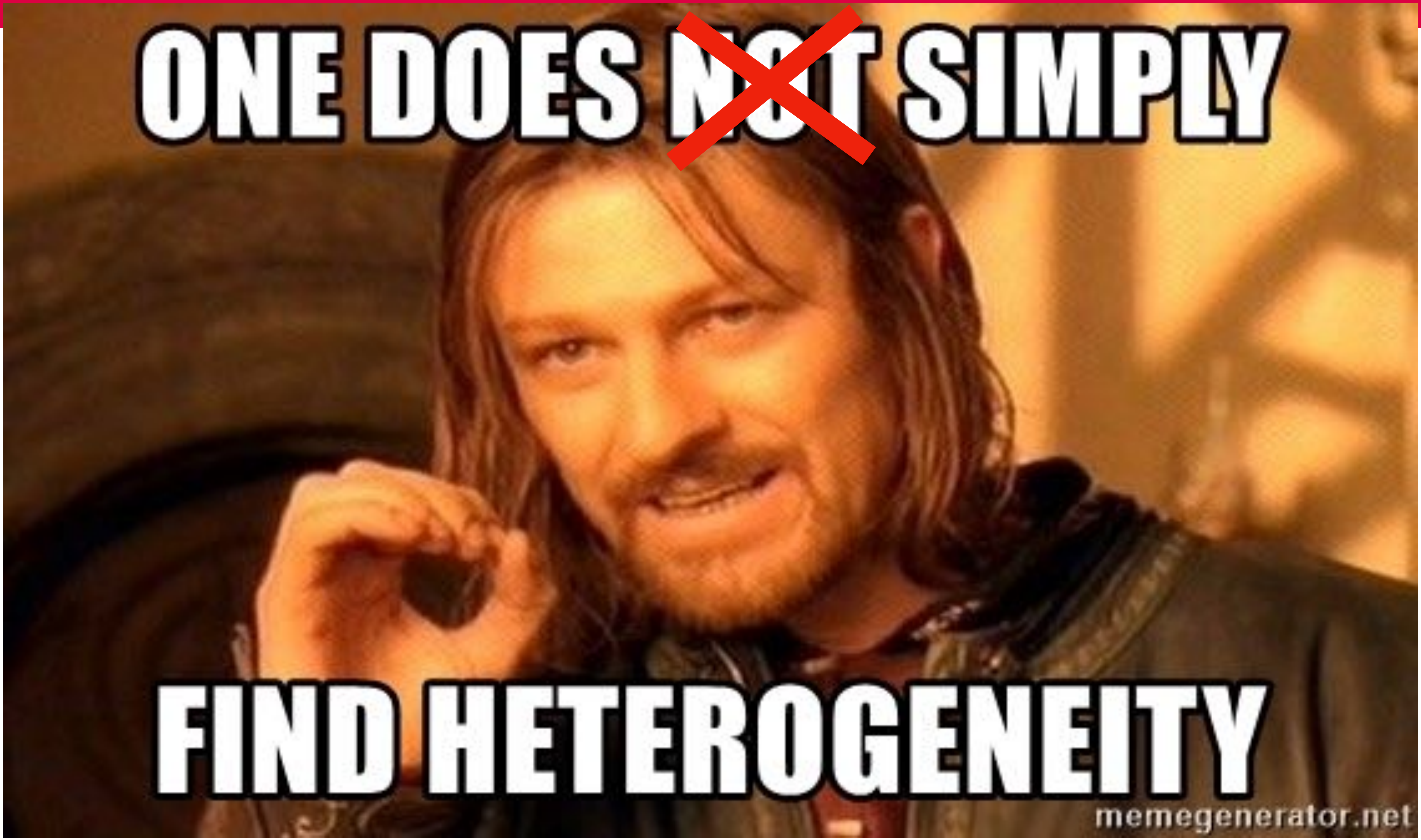
HUBS AND CENTRALITY



Last week recap and some real-world examples

Assortativity: high-degree nodes connect more likely to high-degree nodes.

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Clustering: your friends are friends with each other.

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Clustering: your friends are friends with each other.

Paths: the “steps” it takes to reach other nodes.

Studies on multiple countries (e.g. USA, Italy, Japan, Brazil, Colombia, etc) show that interbank networks have some commonalities worldwide

Low density

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Short paths
Disassortative
High clustering

