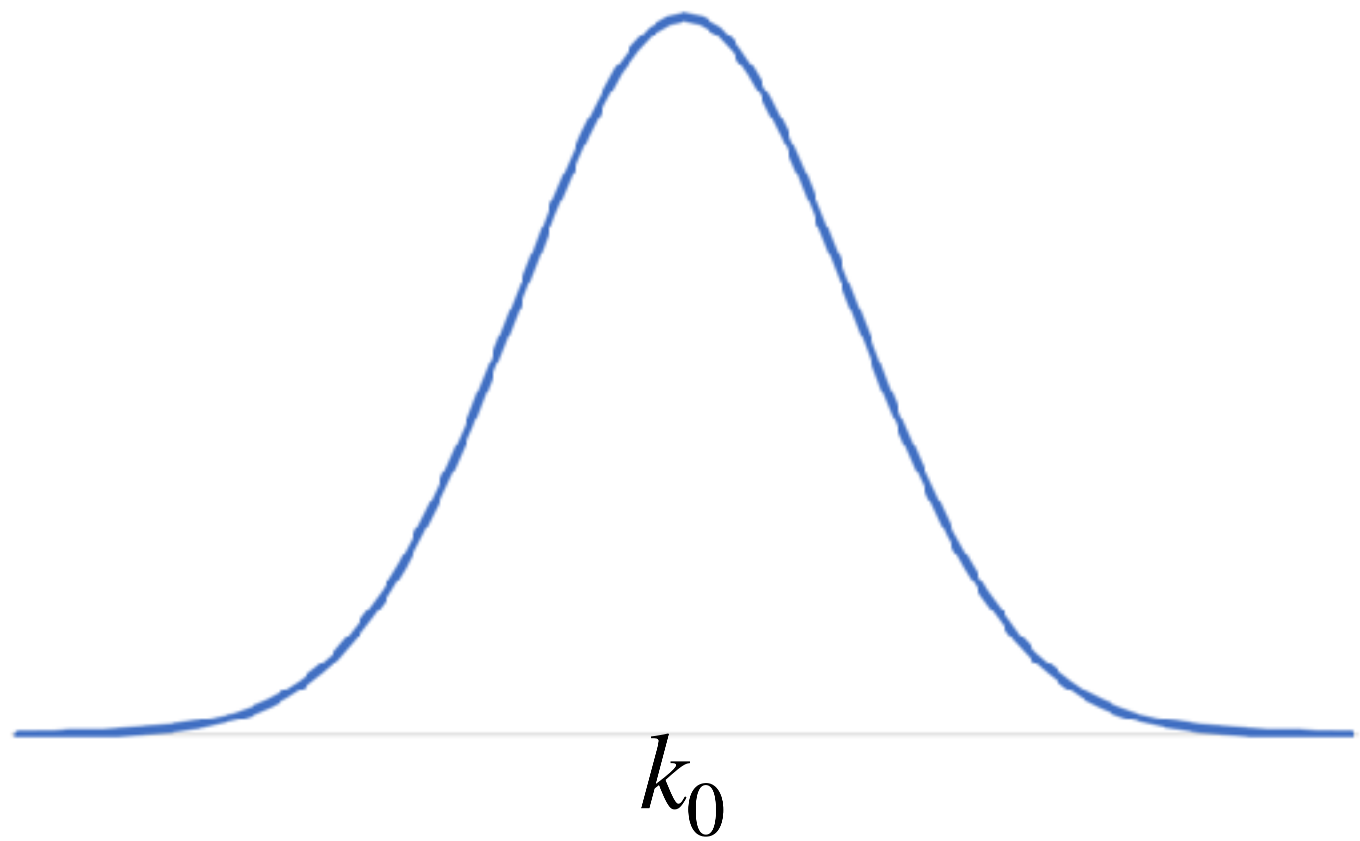
PREVIOUS LECTURE RECAP

HOW TO MEASURE HETEROGENEITY

Degree heterogeneity $\kappa = \dfrac{\langle k^2 \rangle}{\langle k \rangle^2}$

HOW TO MEASURE HETEROGENEITY

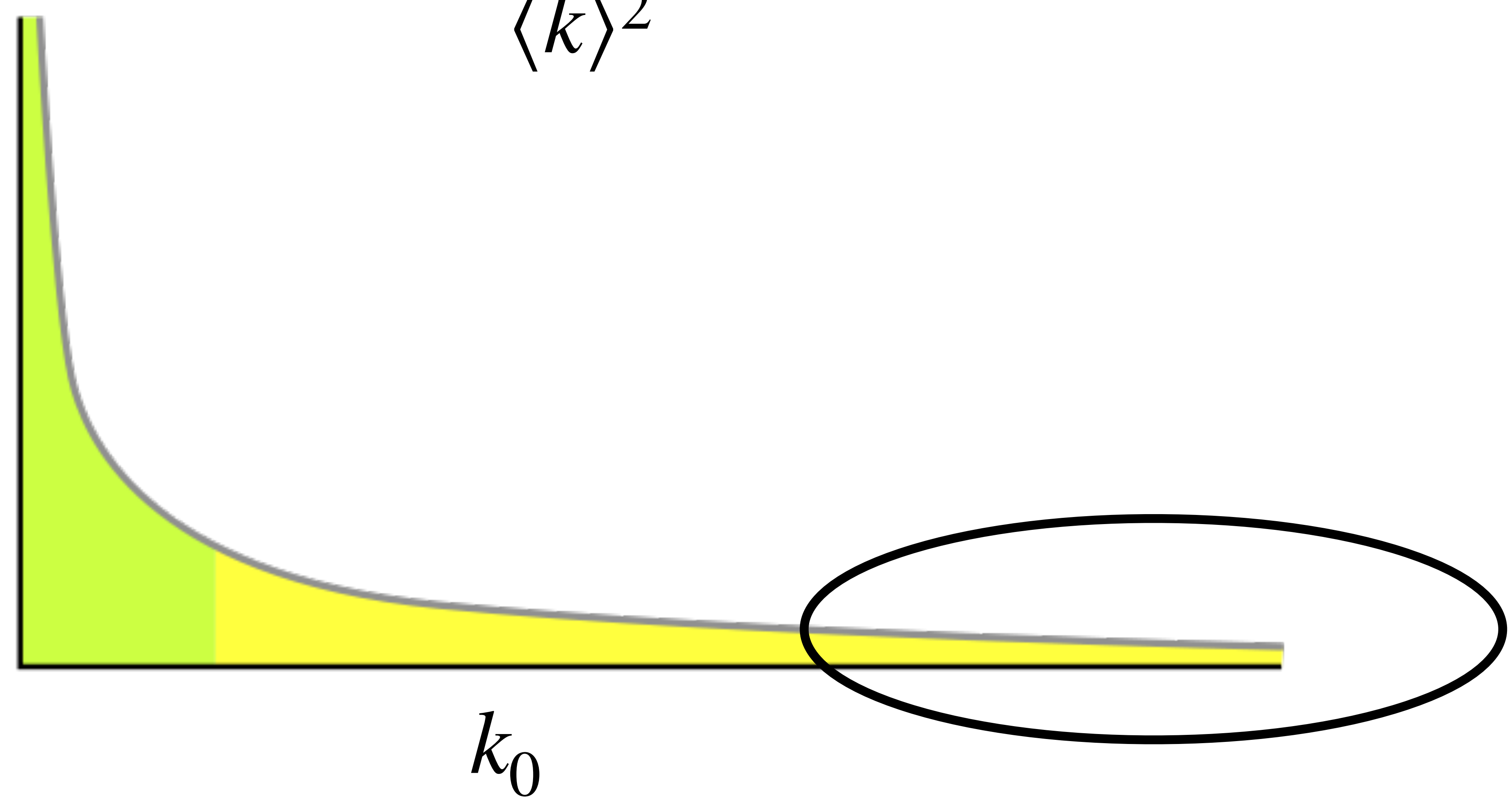
$$\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle^2}$$



If not heterogeneous $\langle k^2 \rangle \approx \langle k \rangle^2 \approx k_0^2$ $\kappa \approx 1$

