

# Community detection



# LEARNING OUTCOMES

UNDERSTAND WHAT **COMMUNITIES** ARE

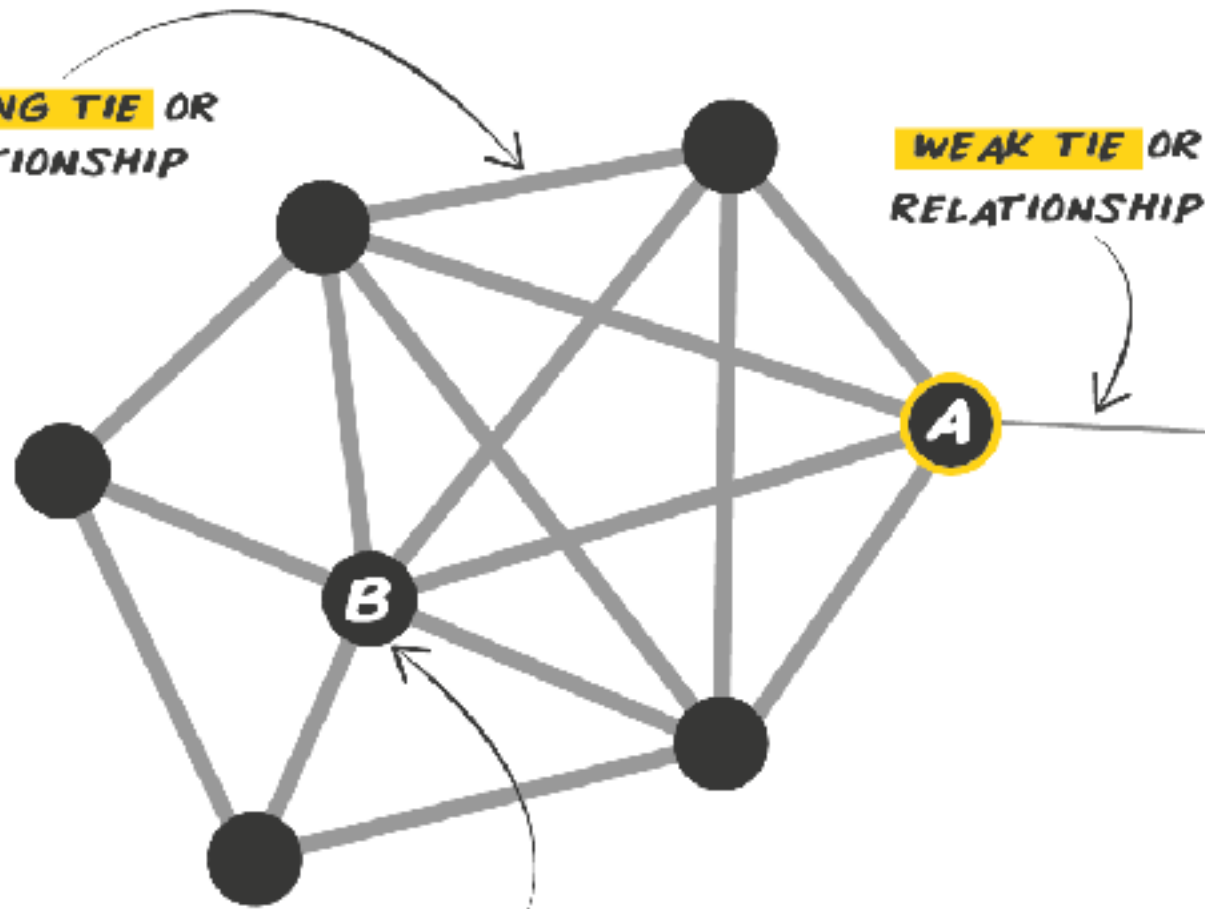
BE ABLE TO **DESCRIBE A NETWORK** IN TERMS OF COMMUNITIES

LEARN DIFFERENT TYPES OF COMMUNITY **CLASSIFICATIONS**



## GRANOVETTER'S STRENGTH OF WEAK TIES

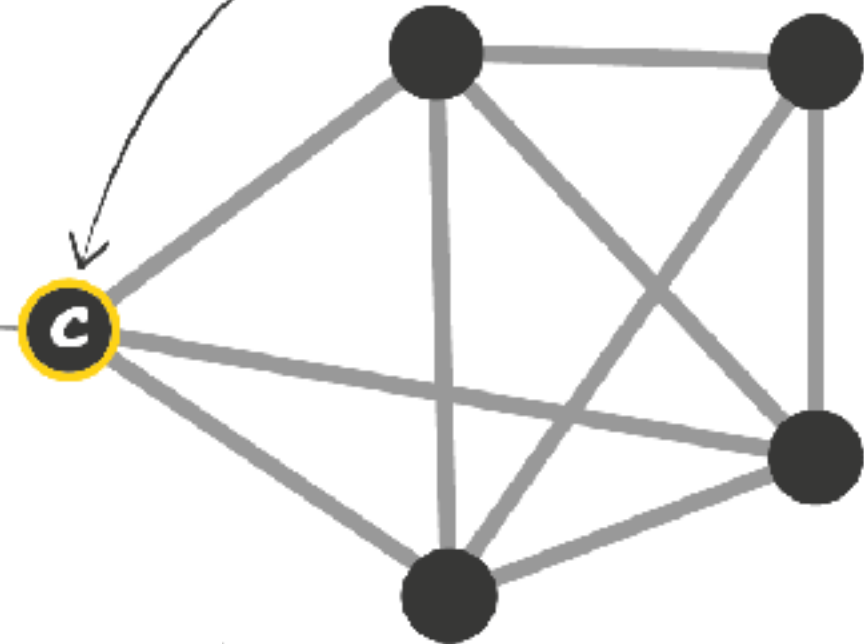
**STRONG TIE** OR  
RELATIONSHIP



EVEN THOUGH **B** HAS MORE  
TIES THAN **A**, ALL THOSE TIES  
LIKELY HAVE THE SAME INFORMATION  
BECAUSE THEY ALL KNOW EACH OTHER WELL

**WEAK TIE** OR  
RELATIONSHIP

IT'S VALUABLE TO HAVE  
A COMBINATION OF STRONG  
AND WEAK TIES



FOR EXAMPLE, **A** CAN SHARE INFORMATION  
WITH **C** THAT **C** WOULDN'T GET FROM ANYONE  
ELSE IN THEIR GROUP, AND VICE VERSA.