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Low density

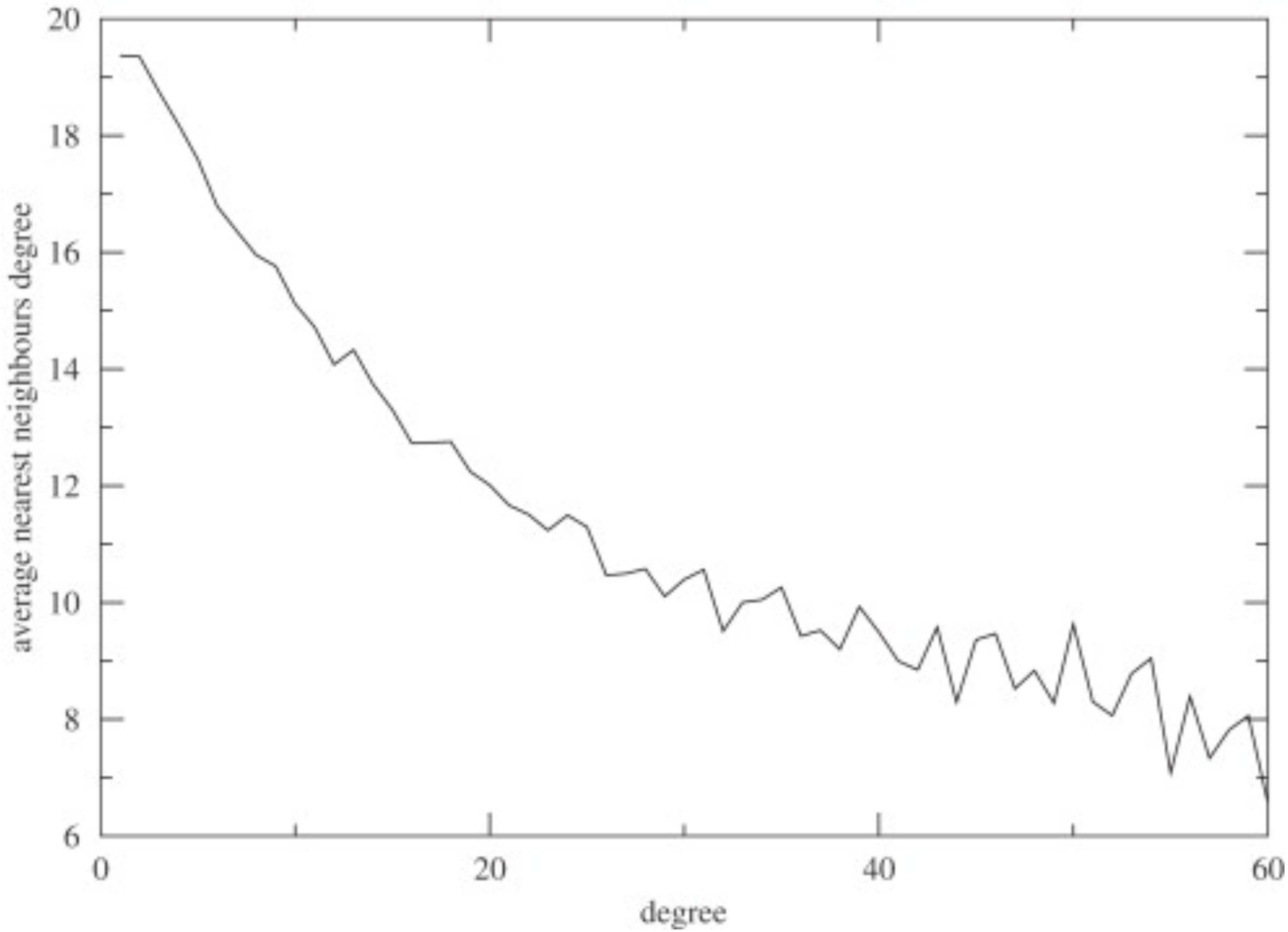
Short paths

Disassortative

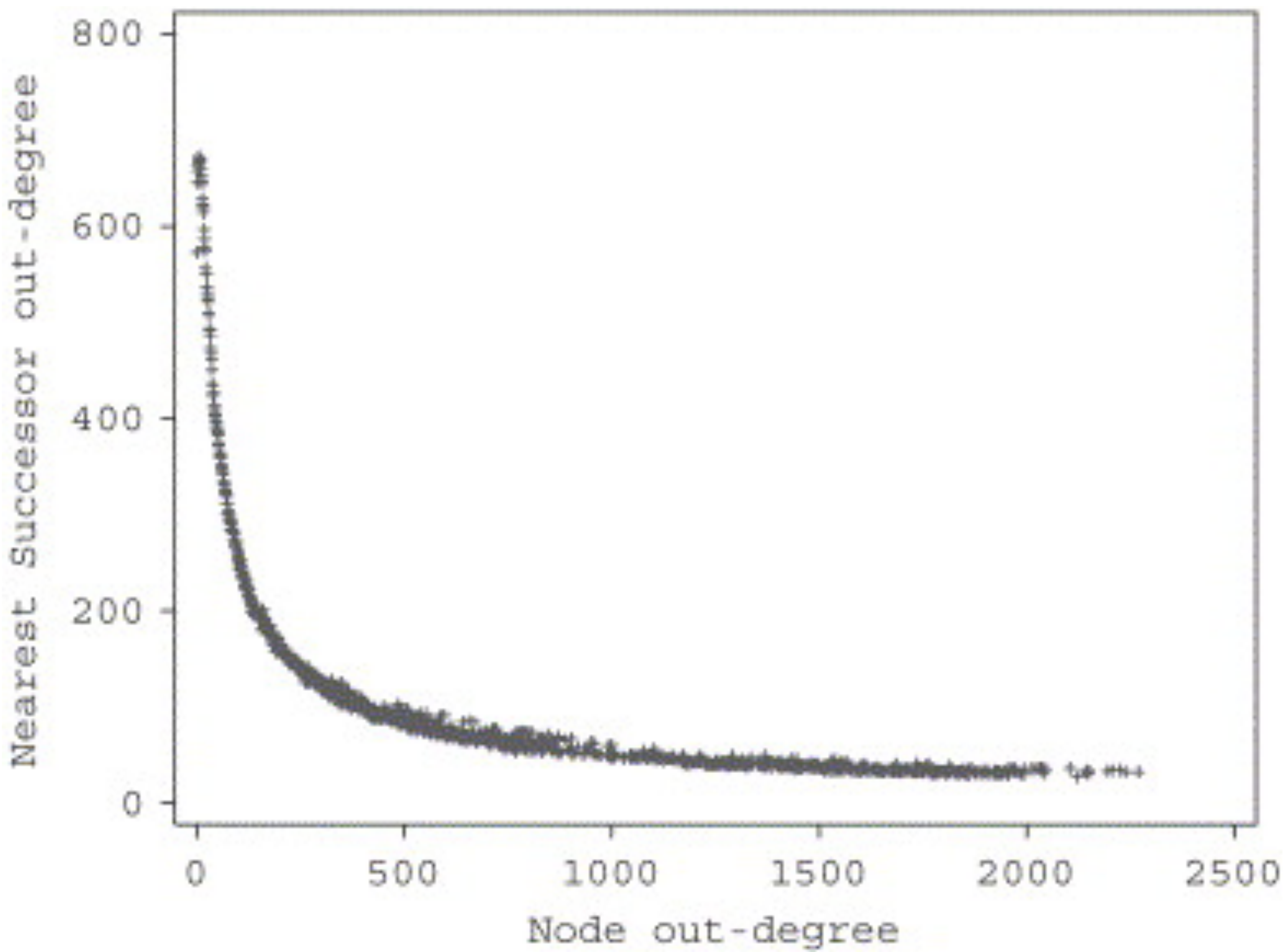
High clustering

Heavy-tailed degree distributions*

Italian interbank overnight market



Interbank payments transferred between commercial banks over the Fedwire Funds Service



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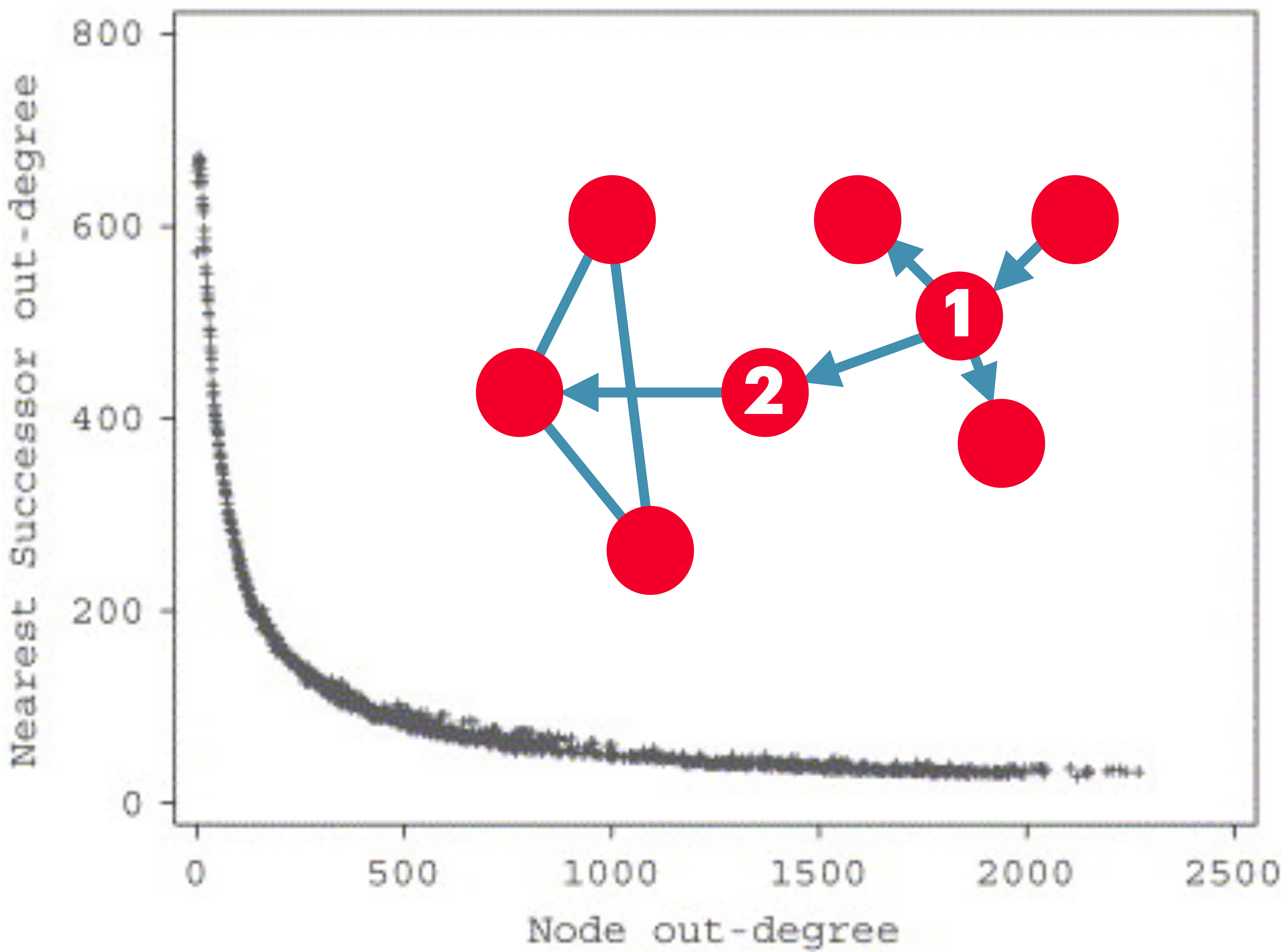

LEARNING OUTCOMES

Learn about **network heterogeneity**
Discover how to find **important nodes**
Find out that **your friends have more friends than you** (really)

HETEROGENEITY

In real-world networks the importance of nodes is **heterogeneous**

Importance is often measured with centrality

There are **several measures** of centrality

DEGREE CENTRALITY

Trivially, this is the degree of a node

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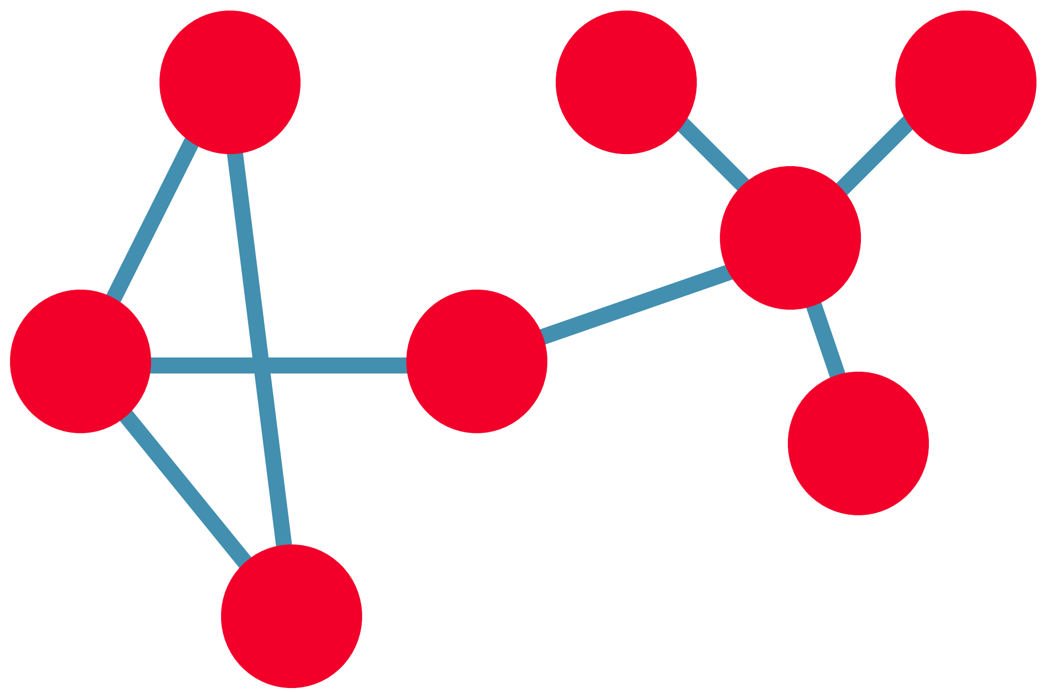

CLOSENESS CENTRALITY

How close a node is to other nodes

$$g_i = \frac{1}{\sum_{i \neq j} \ell_{ij}}$$

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Average distance

BETWEENNESS CENTRALITY

How many shortest paths pass through a node

$$b_i = \sum_{h \neq j \neq i} \frac{\sigma_{hj}(i)}{\sigma_{hj}}$$

BETWEENNESS CENTRALITY

How many shortest paths pass through a node

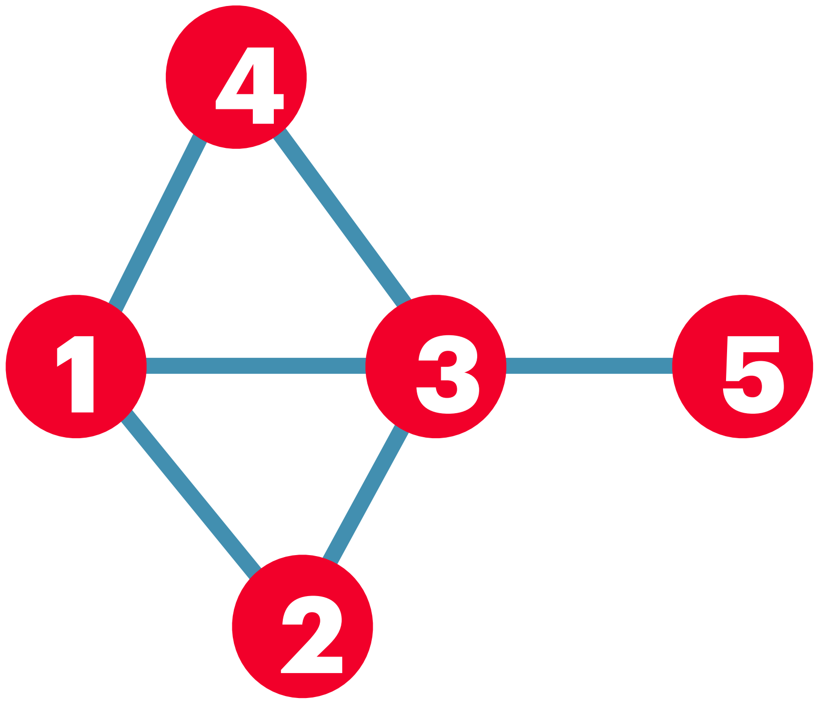
Number of shortest paths between h and j passing through i