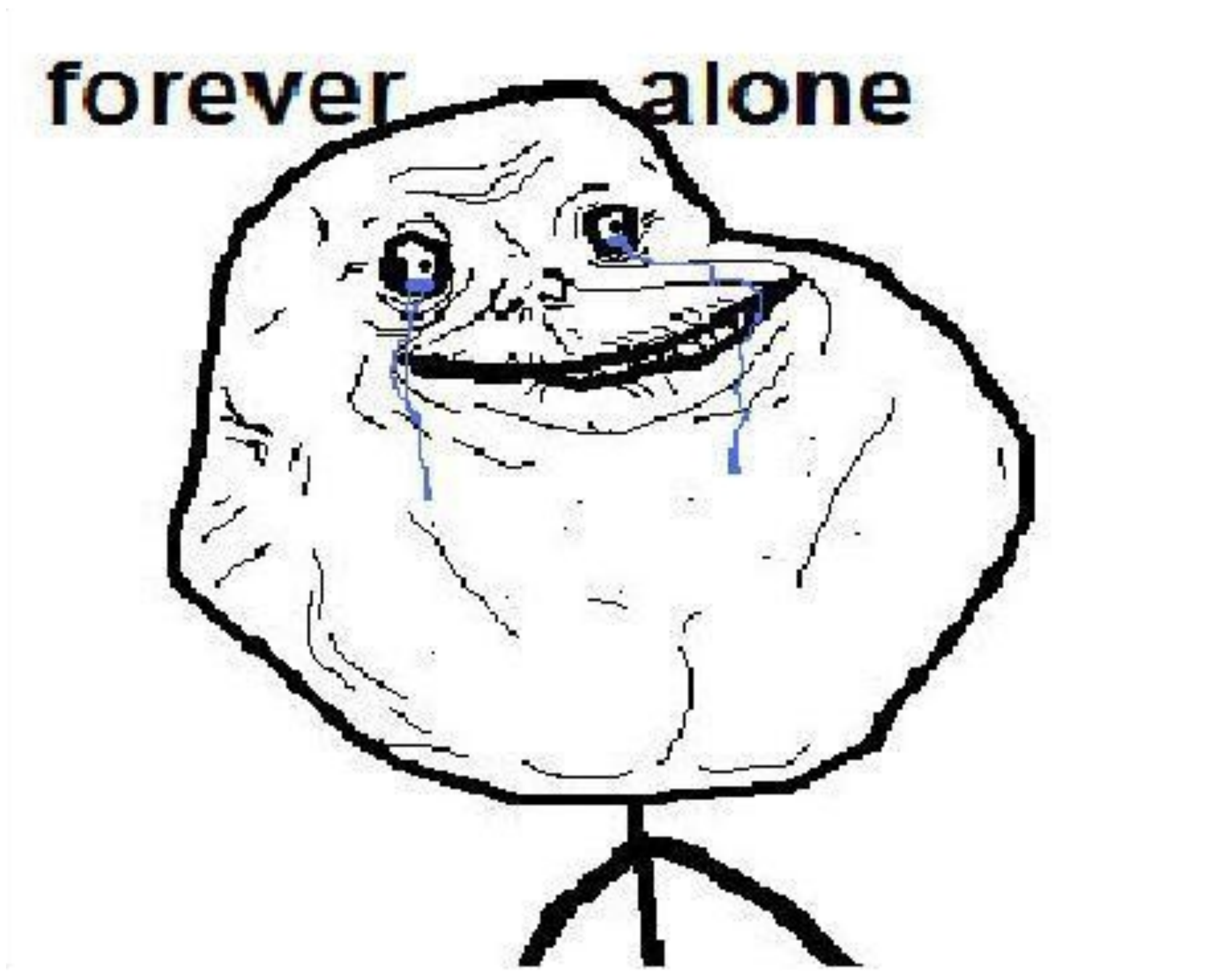
HOW TO MEASURE HETEROGENEITY

$$\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle^2}$$



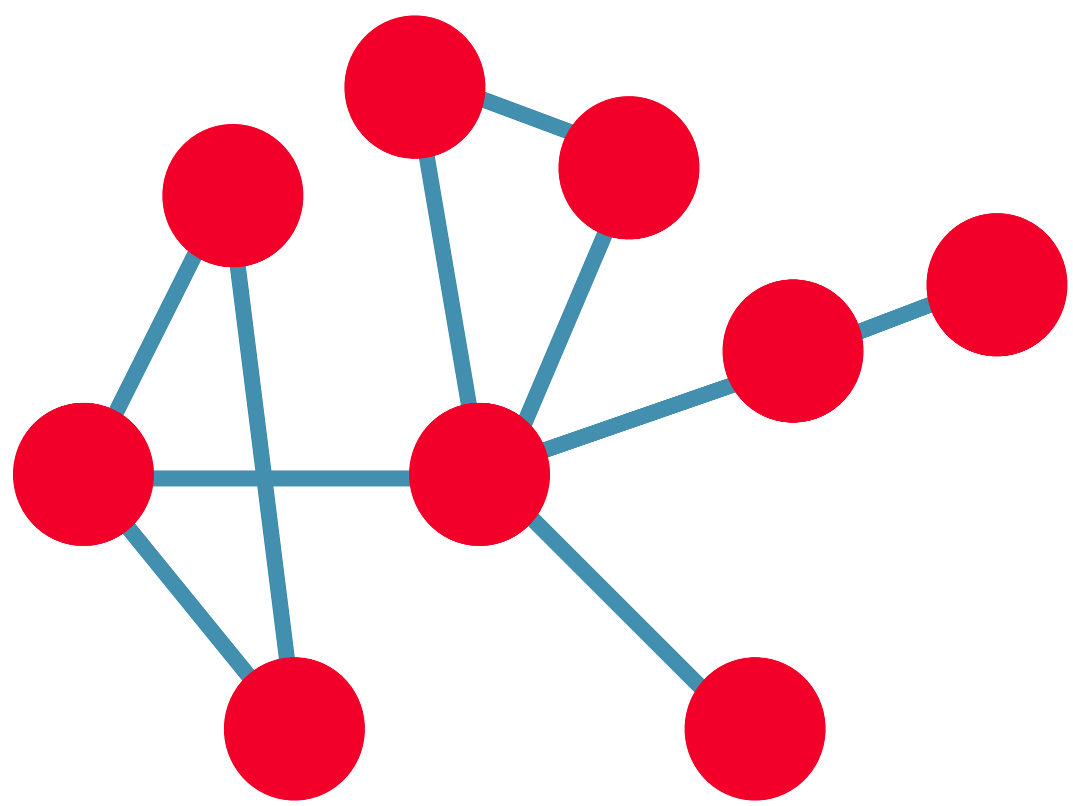
If heterogeneous $\langle k^2 \rangle \gg \langle k \rangle^2$ $\kappa \gg 1$

FRIENDSHIP PARADOX



YOUR FRIENDS HAVE MORE FRIENDS THAN YOU

FRIENDSHIP PARADOX



Trees

LEARNING OUTCOMES

How to build trees

When to build trees

Why to build trees

What is a tree?

Connected, acyclic, graph

It has N nodes and N-1 edges

What is a spanning tree?

It's a subgraph T - that is a tree - of a network G

What is a minimum (maximum) spanning tree?

A spanning tree of a weighted network

For each node, only the edge with minimum (maximum) weight is kept.

Kruskal's algorithm

Finds the minimum spanning forest

Minimum spanning forest = Minimum spanning tree if G is connected

Kruskal's algorithm

1) Create a forest where each node is a tree

Kruskal's algorithm

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2) Sort all edges (in ascending order) by weight

Kruskal’s algorithm

1) Create a forest where each node is a tree

2) Sort all edges (in ascending order) by weight

3) Loop through the edges. For each edge:

Kruskal's algorithm

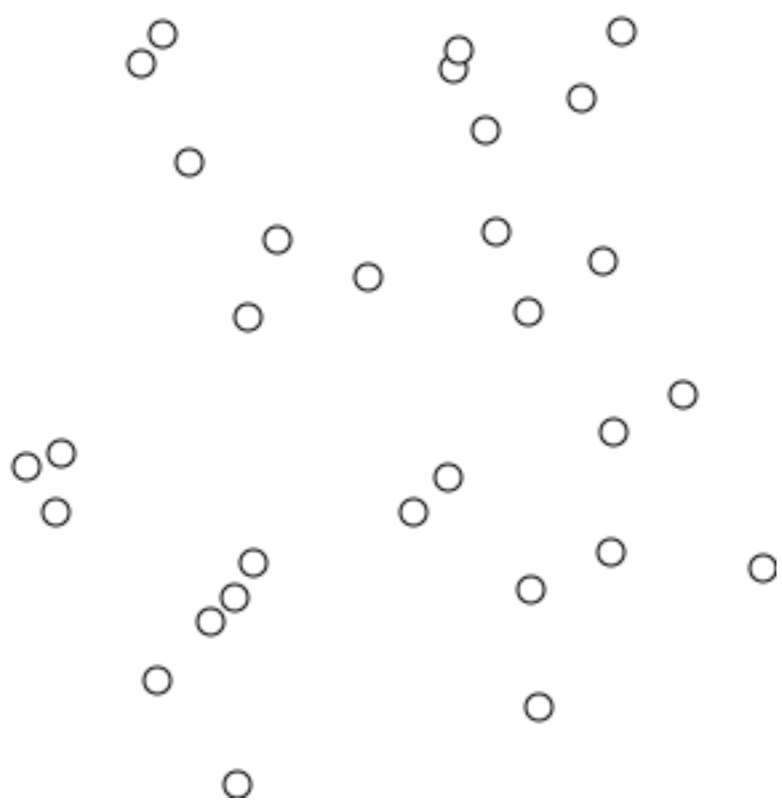
1) Create a forest where each node is a tree

2) Sort all edges (in ascending order) by weight

3) Loop through the edges. For each edge:

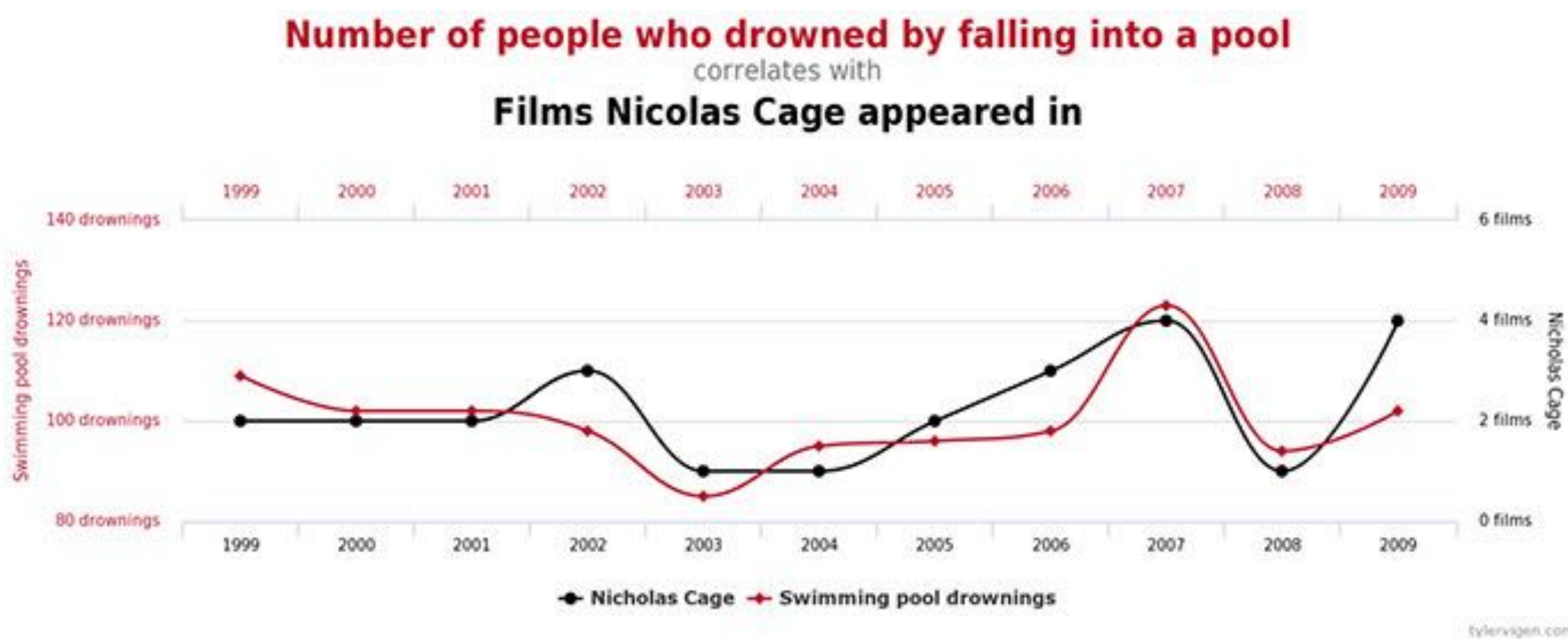
    If adding the edge creates a loop, move on

    Else, add it to the forest



Visualisation from https://en.wikipedia.org/wiki/Kruskal's_algorithm

It all starts with correlations



Spanning tree of correlations

Issues?

Spanning tree of correlations

Negative correlations are missed in maximum spanning trees

Positive correlations are missed in minimum spanning trees

Both are important, what should we do?

