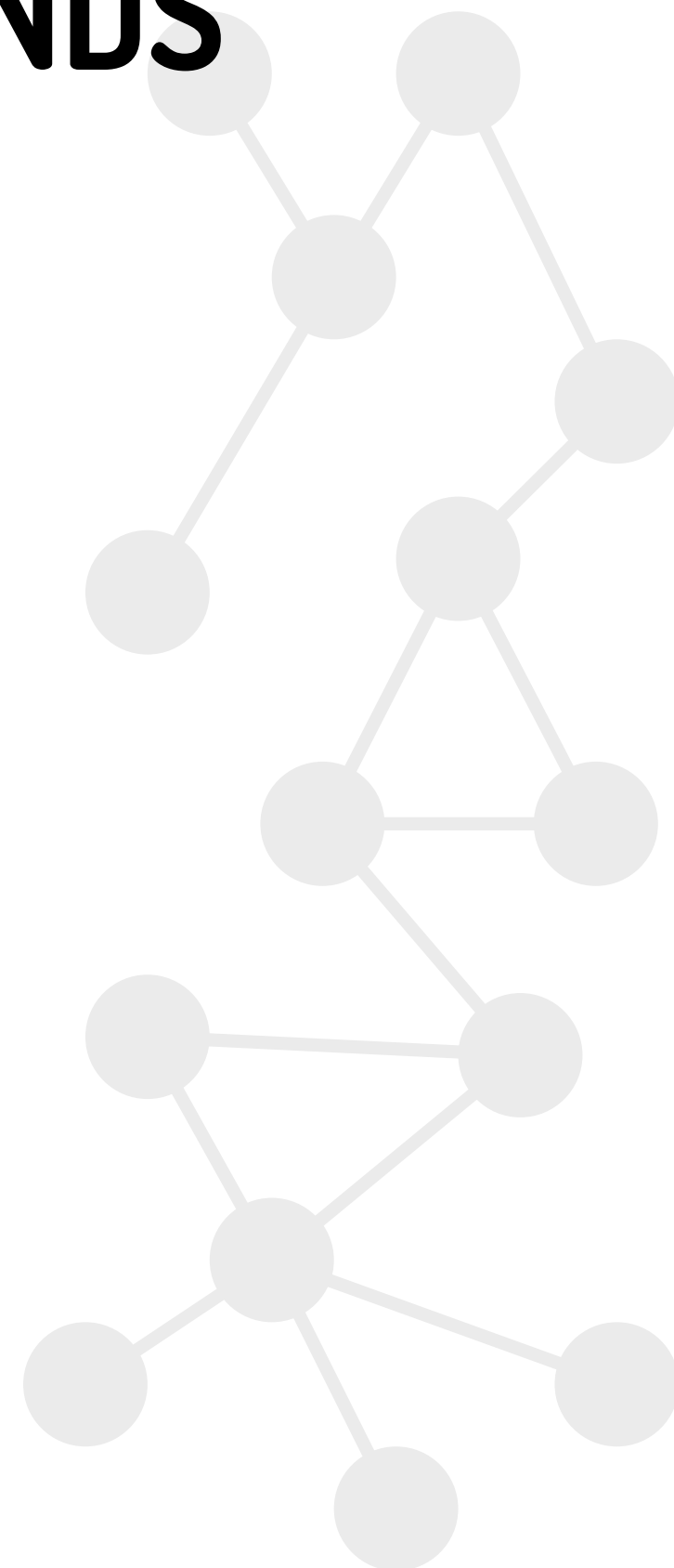
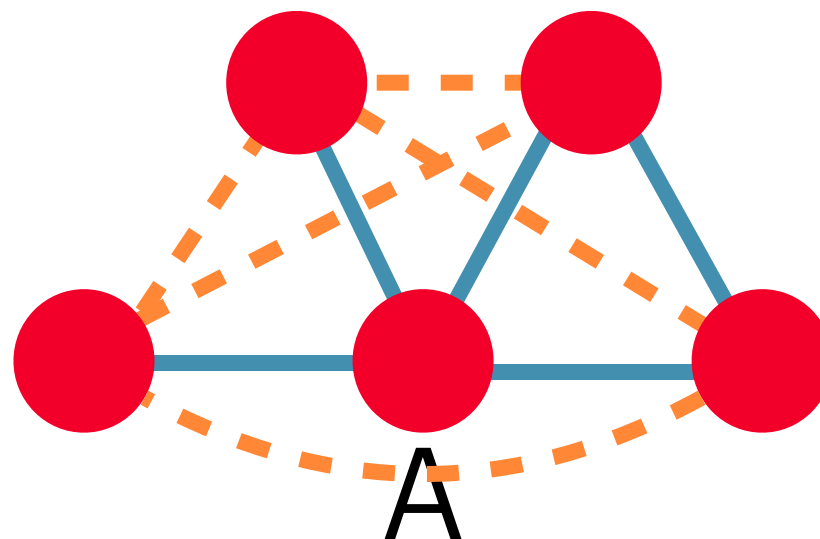
TRIANGLES AND FRIENDS