$$b_i = \sum_{h \neq j \neq i} \frac{\sigma_{hj}(i)}{\sigma_{hj}}$$

Number of shortest paths between h and j

EXERCISE

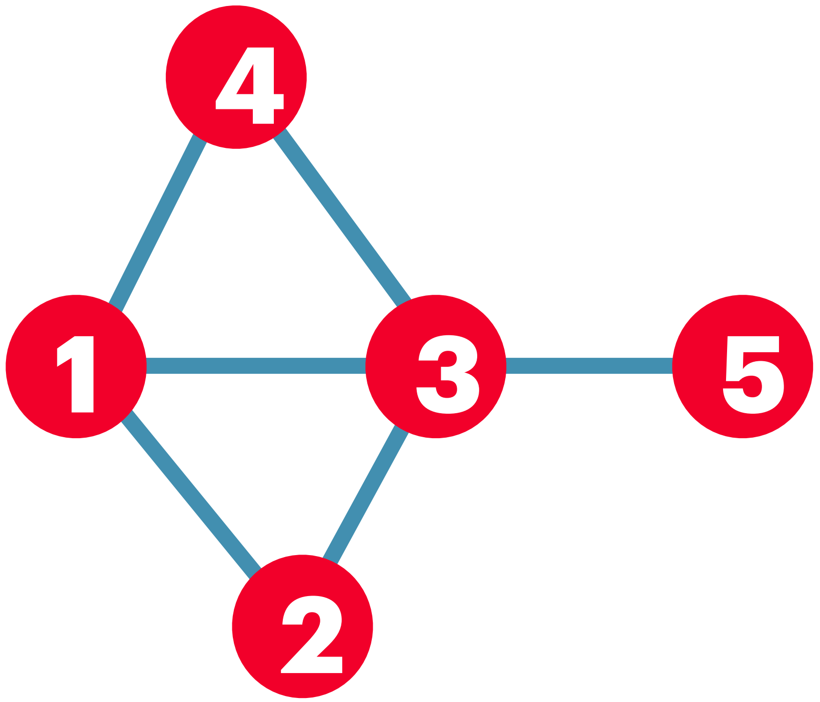
$$k_3 = ?$$



EXERCISE

$$k_3 = 4$$

$$g_3 = ?$$



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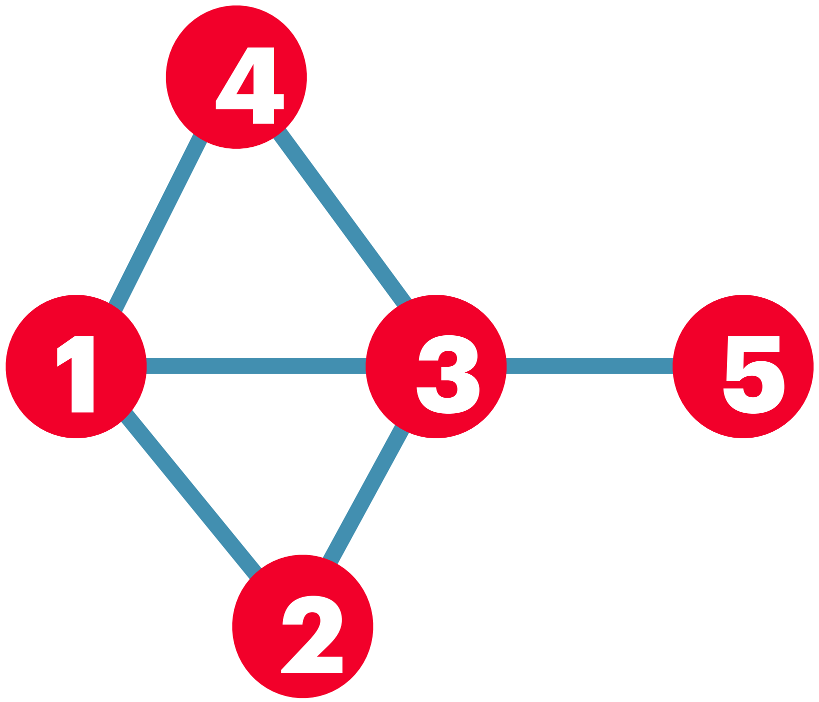


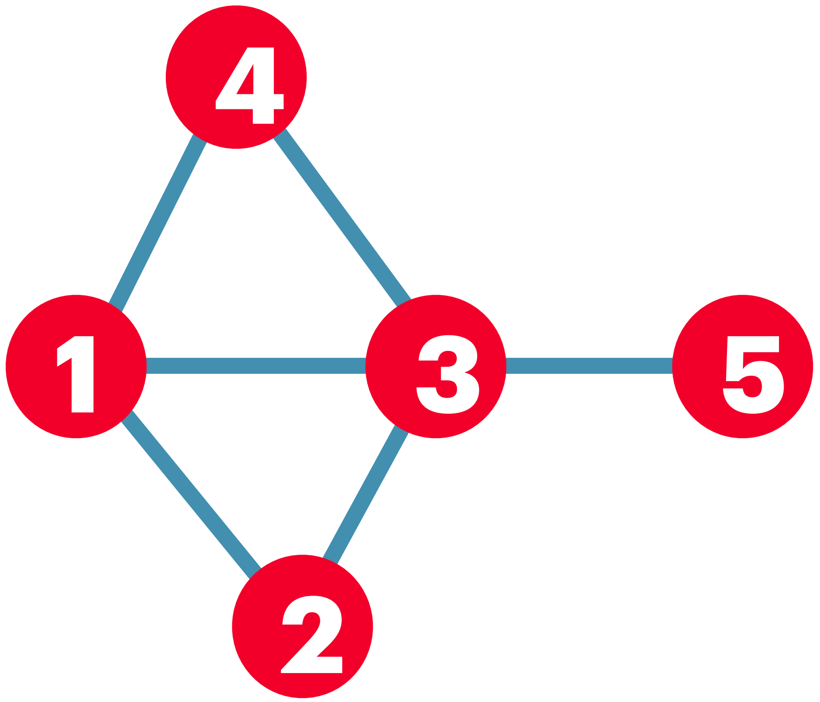
$$g_3 = \frac{1}{\ell_{1,3} + \ell_{2,3} + \ell_{4,3} + \ell_{5,3}} = \frac{1}{1+1+1+1} = \frac{1}{4}$$

EXERCISE

$$k_3 = 4$$

$$g_3 = \frac{1}{4}$$

$$b_3 = ?$$



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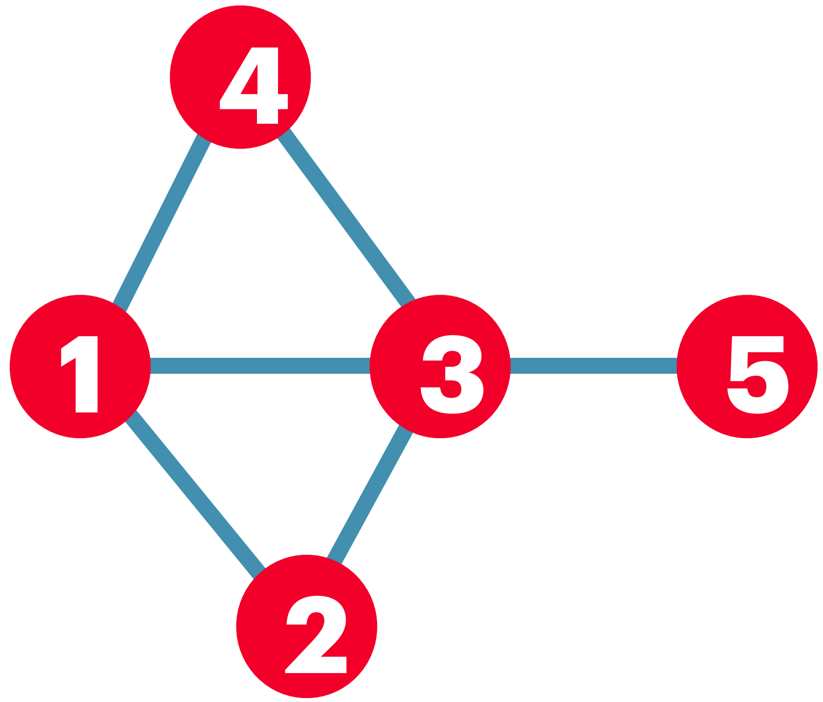

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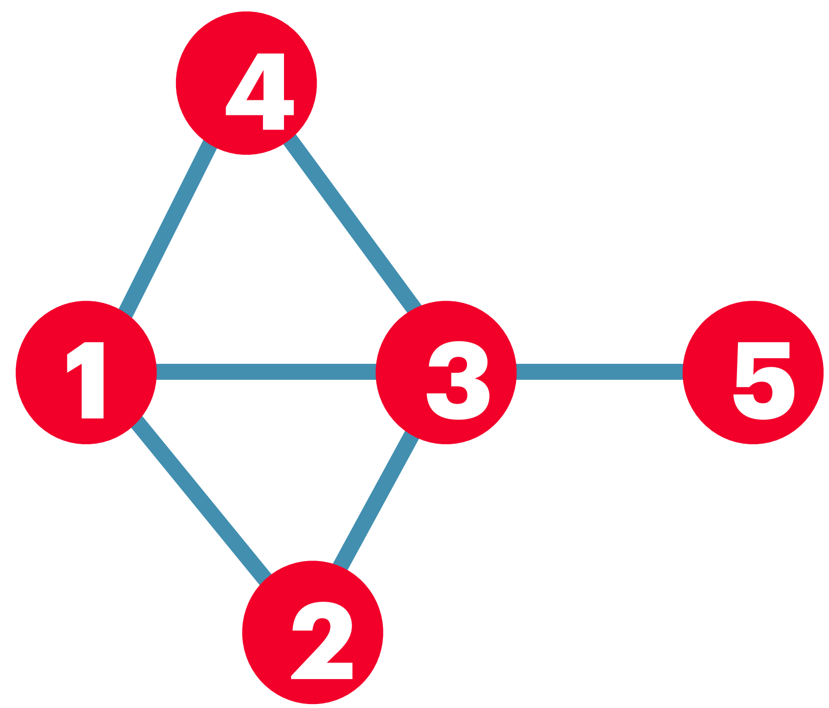