Euclidean distance transformation

$$d(i,j) = 1 - \rho(i,j)^2$$

Properties of Euclidean metric:

(i) $d(i,j) = 0$ if and only if $i = j$

(ii) $d(i,j) = d(j,i)$

(iii) $d(i,j) \leq d(i,k) + d(k,j)$

R. MANTEGNA **"Information and hierarchical structure in financial markets"** https://doi.org/10.1016/S0010-4655(99)00302-1

$$d(i, j) = 1 - \rho(i, j)^2$$

d(**LOW DISTANCE = HIGH CORRELATION**

$$d(i, j) = d(j, i)$$

$$d(i, j) \leq d(i, k) + d(k, j)$$

Euclidean distance transformation

Other proposed metrics

$$d(i,j) = \sqrt{2(1 - \rho(i,j))}$$

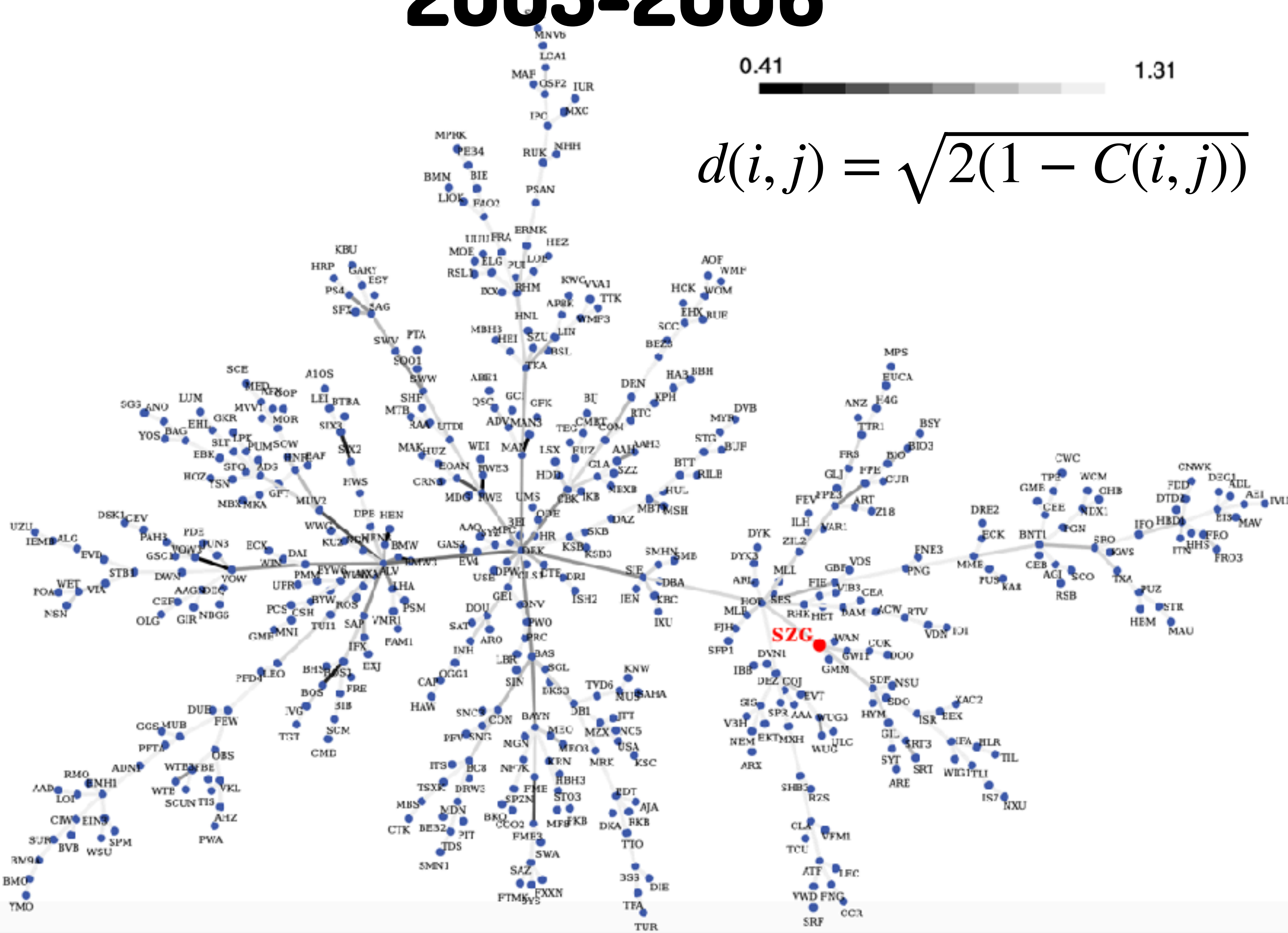
$$d(i,j) = 1 - |\rho(i,j)|$$

CASE STUDY I - HOW TO BECOME MILLIONAIRES WITH NETWORKS

- THIS PAPER STUDIES THE TOPOLOGICAL FEATURES OF THE **CORRELATION NETWORK** OF THE FRANKFURT STOCK EXCHANGE (FSE)
- THE AUTHORS SHOW THAT THERE ARE **PHASE TRANSITIONS**
- **BEFORE AND AFTER** THE 2008 **CRISIS**

M. WILINSKI ET AL. **"STRUCTURAL AND TOPOLOGICAL PHASE TRANSITIONS ON THE GERMAN EXCHANGE STOCK MARKET"** https://arxiv.org/pdf/1301.2530.pdf

2005-2006



$$d(i,j) = \sqrt{2(1 - C(i,j))}$$

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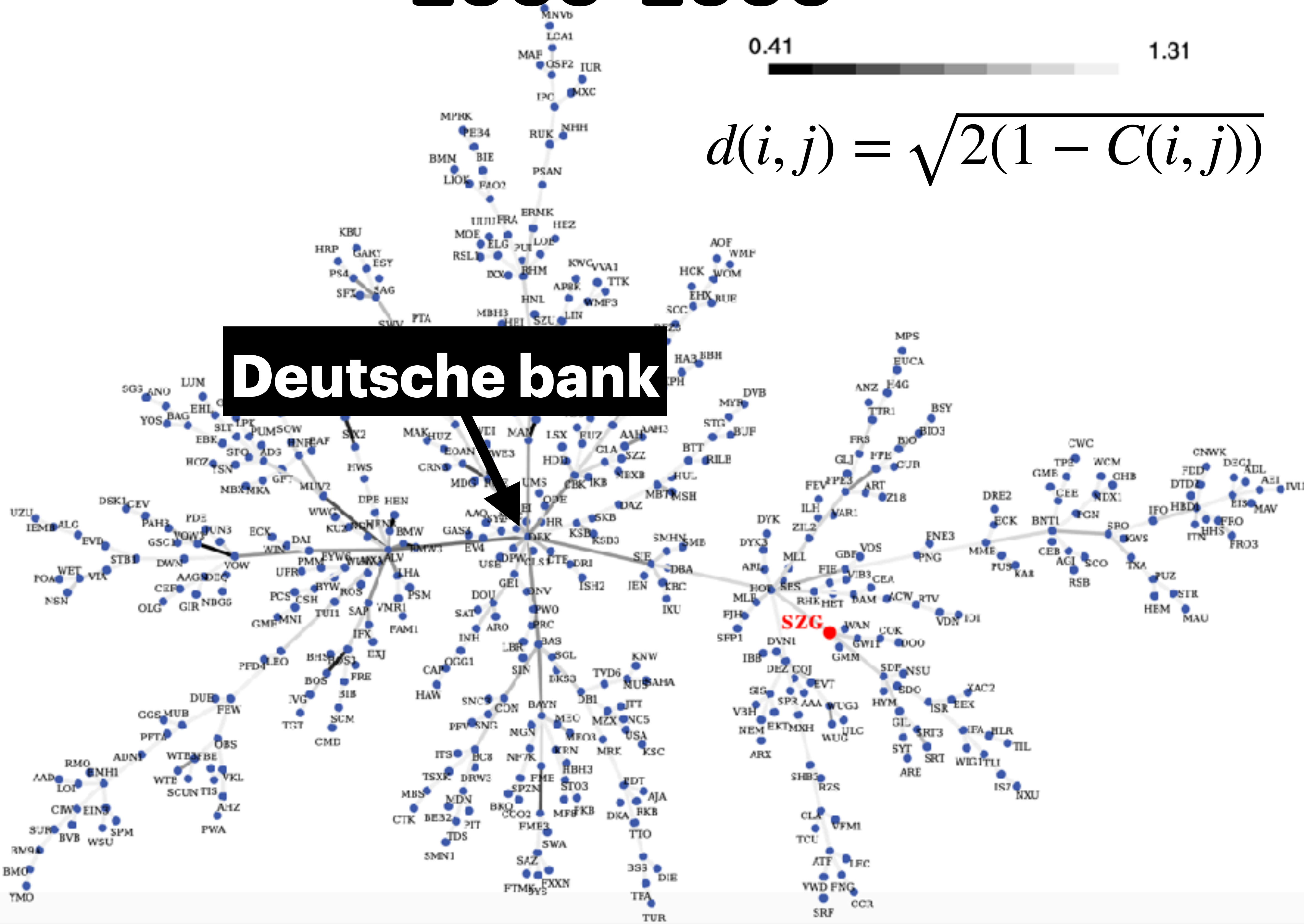