TRIANGLES AND FRIENDS

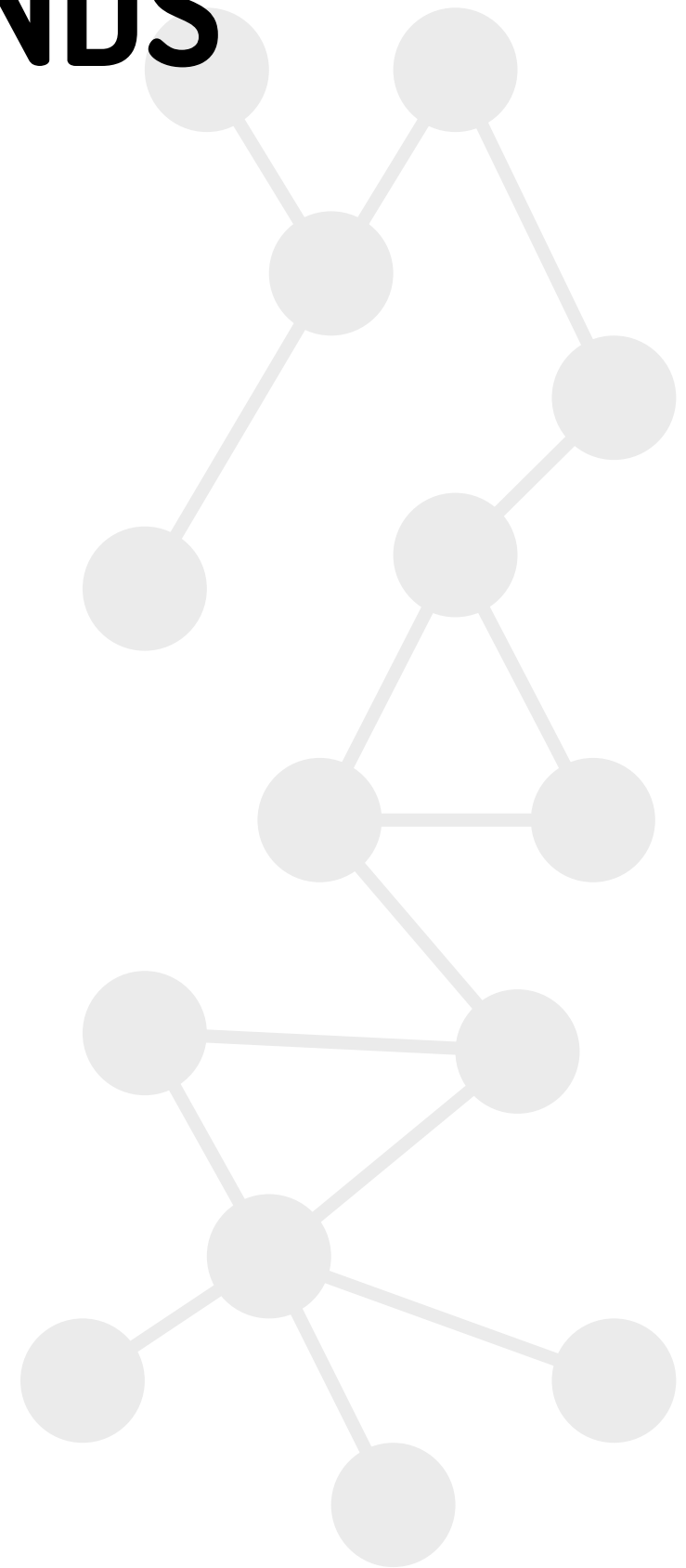
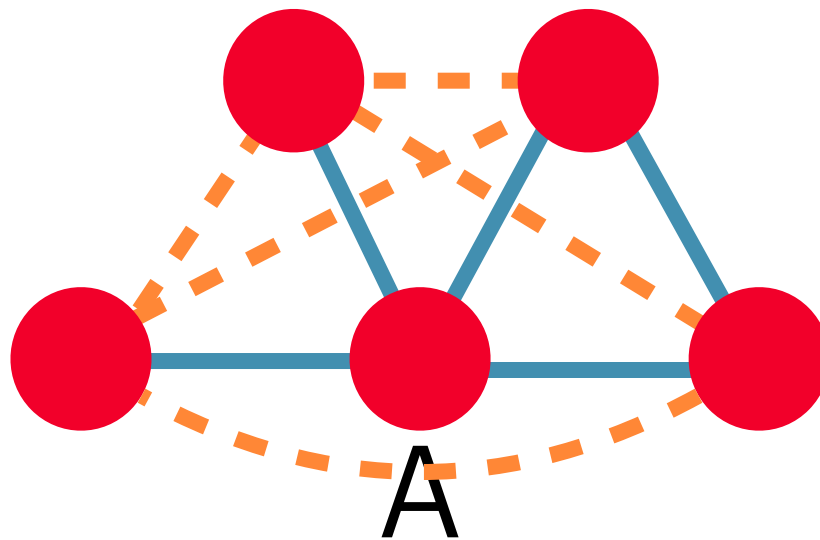