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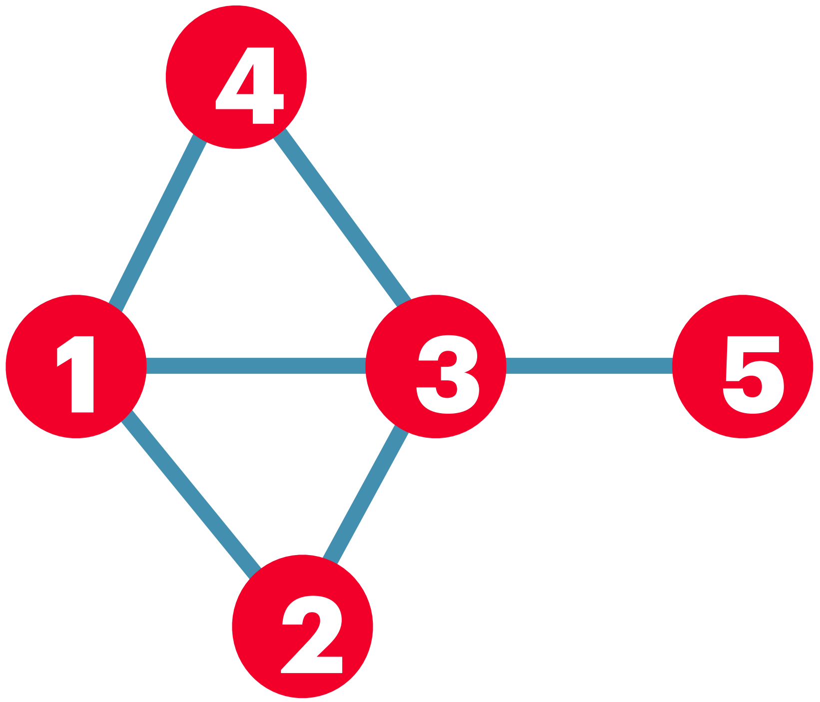
Possible node pairs

1,2   1,3   1,4   1,5
2,3   2,4   2,5
3,4   3,5
4,5

EXERCISE



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We need to exclude some pairs

1,2   1,3   1,4   1,5
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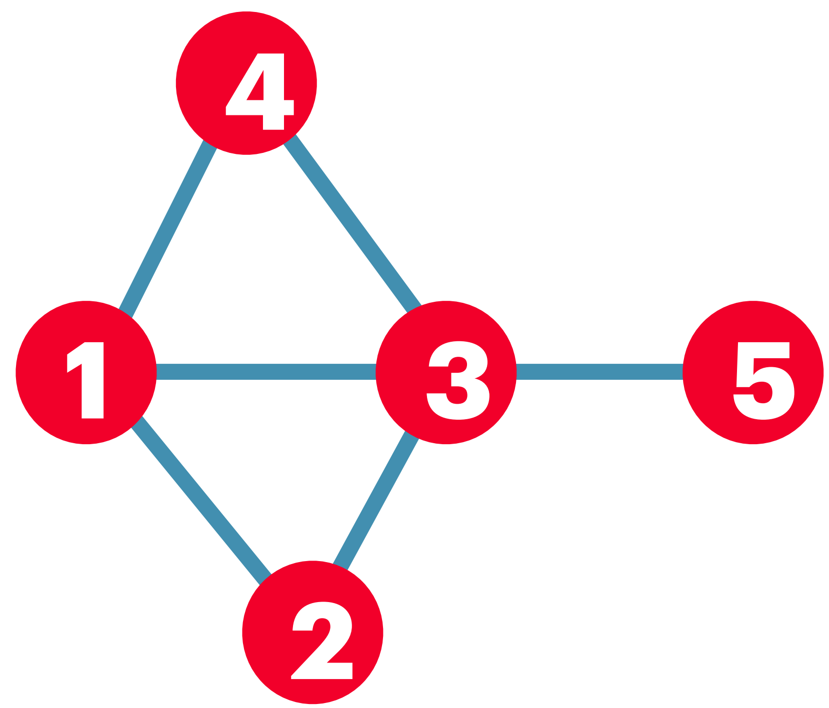
EXERCISE



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1,2 ~~1,3~~ 1,4 1,5

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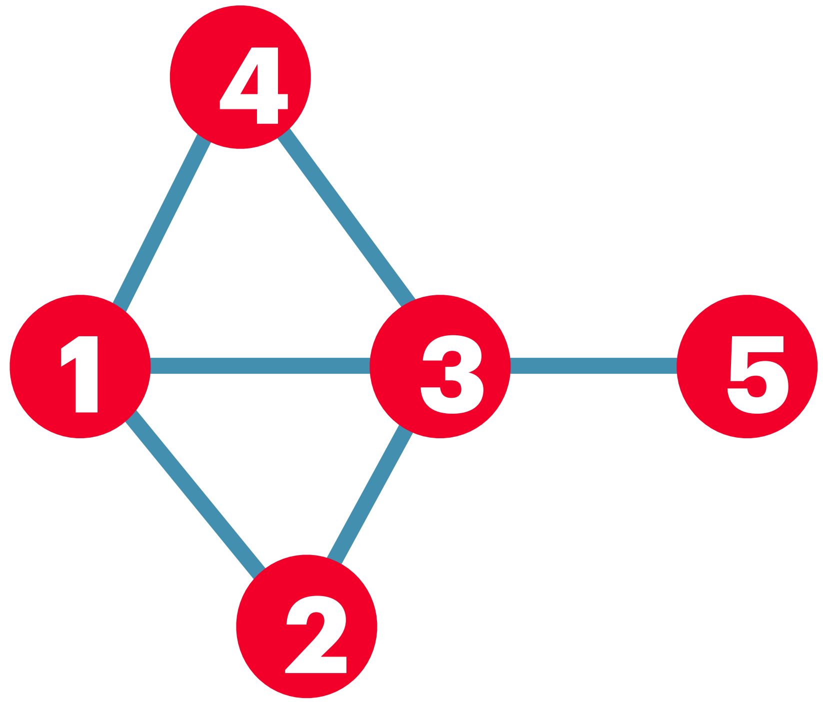
EXERCISE



$$k_3 = 4$$

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No s.p. through i = 3

$$b_i = \sum_{h \neq j \neq i} \frac{\sigma_{hj}(i)}{\sigma_{hj}}$$

1,2 ~~1,3~~ 1,4 1,5

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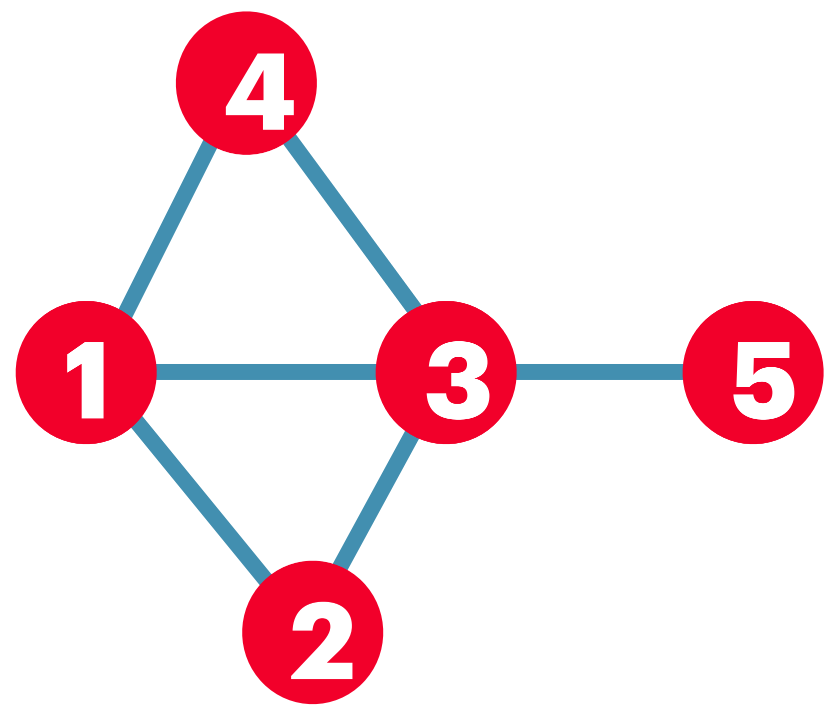
EXERCISE



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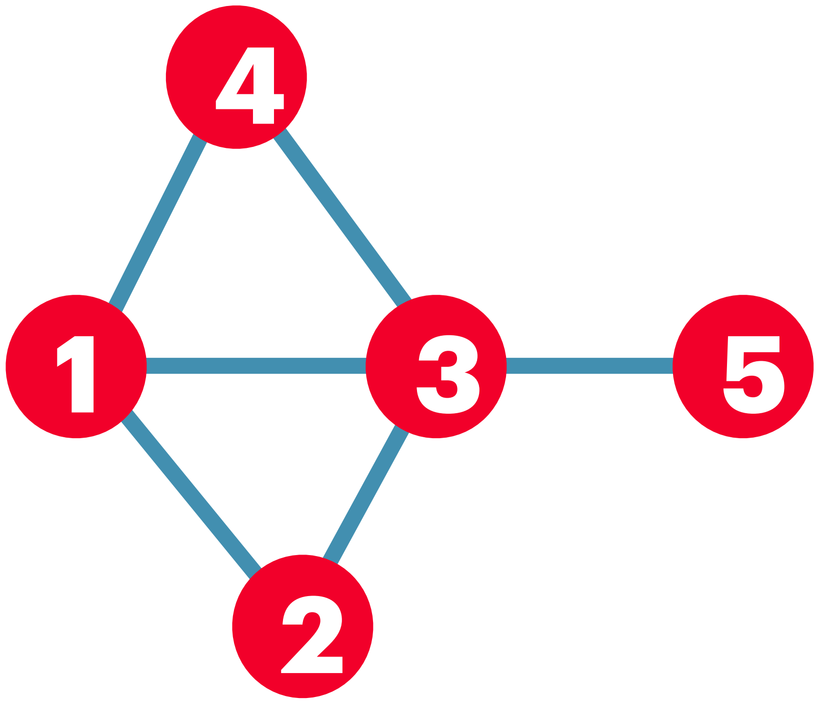
EXERCISE