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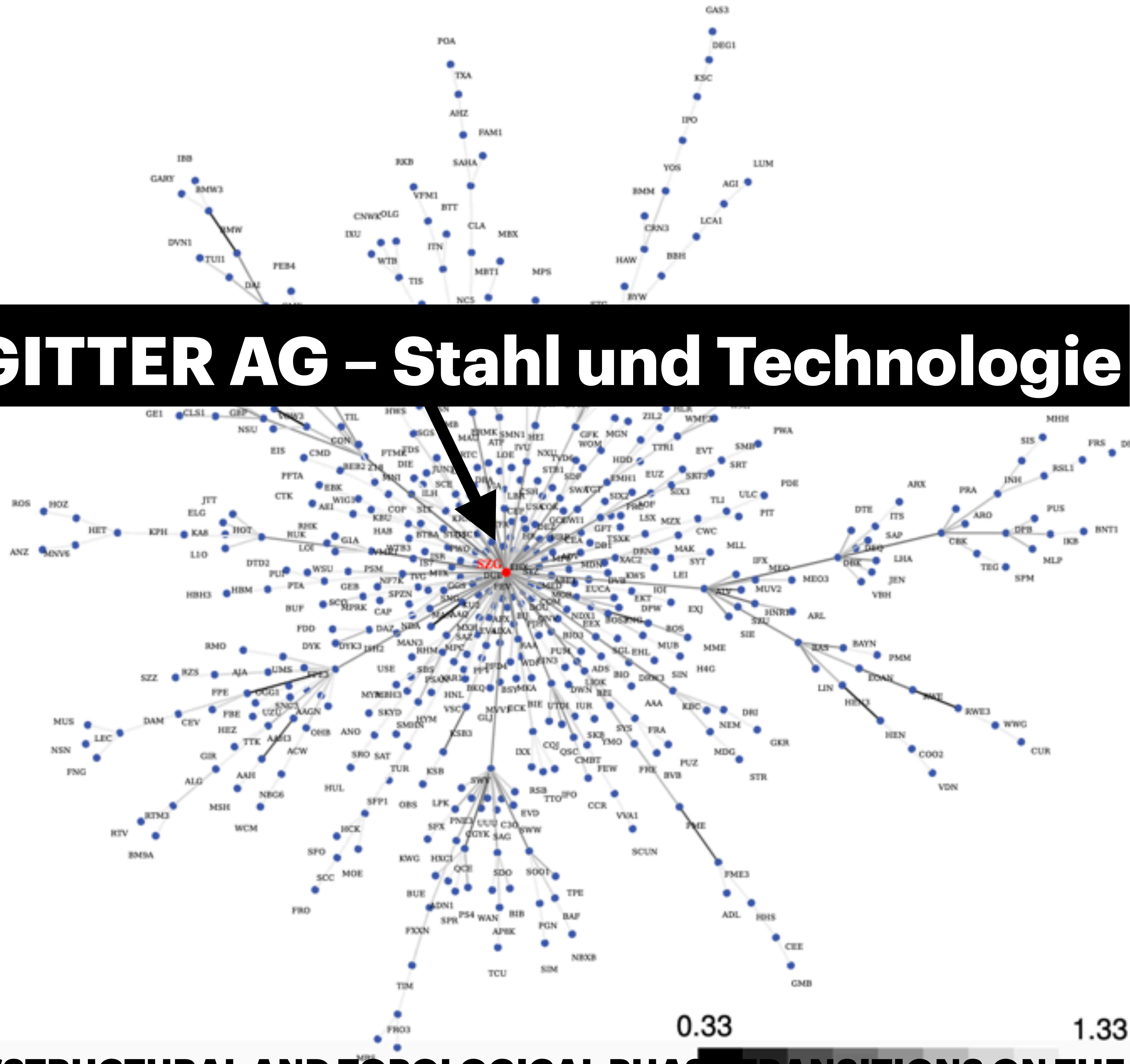

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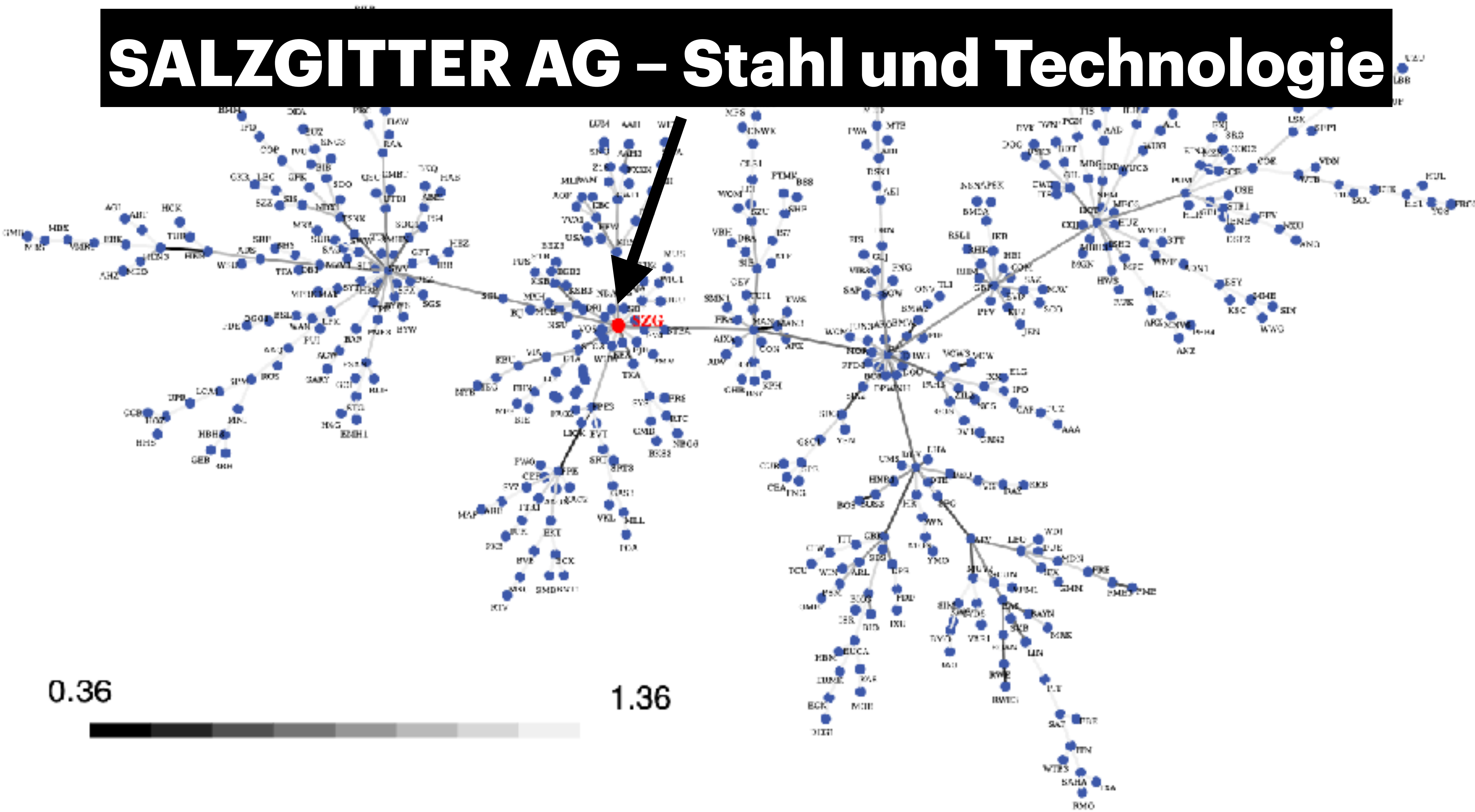
2007-2008



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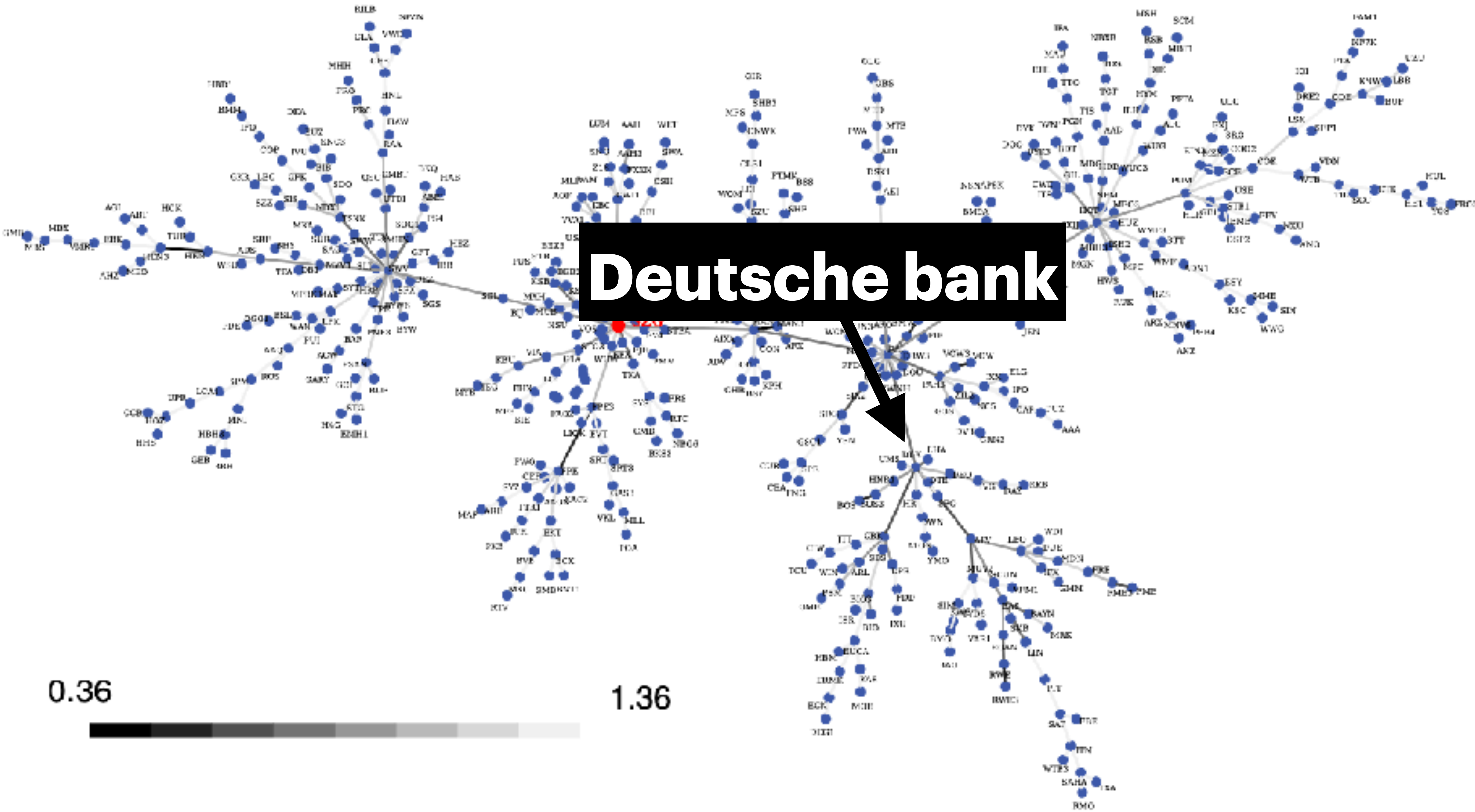