TRIANGLES AND FRIENDS

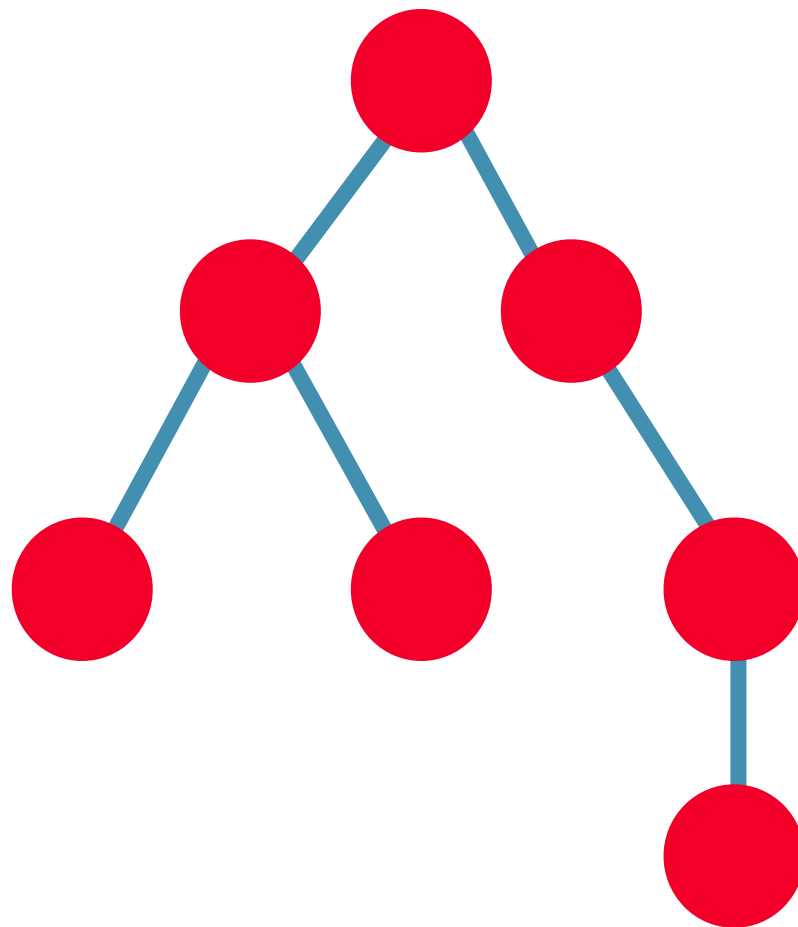


TRIANGLES AND FRIENDS