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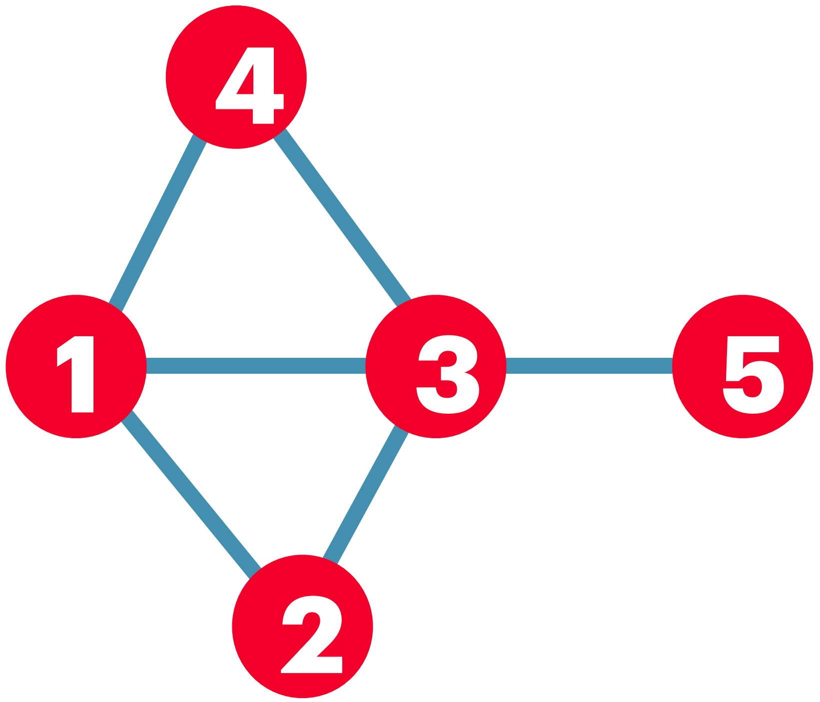
EXERCISE



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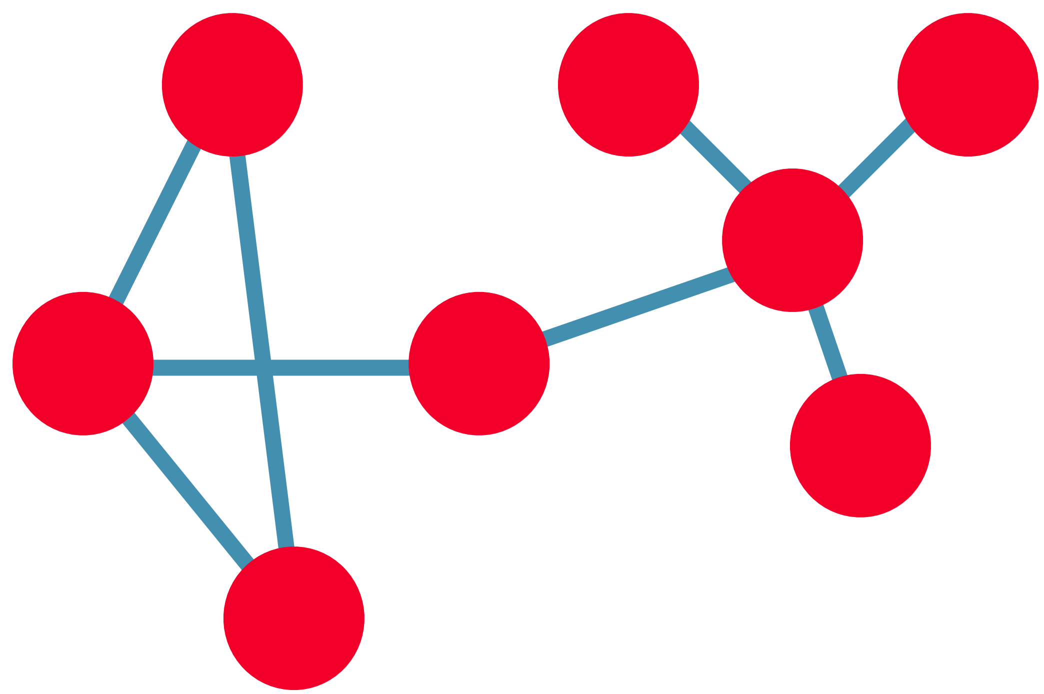
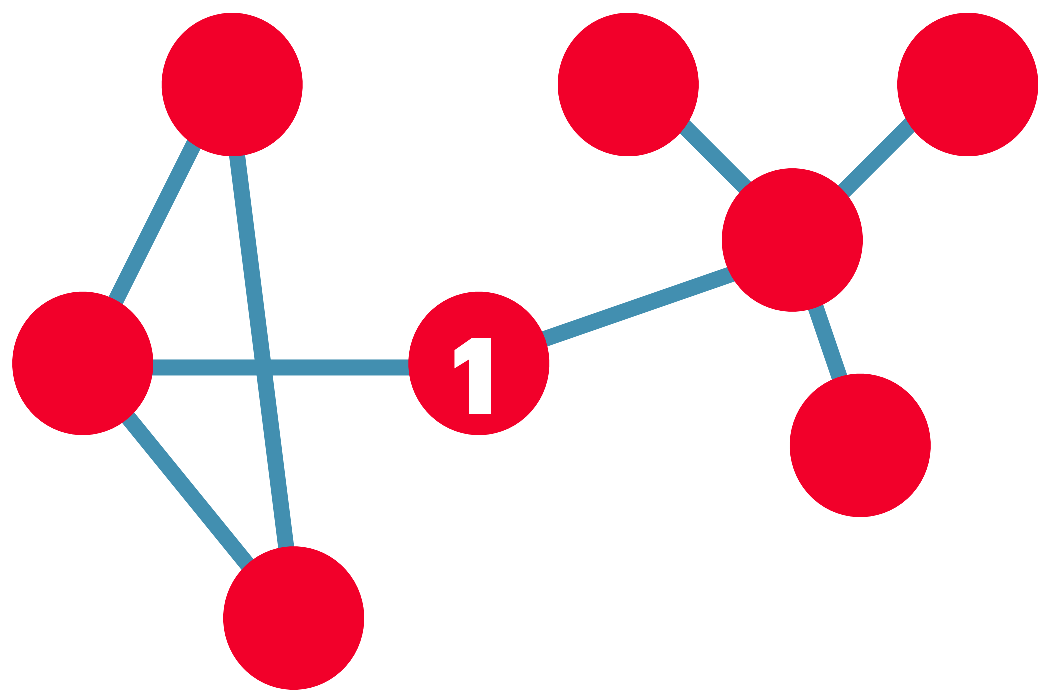
EXERCISE

Node with max betweenness?