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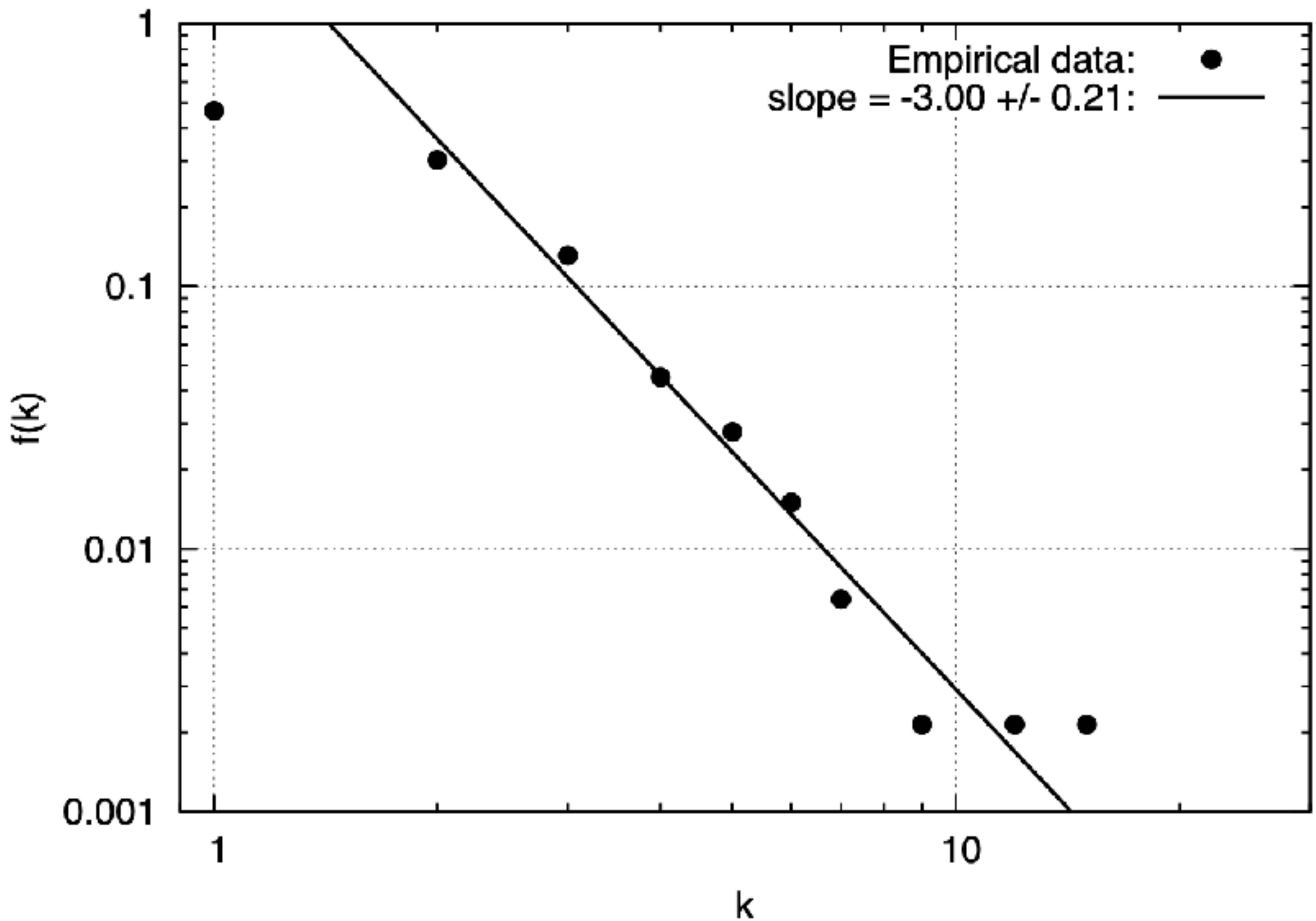

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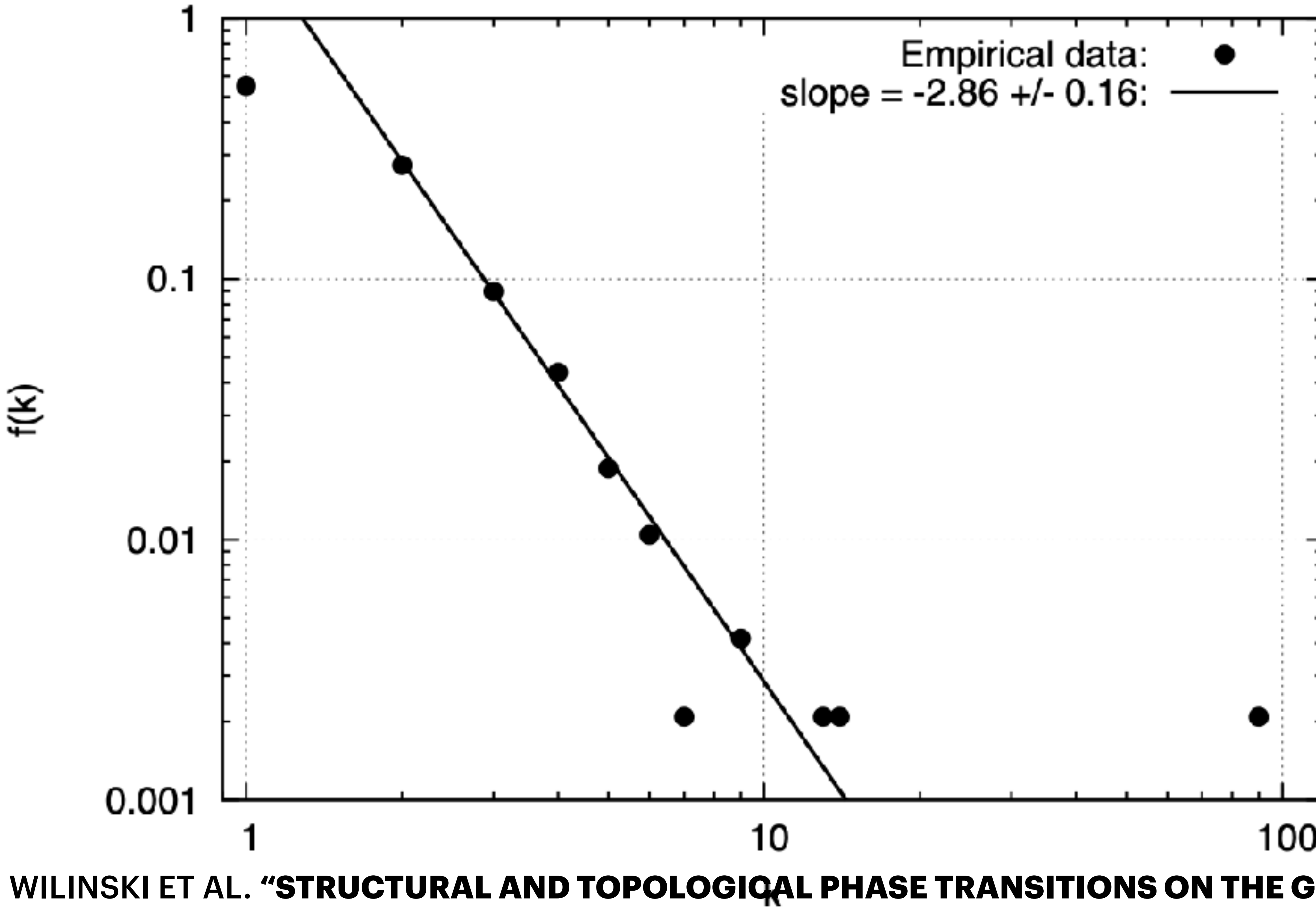

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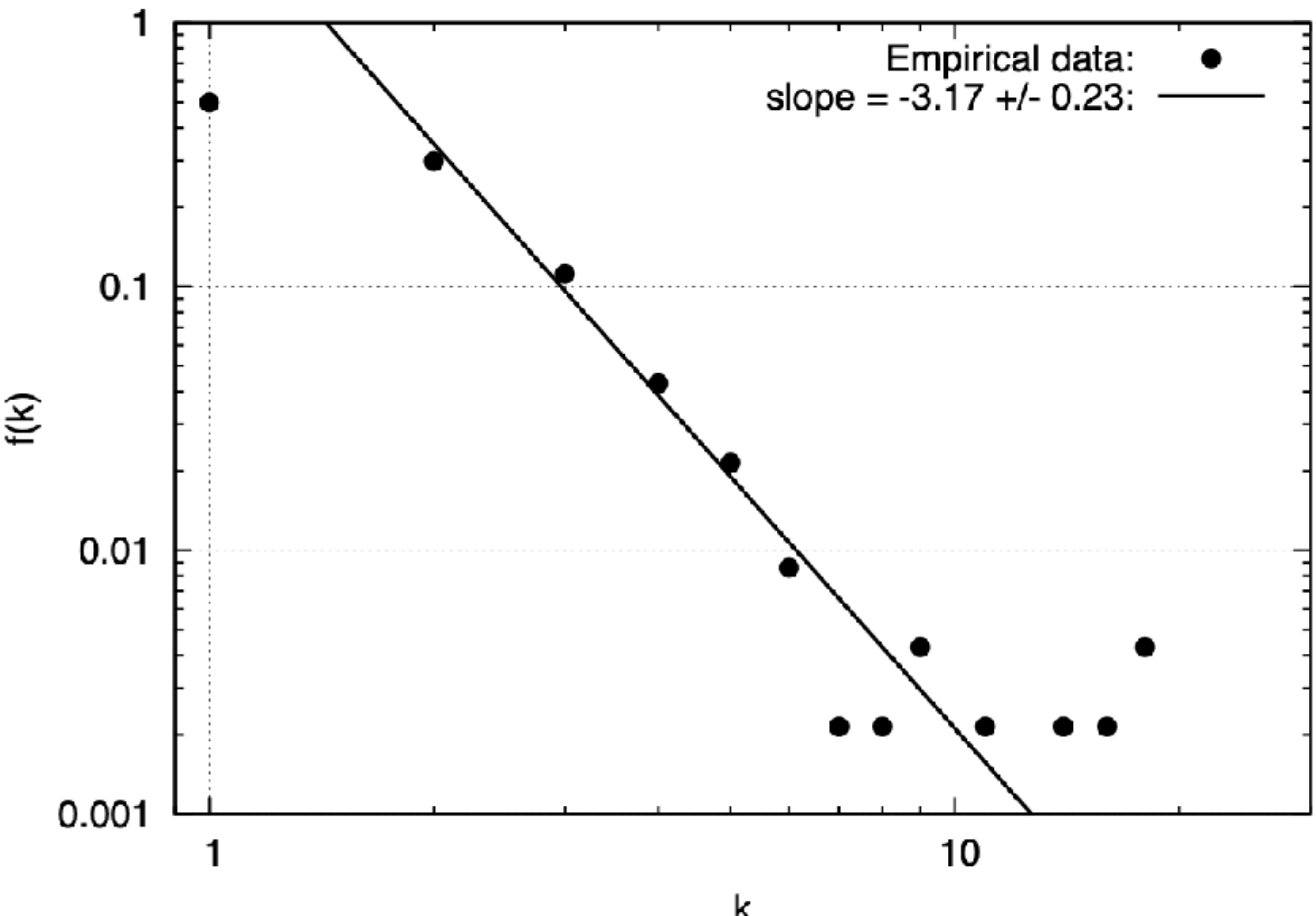

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05/06: Phase of scale-free MST - a (relatively) stable stock market state