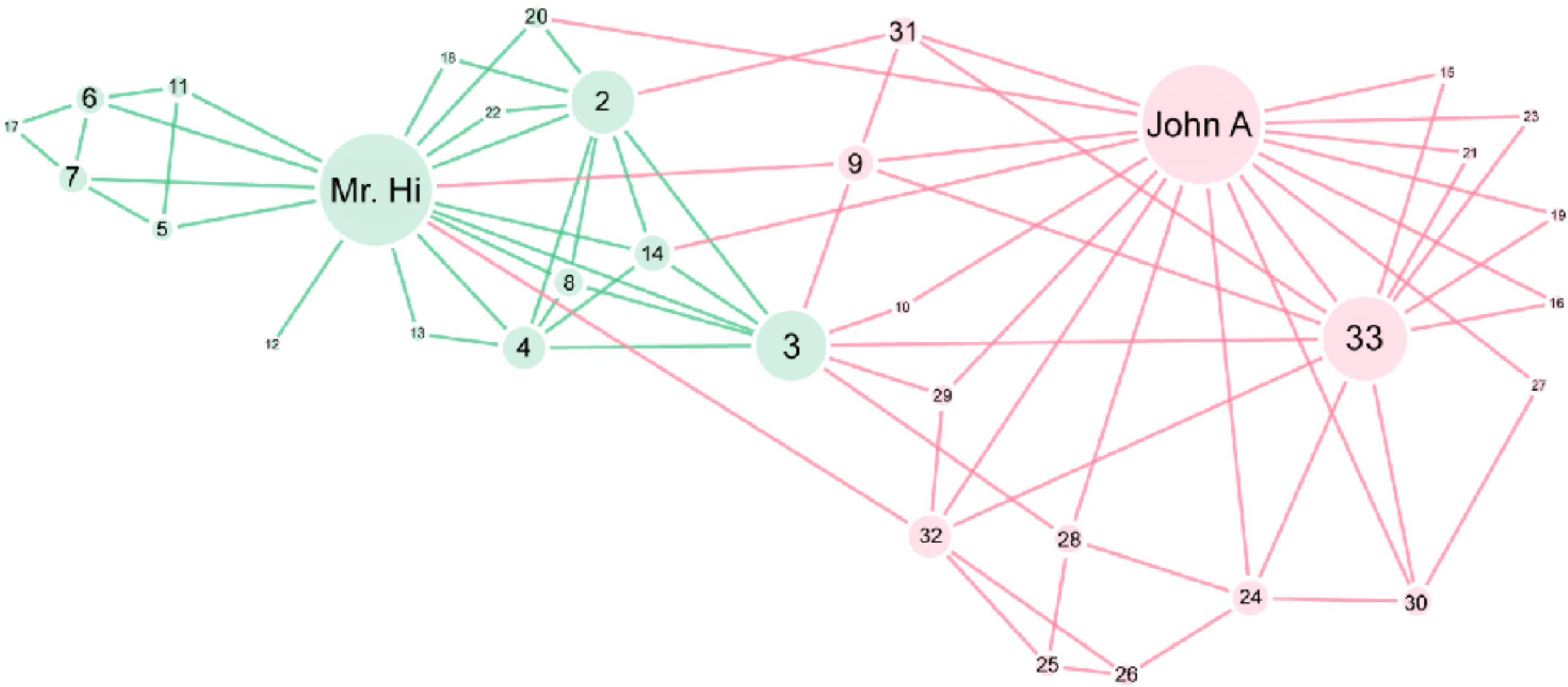
UNDER EIGHTEEN  
**ALL VALLEY KARATE CHAMPIONSHIP**

SEMI-FINALS