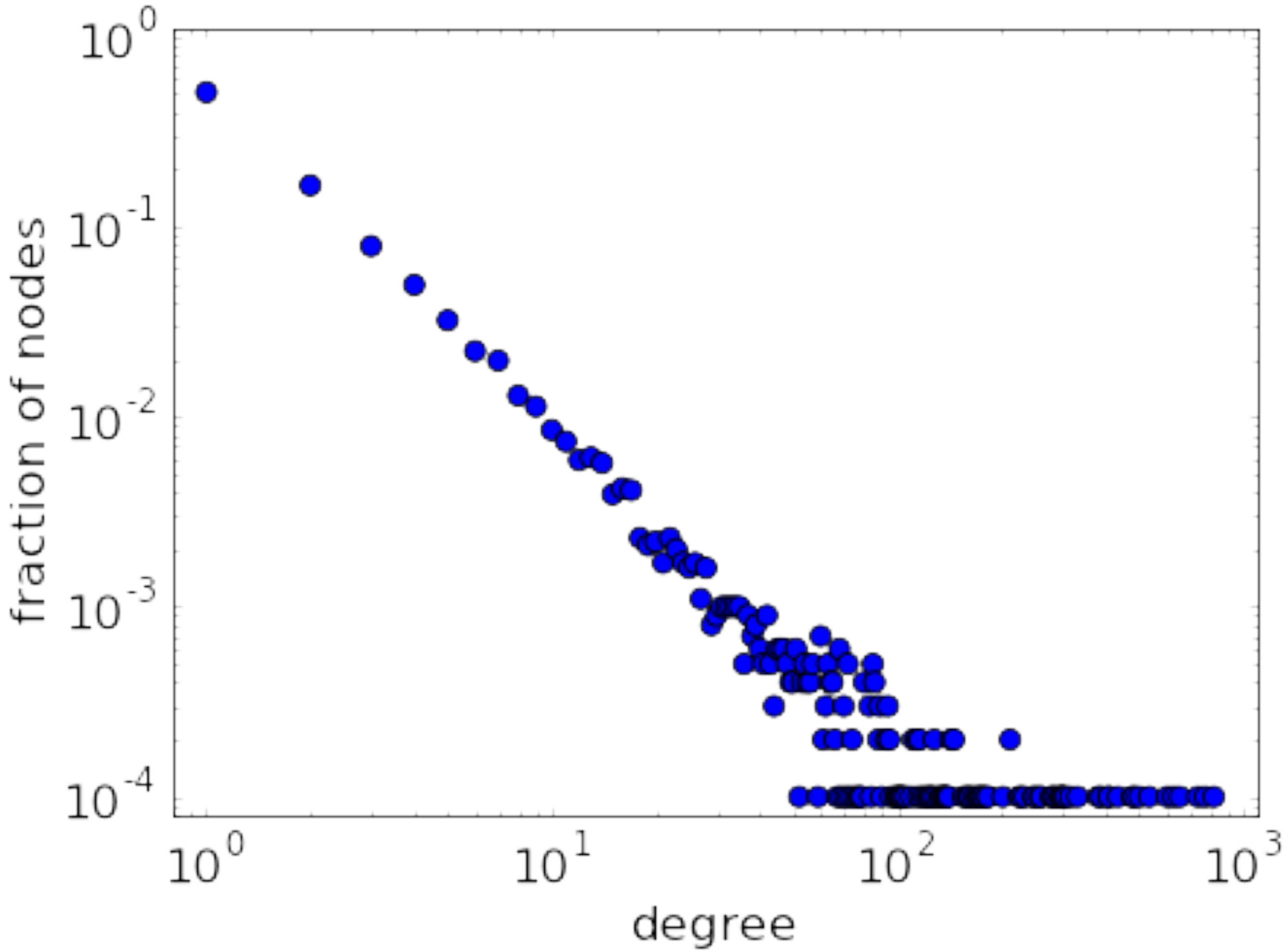


EXERCISE

$b_1 = ?$



DISTRIBUTIONS



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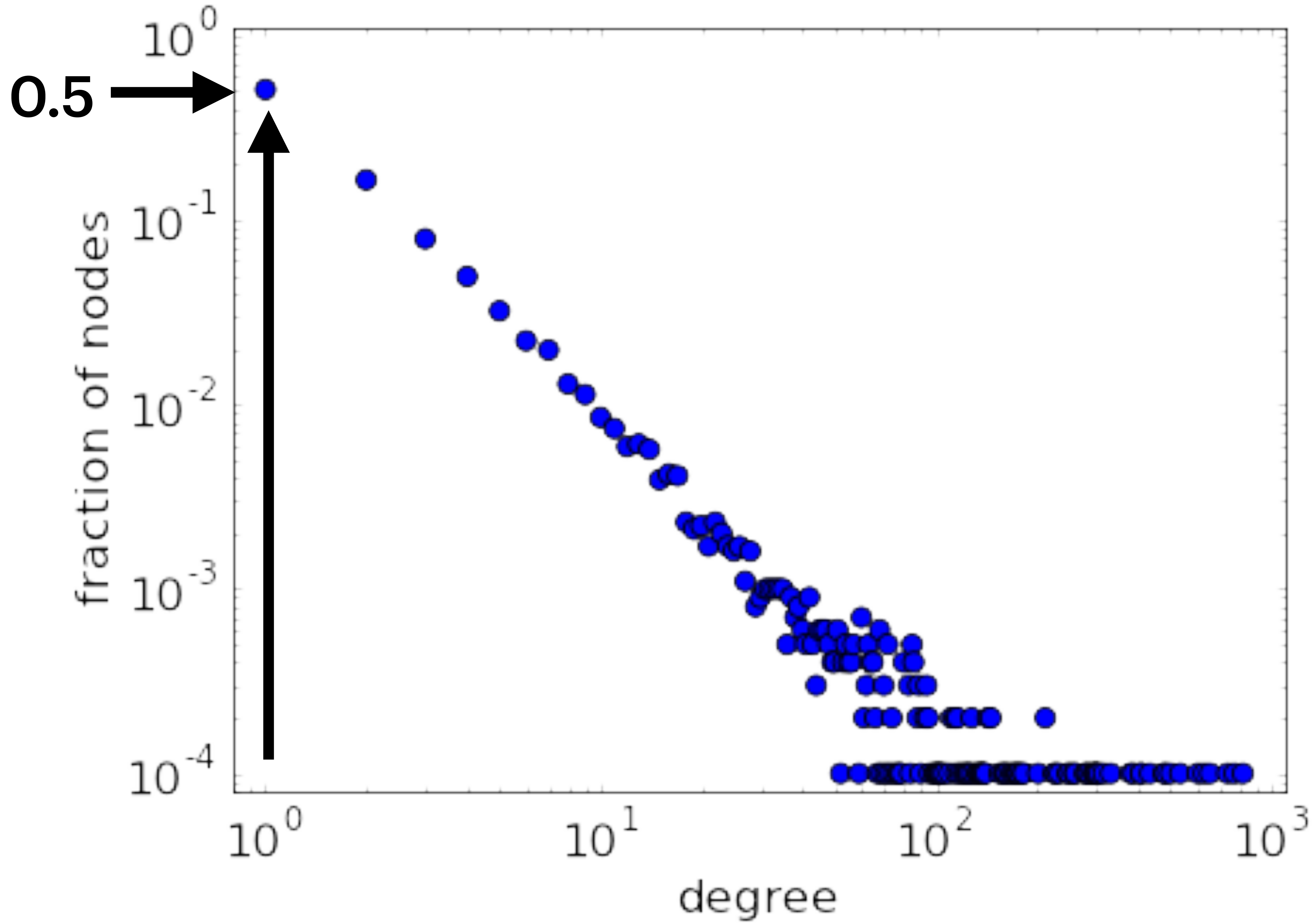