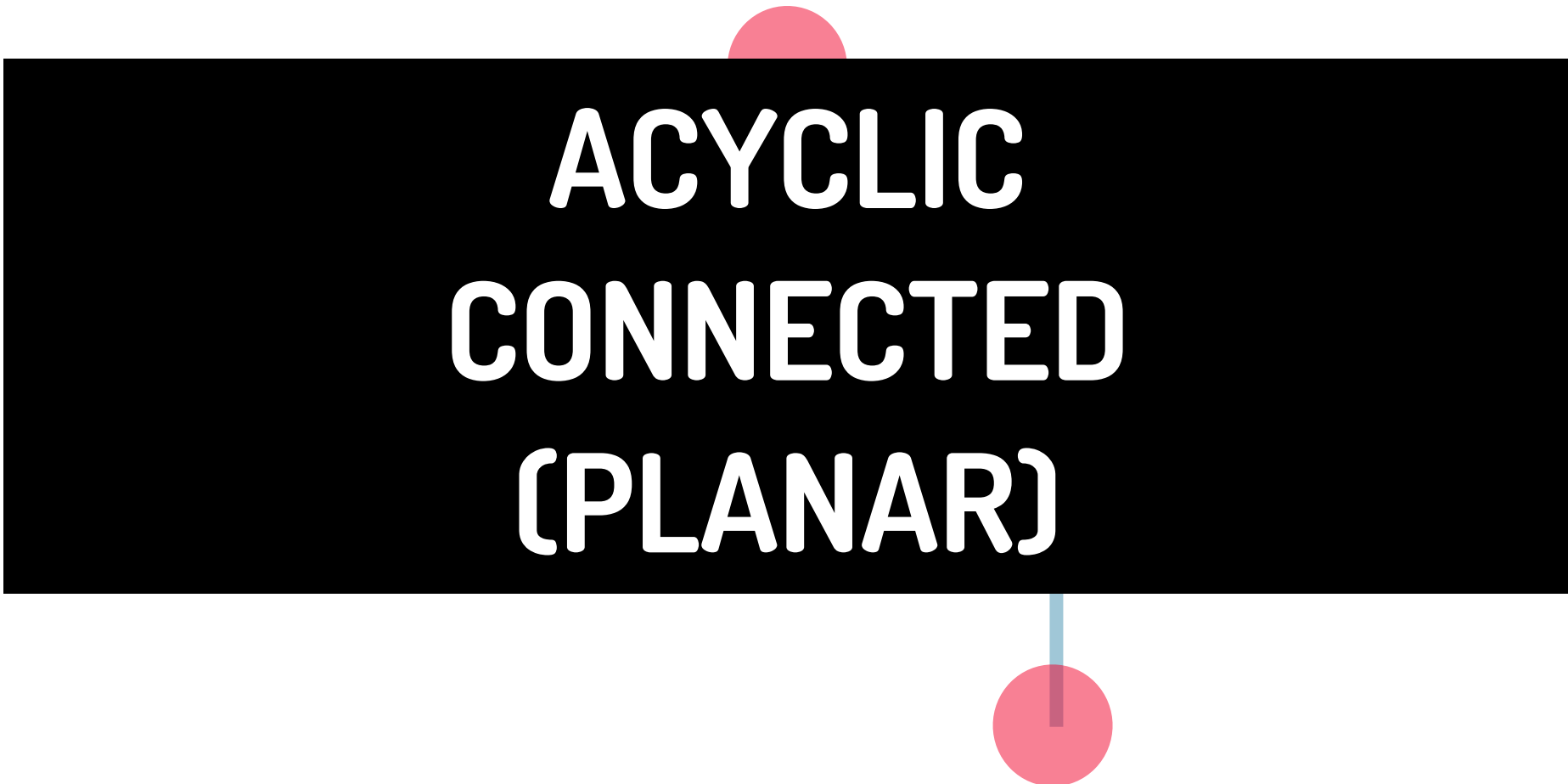
$$C(i) = \frac{\tau(i)}{\tau_{max}(i)} = \frac{2\tau(i)}{k_i(k_i - 1)}$$