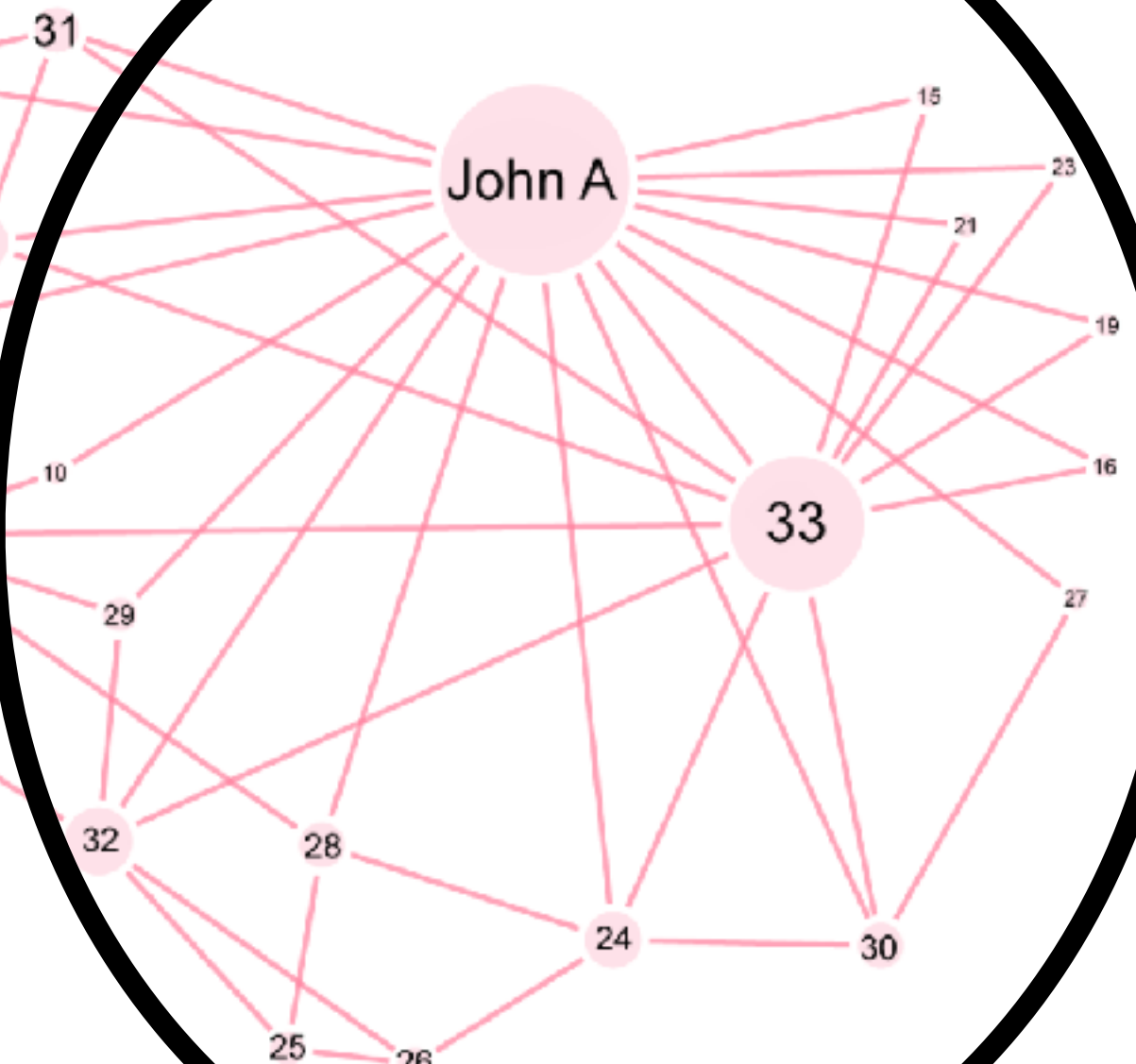
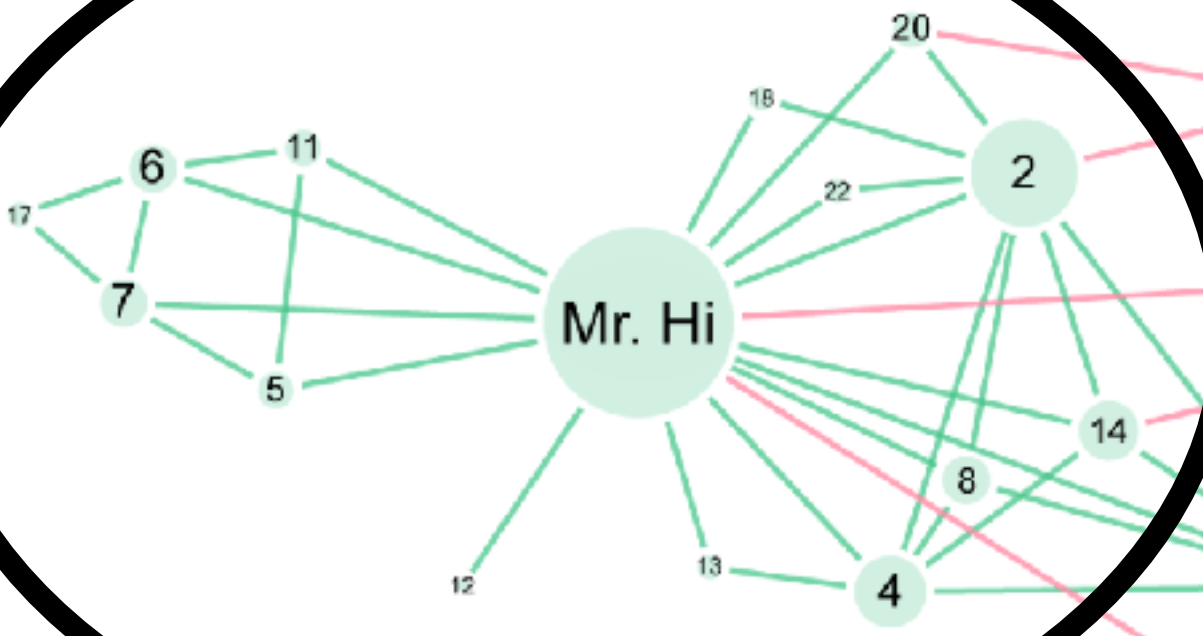
FINALS