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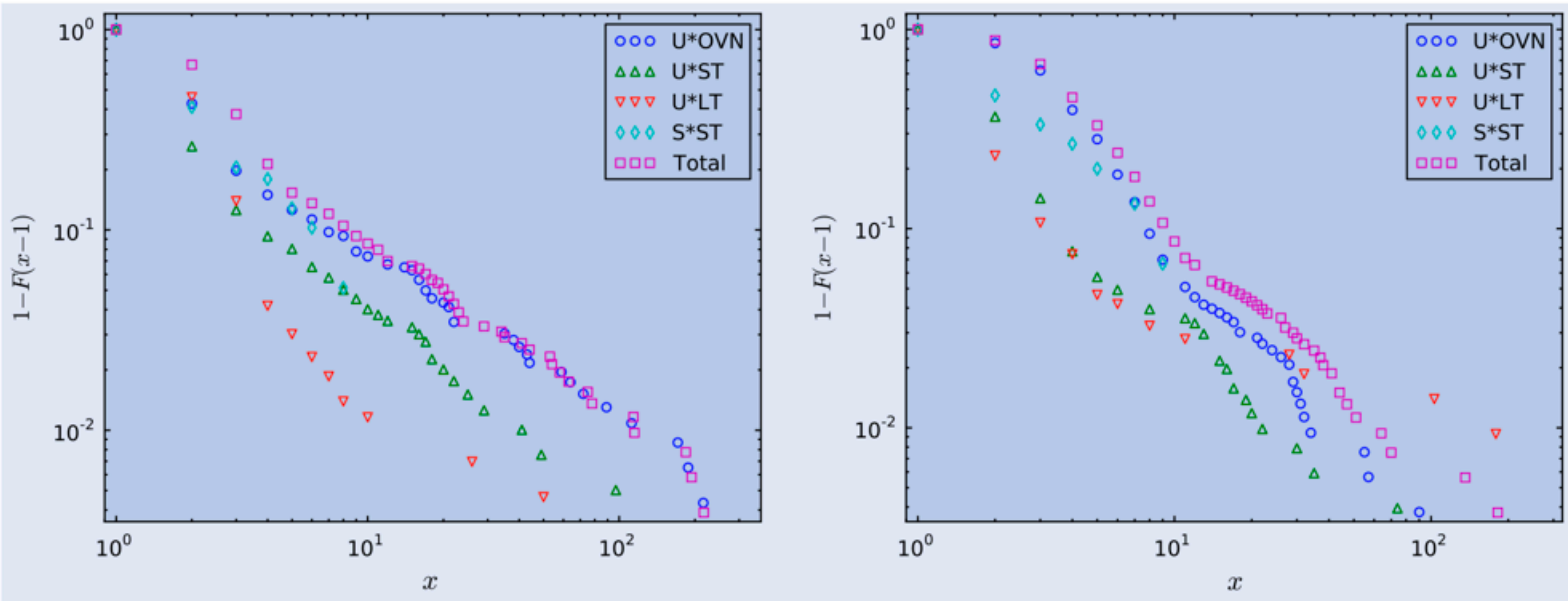
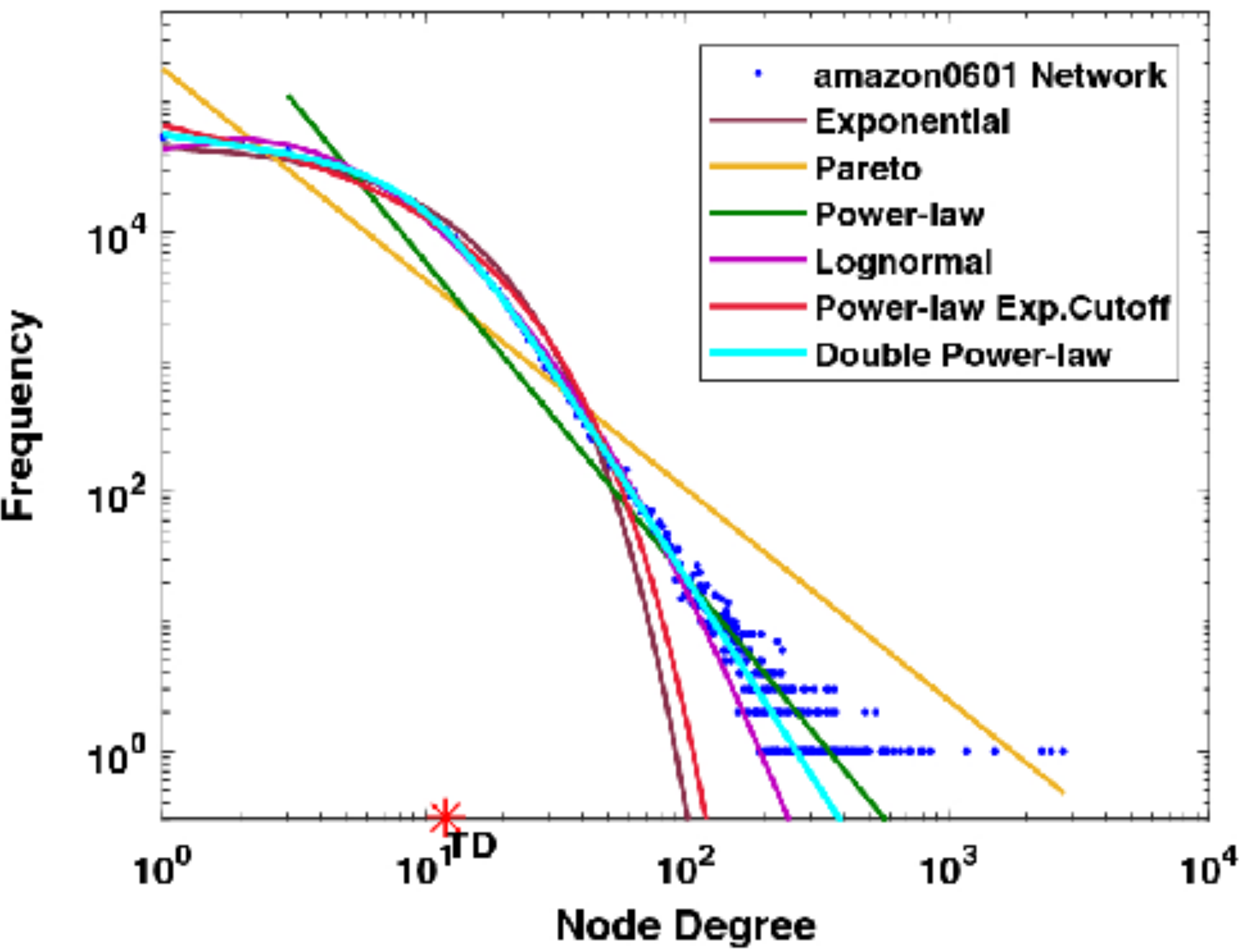
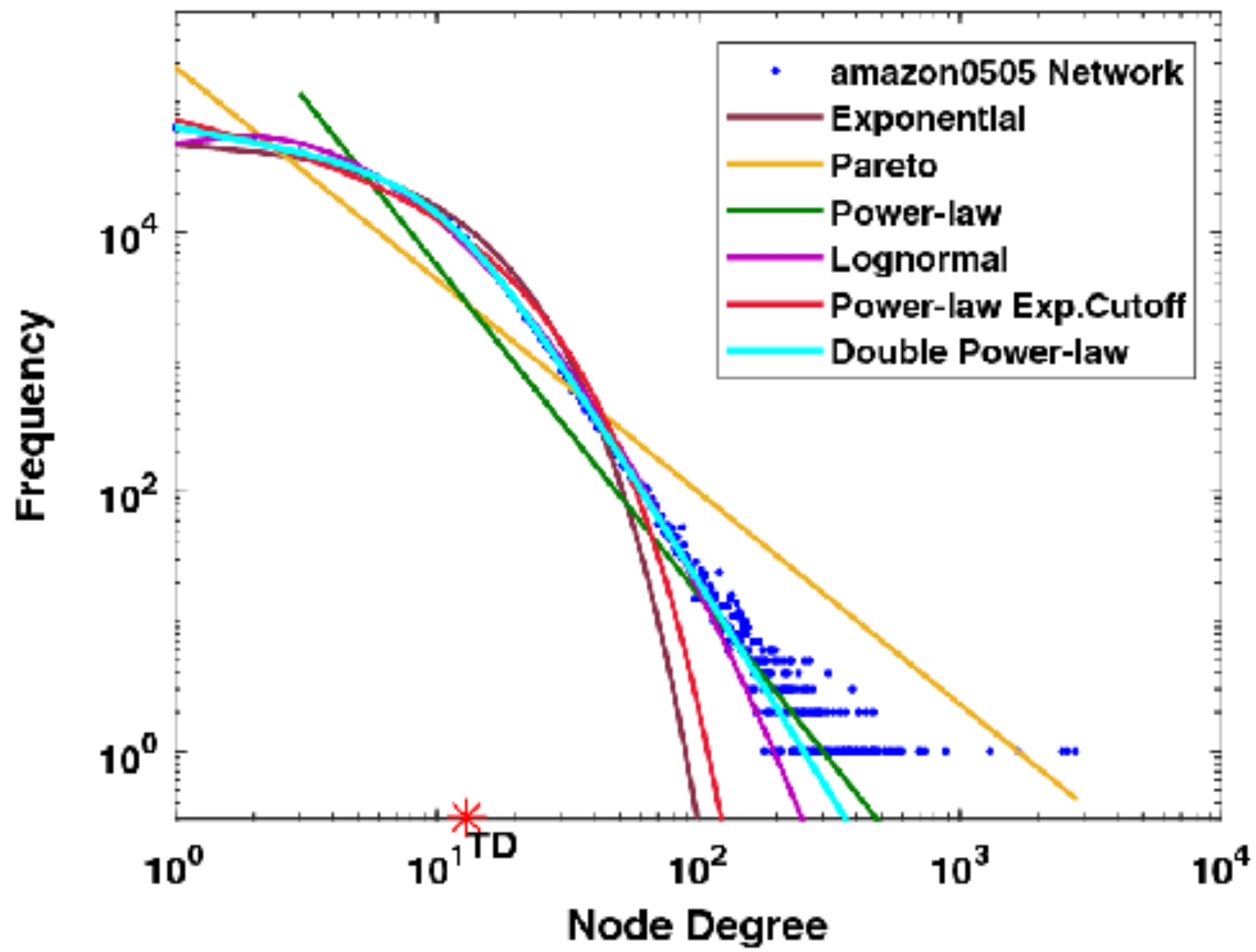


(a) In-degree, 2012

(b) Out-degree, 2012

DISTRIBUTIONS





HOW TO MEASURE HETEROGENEITY

Degree heterogeneity $\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle^2}$

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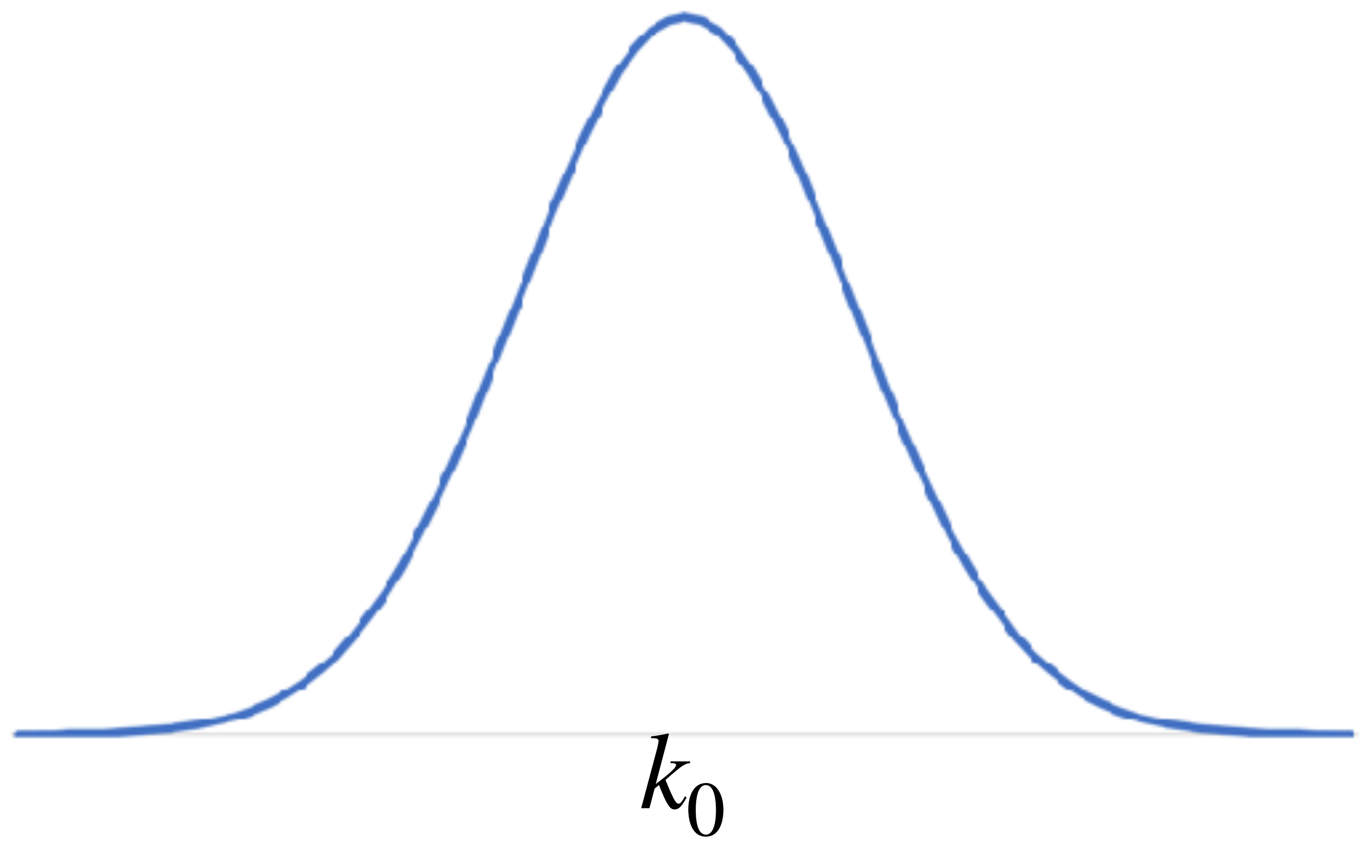
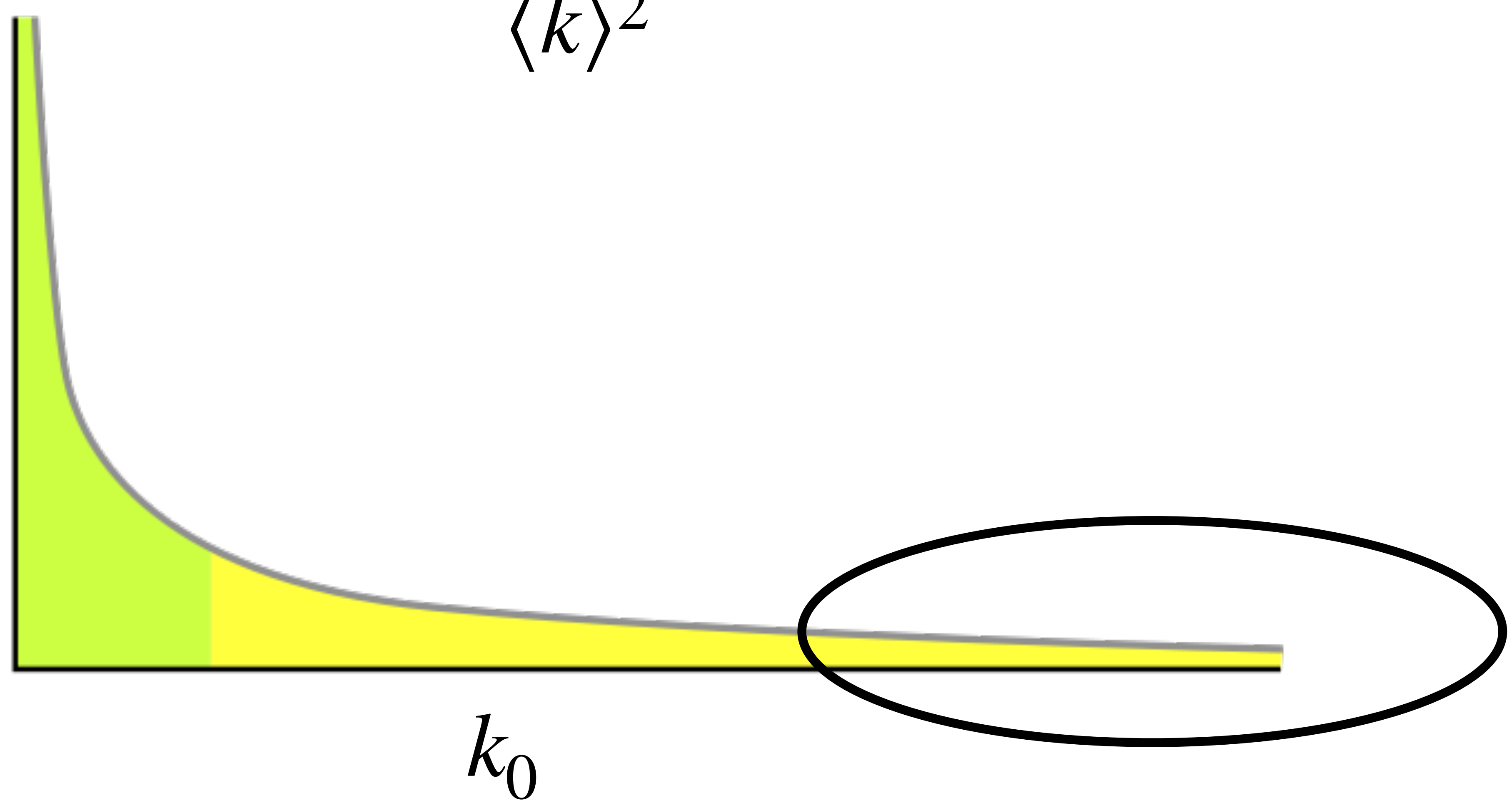
$$\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle^2}$$