TREES



TREES



ACYCLIC
CONNECTED
(PLANAR)

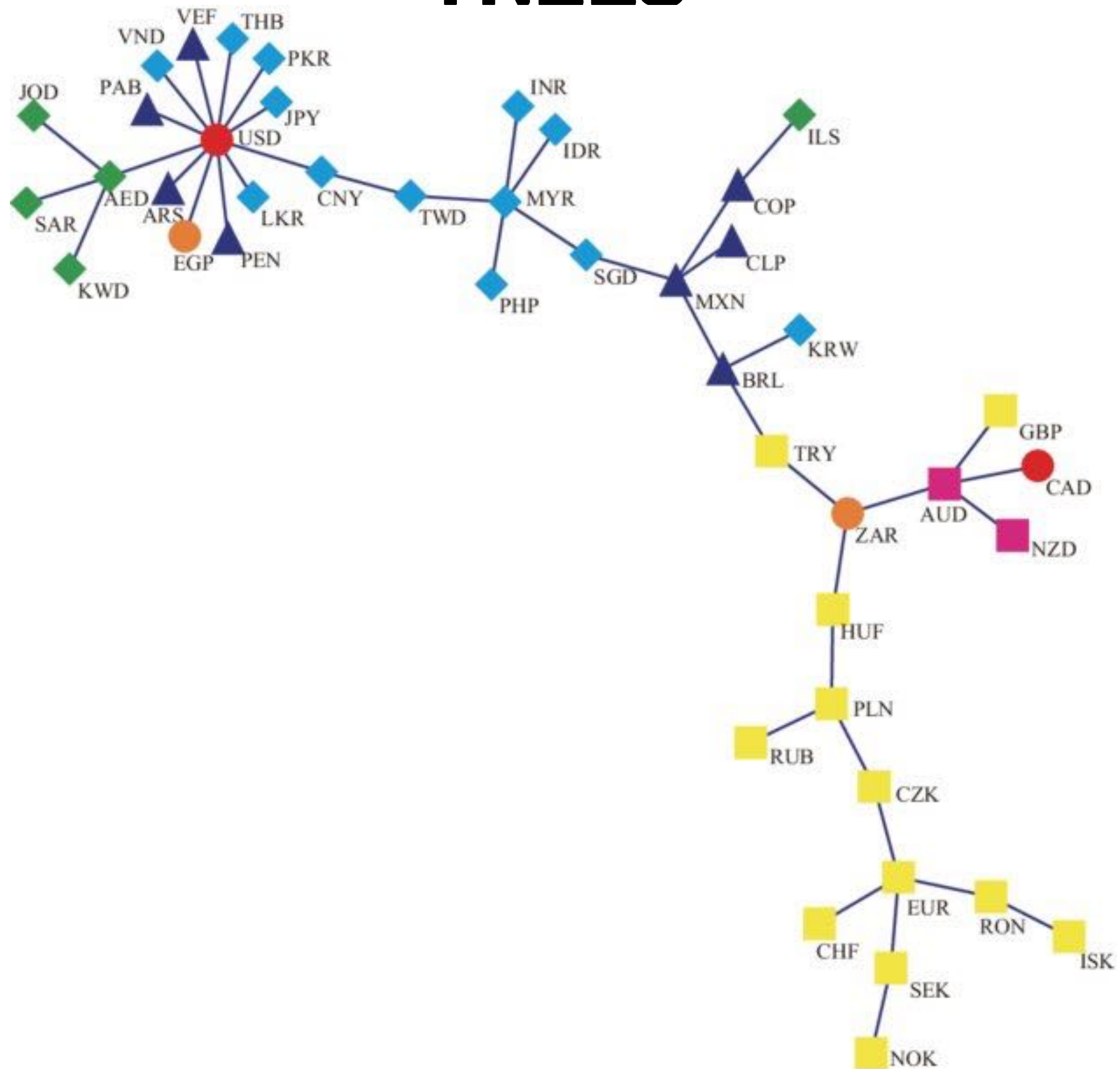
TREES

Maximum (minimum) spanning tree:

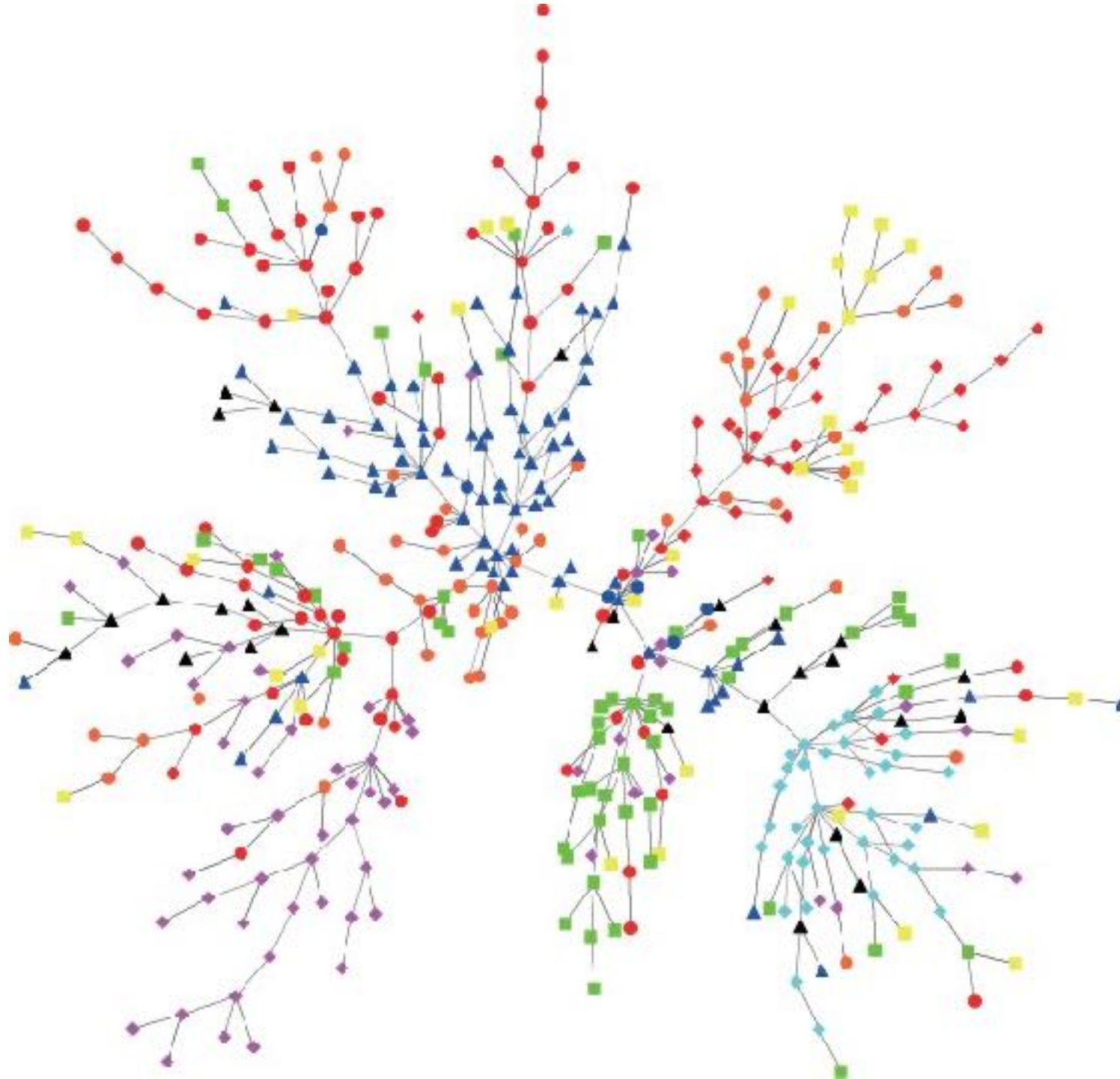
For each node only keep the connection with maximum (minimum) weight



TREES



TREES



SUMMARY

We learned about **clustering and distances**

We learned the **tree topology** and seen its applications

We can now perform basic **network analysis!**