ALL VALLEY  
CHAMPION



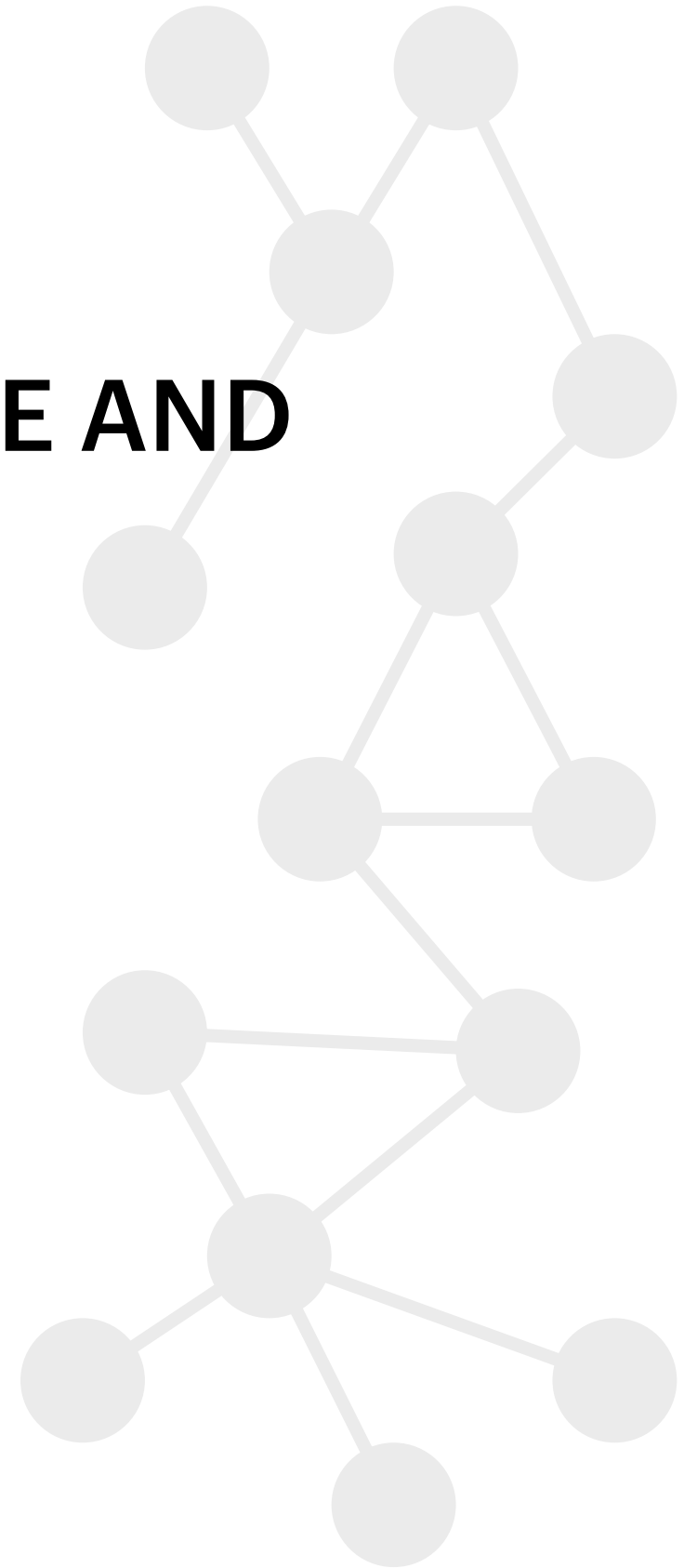






# DEFINITIONS

**INTERNAL AND EXTERNAL DEGREE:  
THE NUMBER OF NEIGHBOURS INSIDE AND  
OUTSIDE THE COMMUNITY**



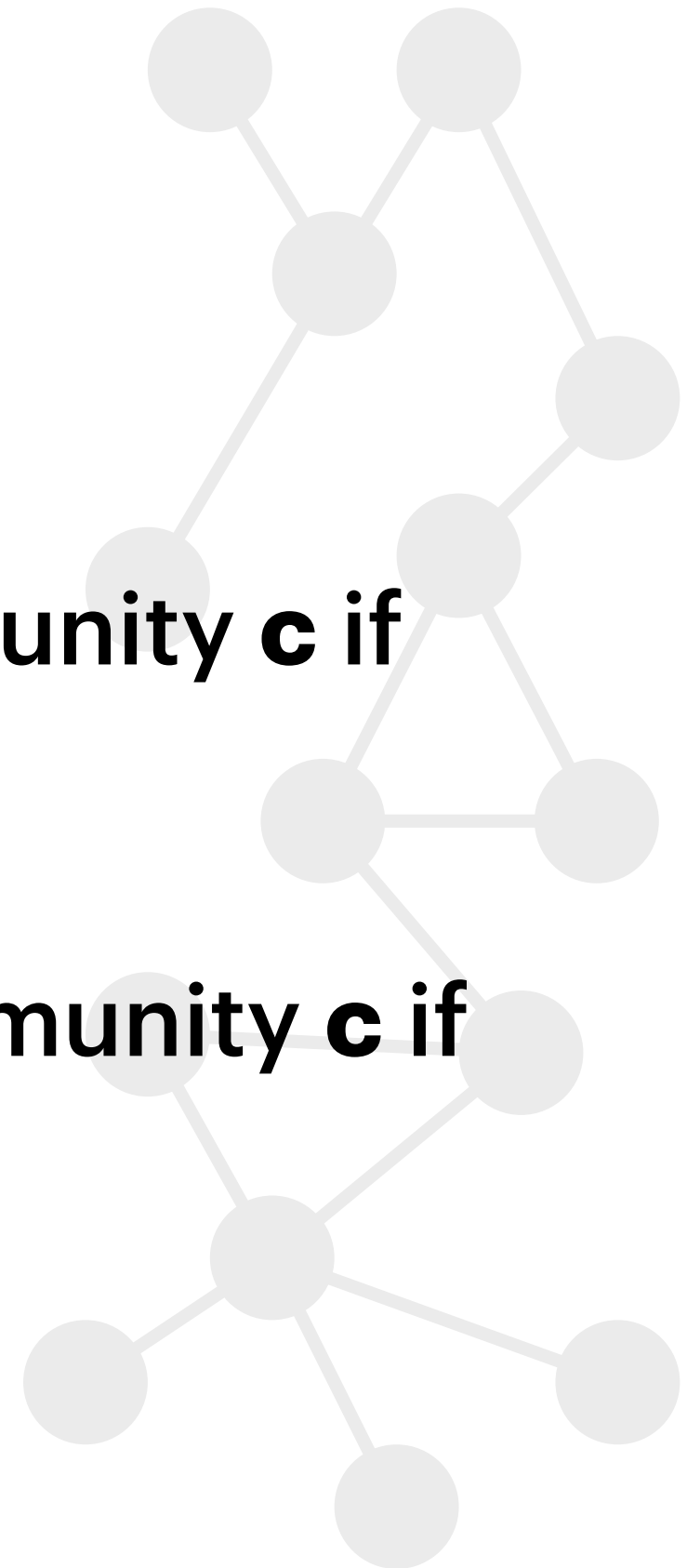
# DEFINITIONS

i is called internal node of community **c** if

$$k_i^{ext} = 0 \text{ \textbf{And} } k_i^{int} > 0$$