06/07: Phase of the superstar-like MST - a transient market state

07/08: Phase of scale-free MST decorated by few local star-like trees - a (relatively) stable stock market state

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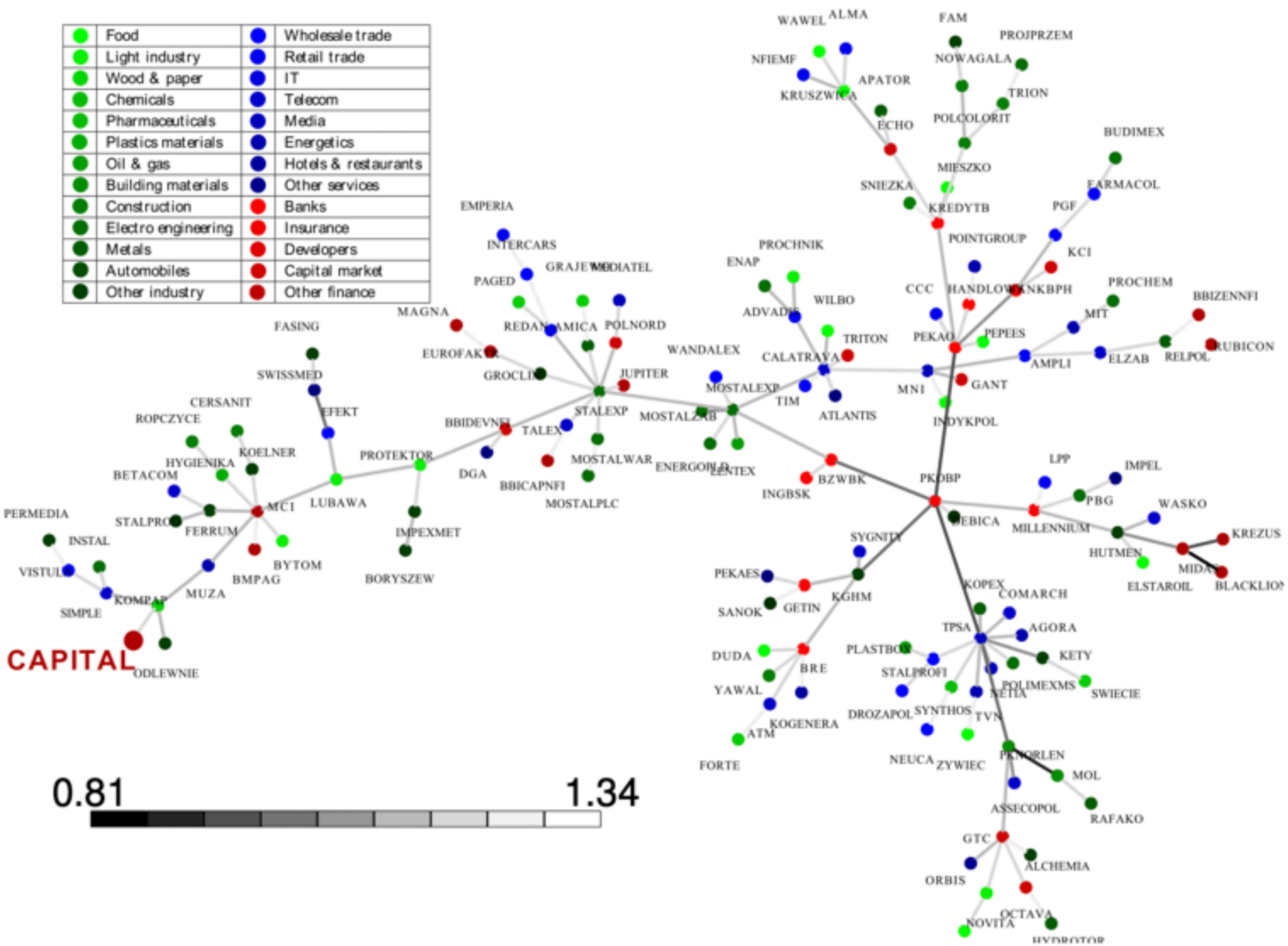
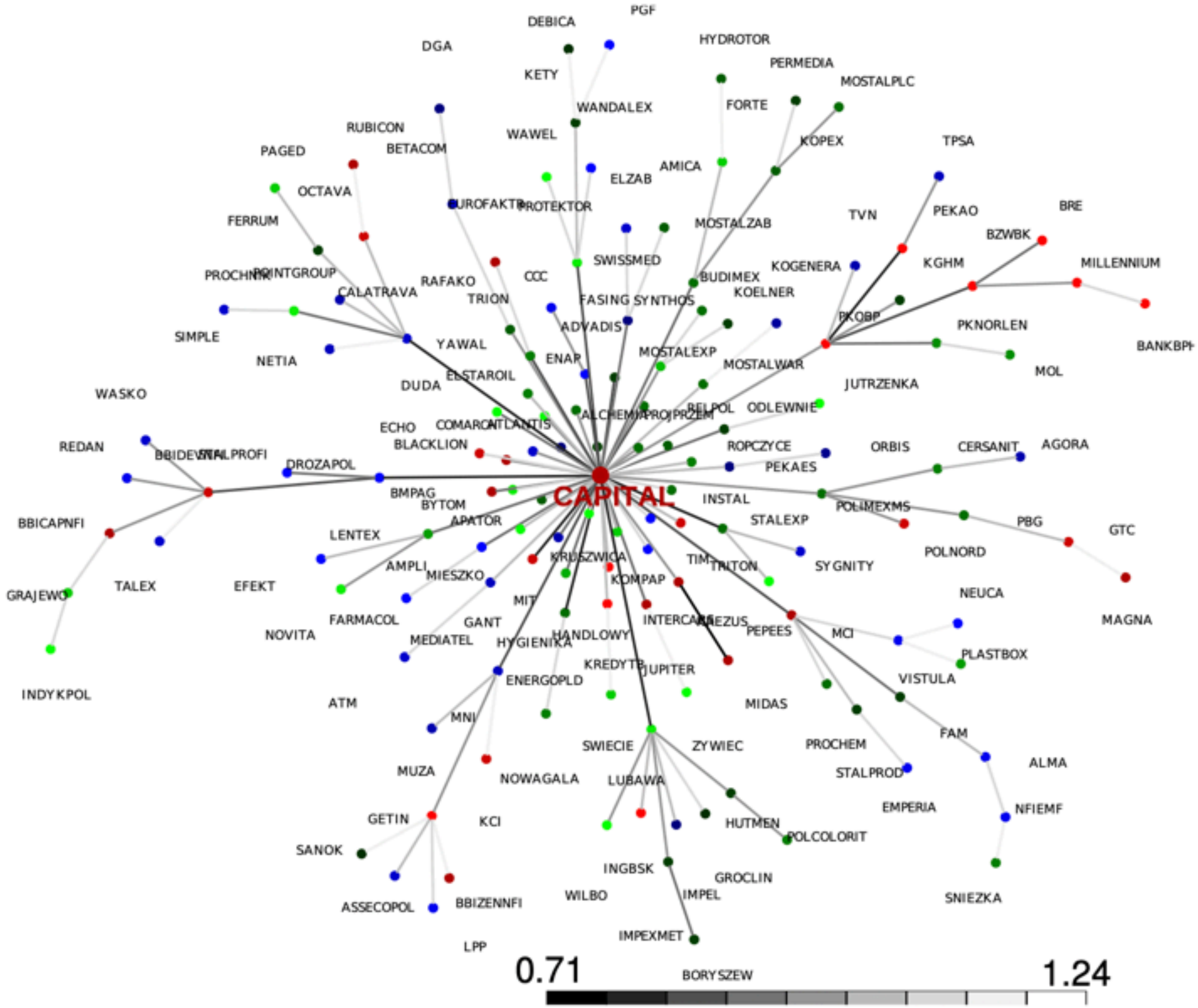


A. SIENKIEWICZ ET AL. **"DYNAMIC STRUCTURAL AND TOPOLOGICAL PHASE TRANSITION ON THE WARSAW STOCK EXCHANGE"** https://arxiv.org/pdf/1301.6506

2006-2007



A. SIENKIEWICZ ET AL. **“DYNAMIC STRUCTURAL AND TOPOLOGICAL PHASE TRANSITION ON THE WARSAW STOCK EXCHANGE”** https://arxiv.org/pdf/1301.6506

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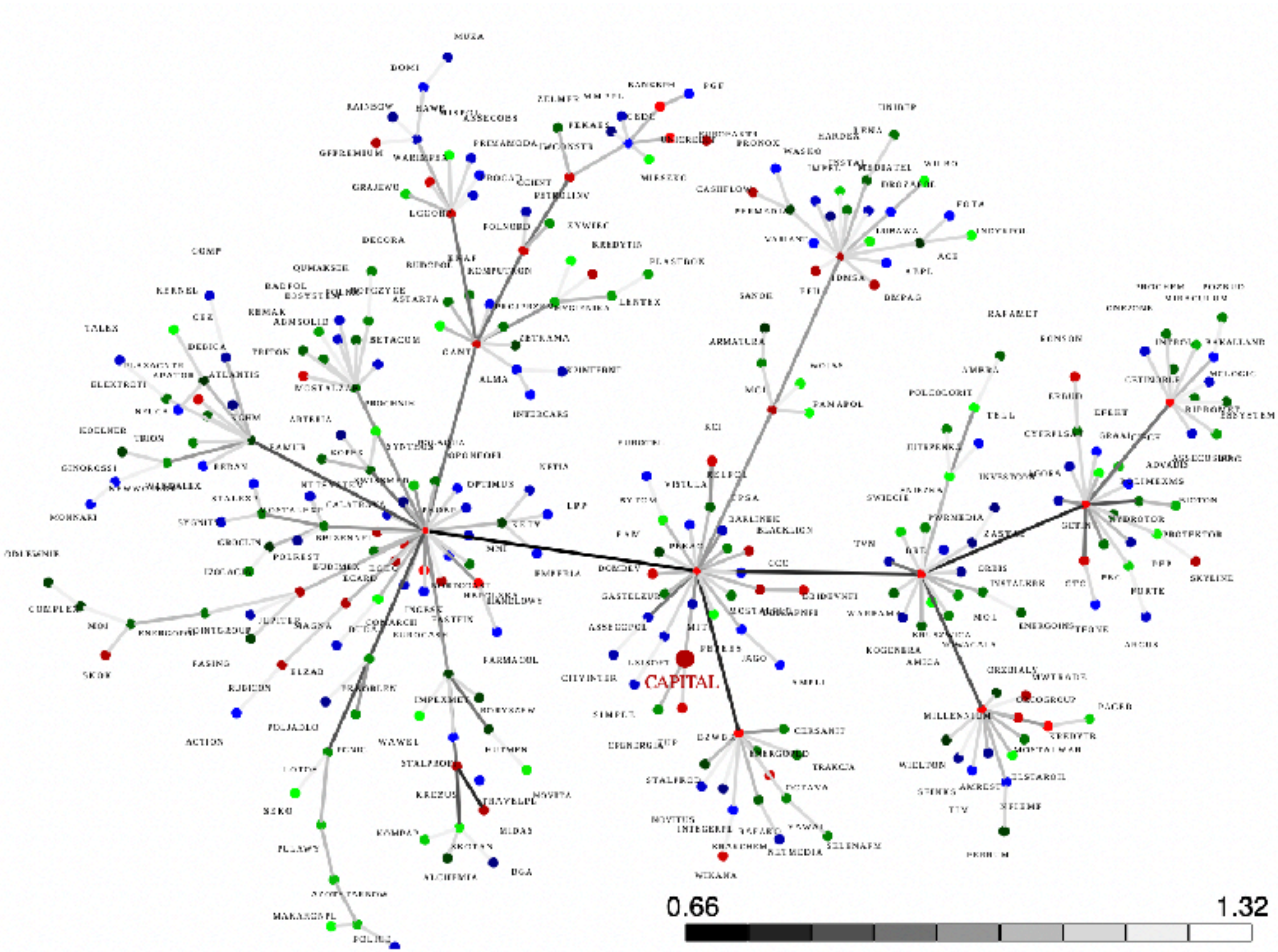