If **not** heterogeneous $\langle k^2 \rangle \approx \langle k \rangle^2 \approx k_0^2$ $\kappa \approx 1$

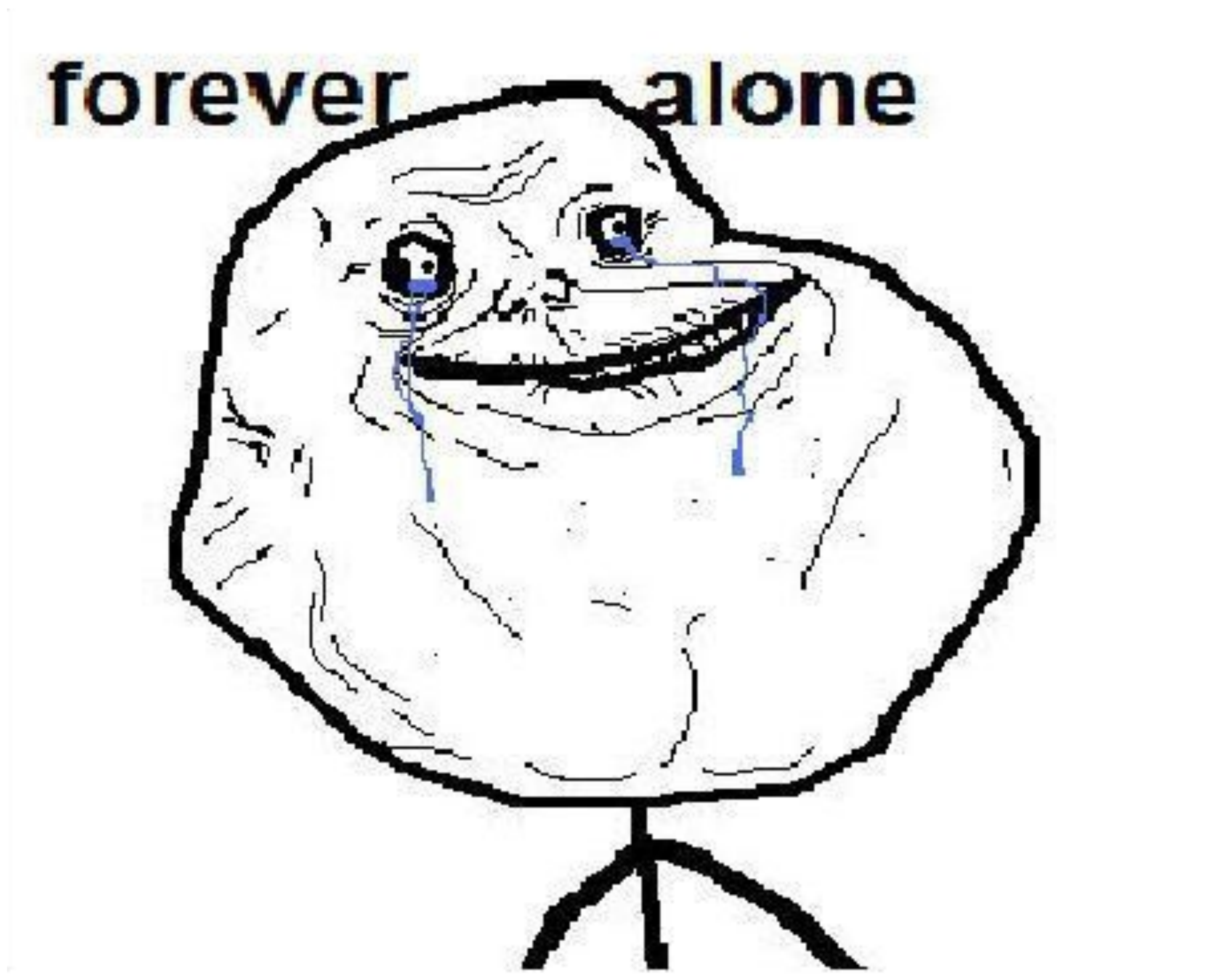
HOW TO MEASURE HETEROGENEITY

$$\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle^2}$$



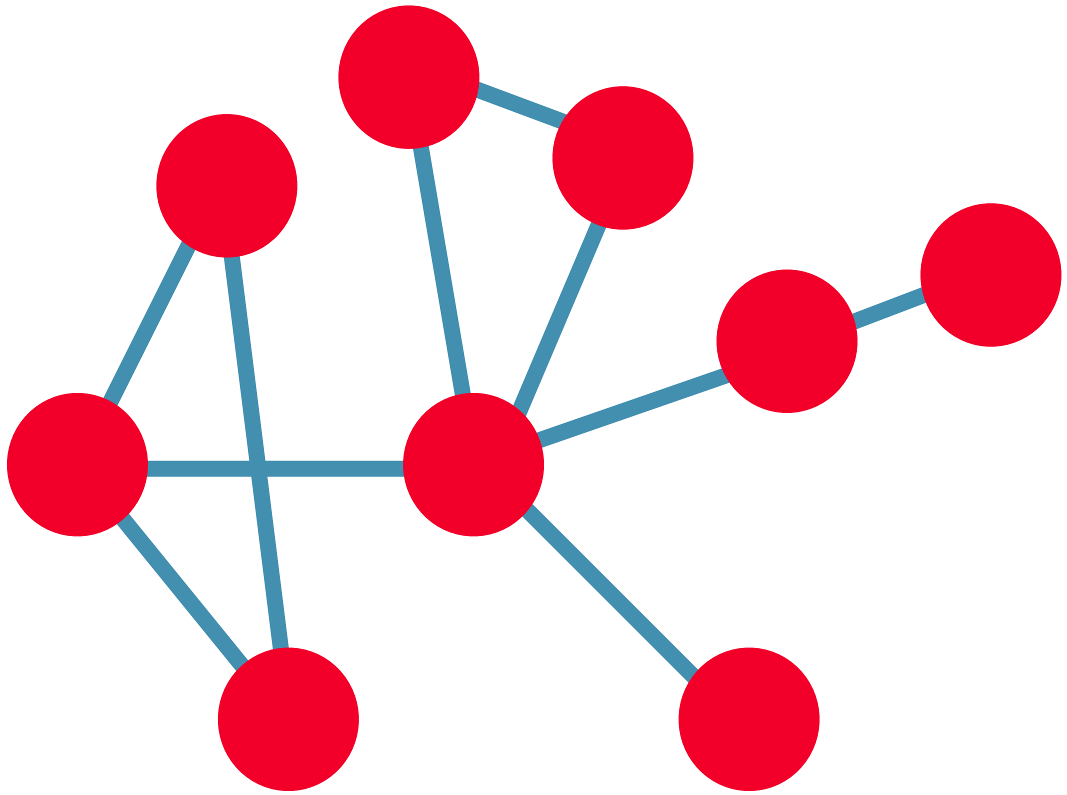
If heterogeneous $\langle k^2 \rangle \gg \langle k \rangle^2$ $\kappa \gg 1$

FRIENDSHIP PARADOX



YOUR FRIENDS HAVE MORE FRIENDS THAN YOU

FRIENDSHIP PARADOX



SUMMARY

Centrality is fundamental to understand the role of nodes

Centrality Distributions represent a great tool to analyse a network

Heterogeneity is a characteristic of **real-world networks**