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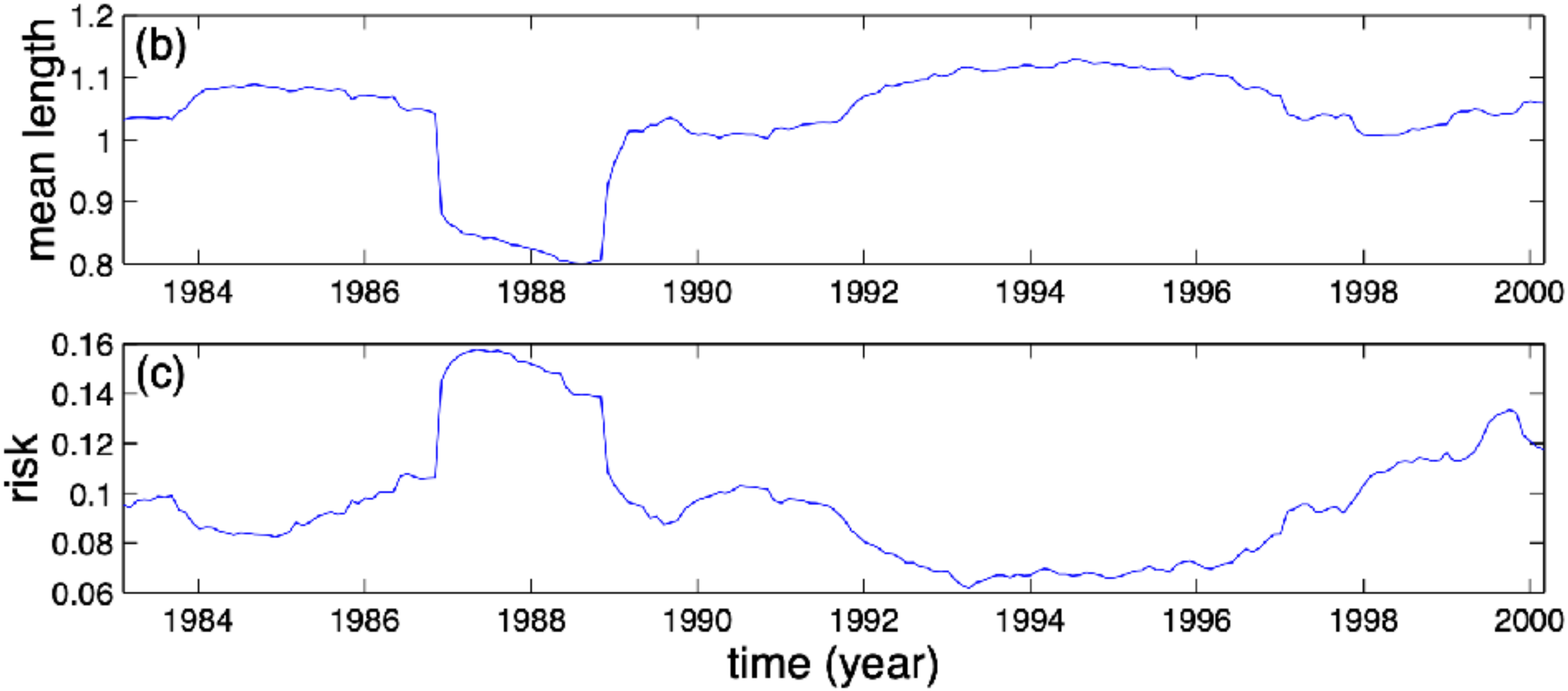
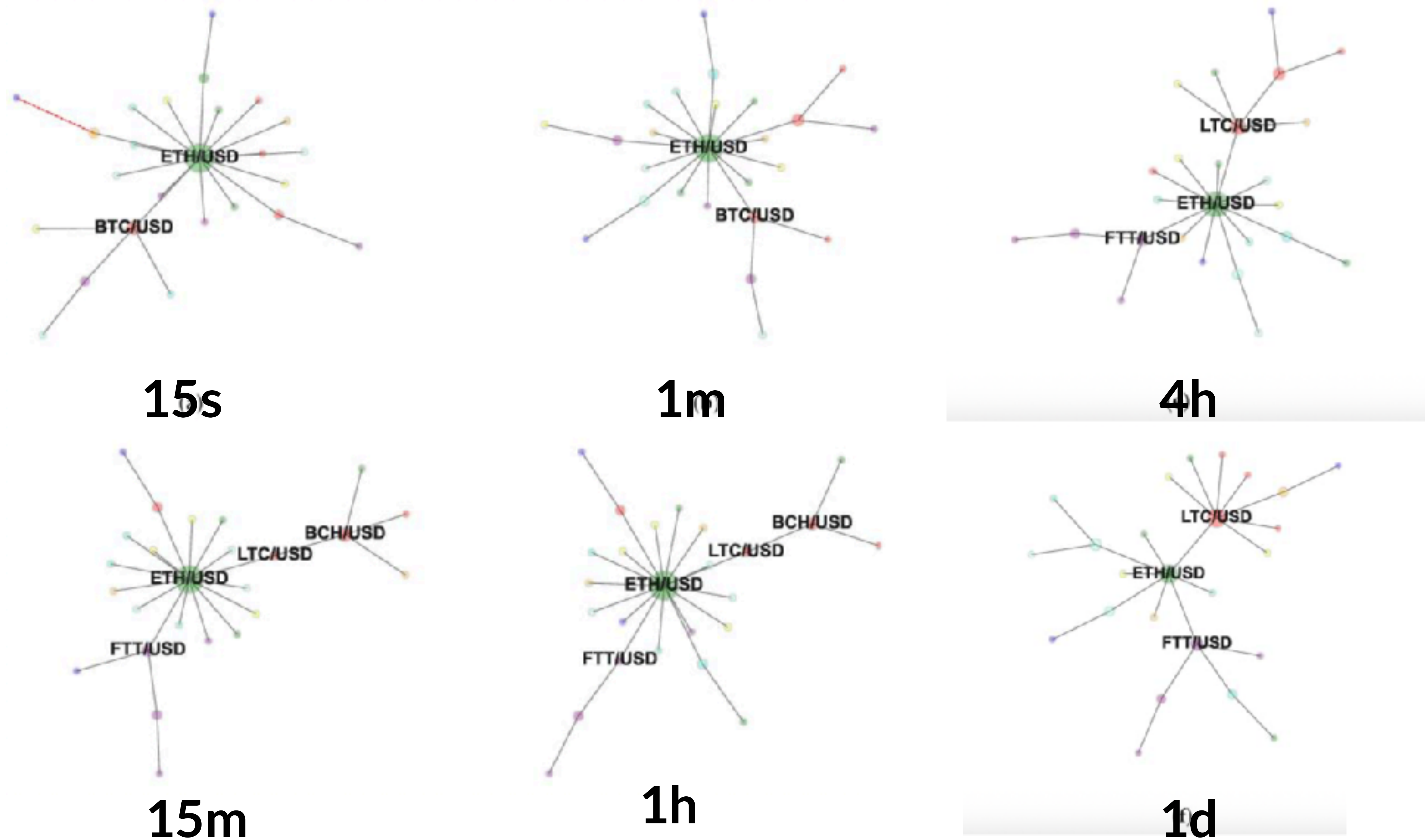


ONNELA ET AL. **“DYNAMIC ASSET TREES AND PORTFOLIO ANALYSIS”** https://arxiv.org/pdf/cond-mat/0208131

Cryptocurrencies



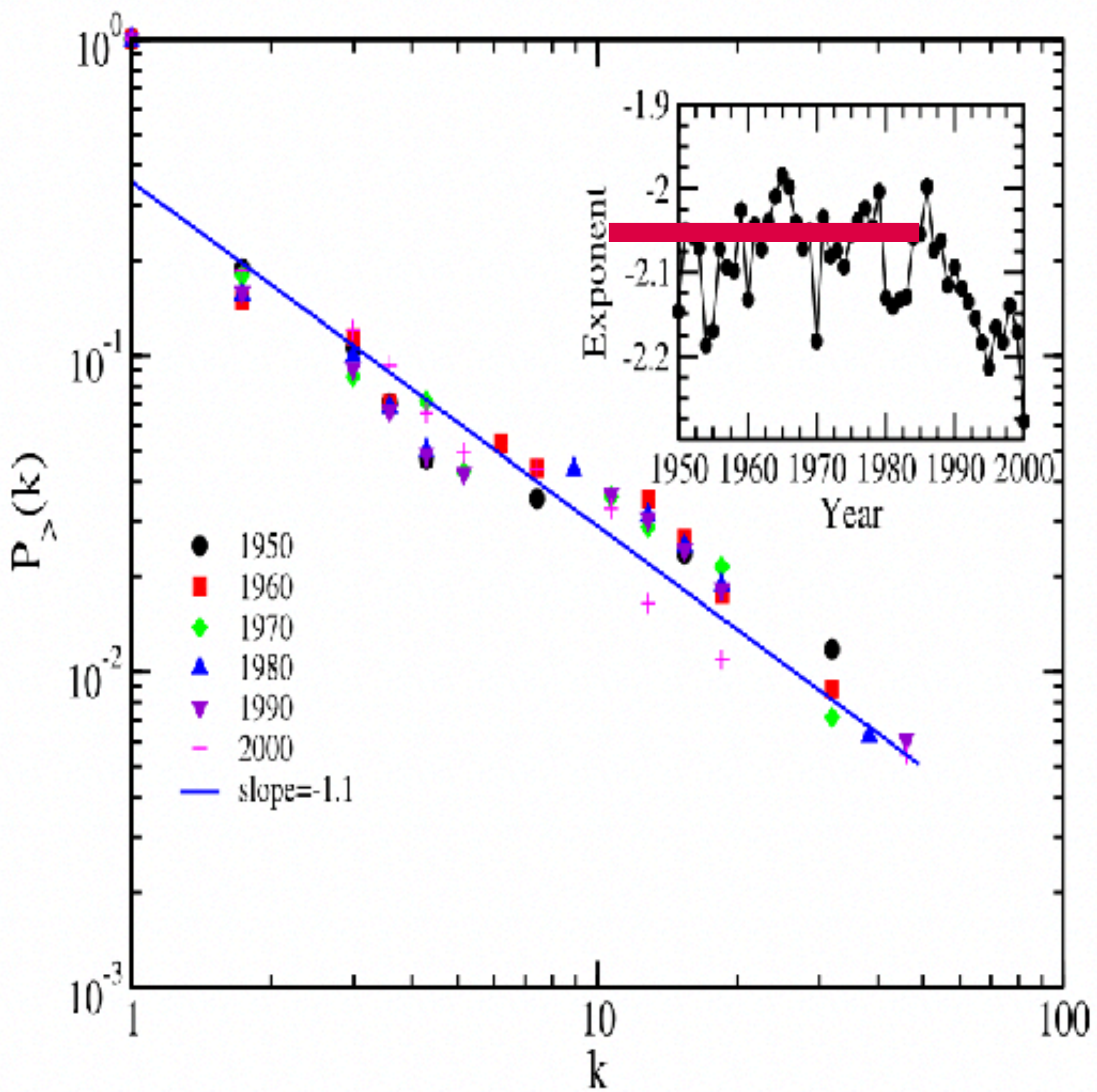
A. BRIOLA ET AL. **"Dependency Structures in Cryptocurrency Market from High to Low Frequency "** https://pmc.ncbi.nlm.nih.gov/articles/PMC9689460/

International trade

Fig. 2. Minimal spanning tree of the international trade network at year 2000. In this network the size of the node represent the relative value of the GDP among the countries. The thickness of the link corresponds to the relative trading strength of the countries. The symbols inside nodes indicate the name of the countries: USA(the United State of America), JPN(Japan), GFR(Germany), UKG(the United Kingdom), RUS(Russia), IND(India), ITA(Italy), FRN(France), SPN(Spain), BRA(Brazil), etc.

Seong Eun Meng ET AL. **"COMPLEX NETWORKS AND MINIMAL SPANNING TREES IN INTERNATIONAL TRADE NETWORK"** https://www.worldscientific.com/doi/epdf/10.1142/S2010194512007775

International trade



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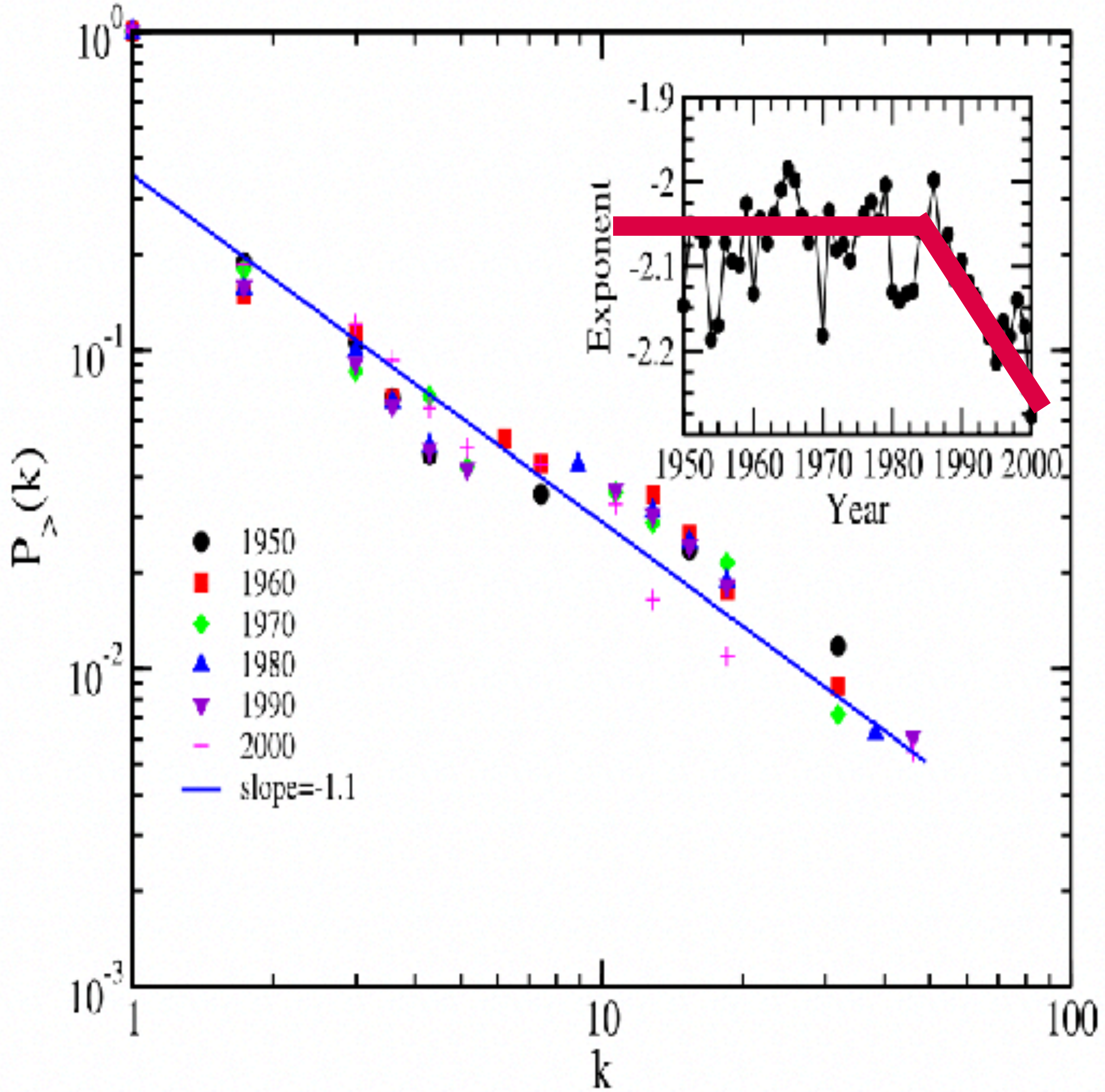

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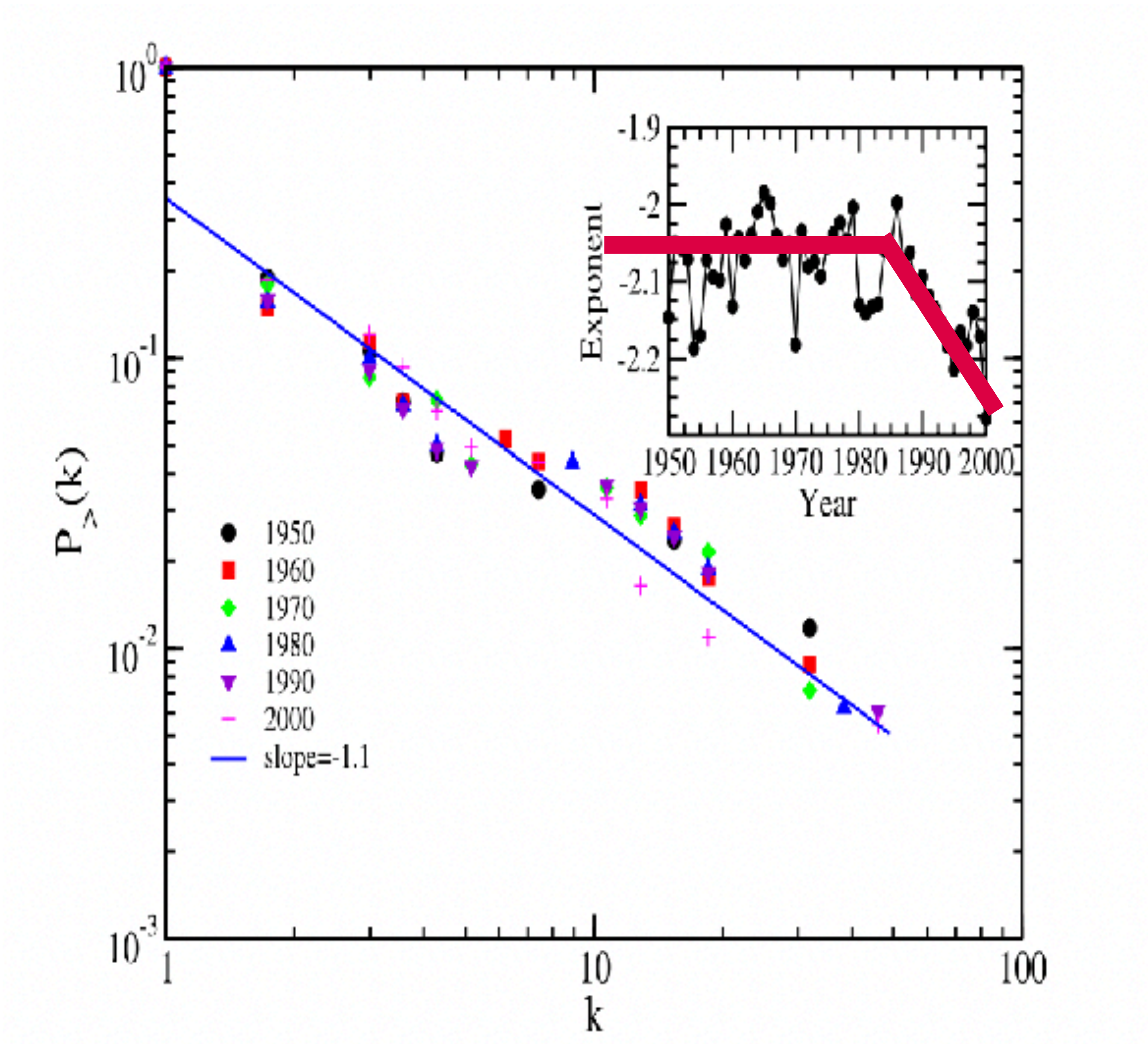

J. DIAS. **"SPANNING TREES AND THE EUROZONE CRISIS"** https://www.sciencedirect.com/science/article/pii/S0378437113007127

SUMMARY

What trees are
What they can reveal
Real-world examples