i is called boundary node of community **c** if

$$k_i^{ext} > 0 \text{ \textbf{And} } k_i^{int} > 0$$

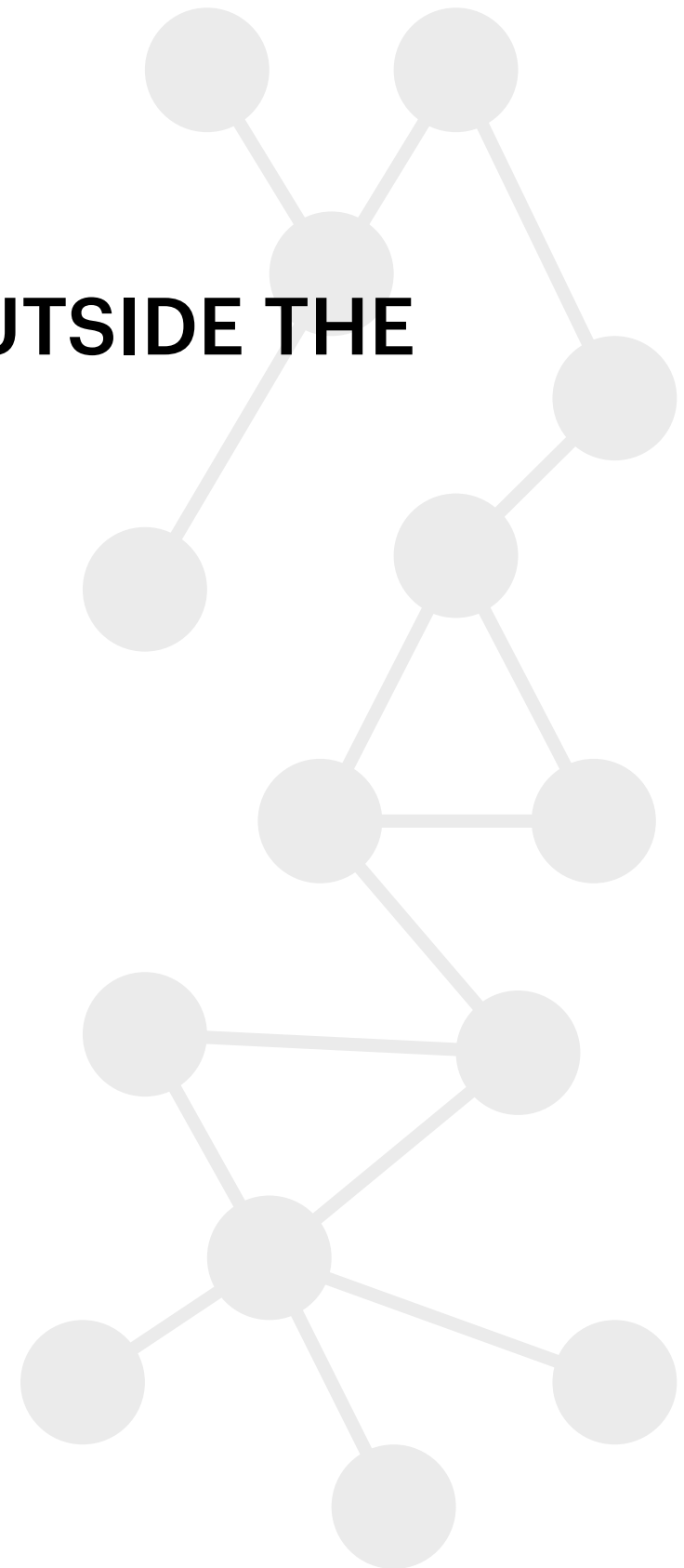




# DEFINITIONS

**INTERNAL AND EXTERNAL DEGREE:**  
THE NUMBER OF NEIGHBOURS INSIDE AND OUTSIDE THE  
COMMUNITY

$$k_i = k_i^{int} + k_i^{ext}$$



# DEFINITIONS

## **INTERNAL AND EXTERNAL DEGREE:**

**THE NUMBER OF NEIGHBOURS INSIDE AND OUTSIDE THE COMMUNITY**

## **NUMBER OF INTERNAL LINKS:**

**THE NUMBER OF LINKS BETWEEN NODES WITHIN THE COMMUNITY**



# DEFINITIONS

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## **NUMBER OF INTERNAL LINKS:**

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## **COMMUNITY DEGREE:**

**THE SUM OF DEGREE OF ALL THE NODES IN THE COMMUNITY**

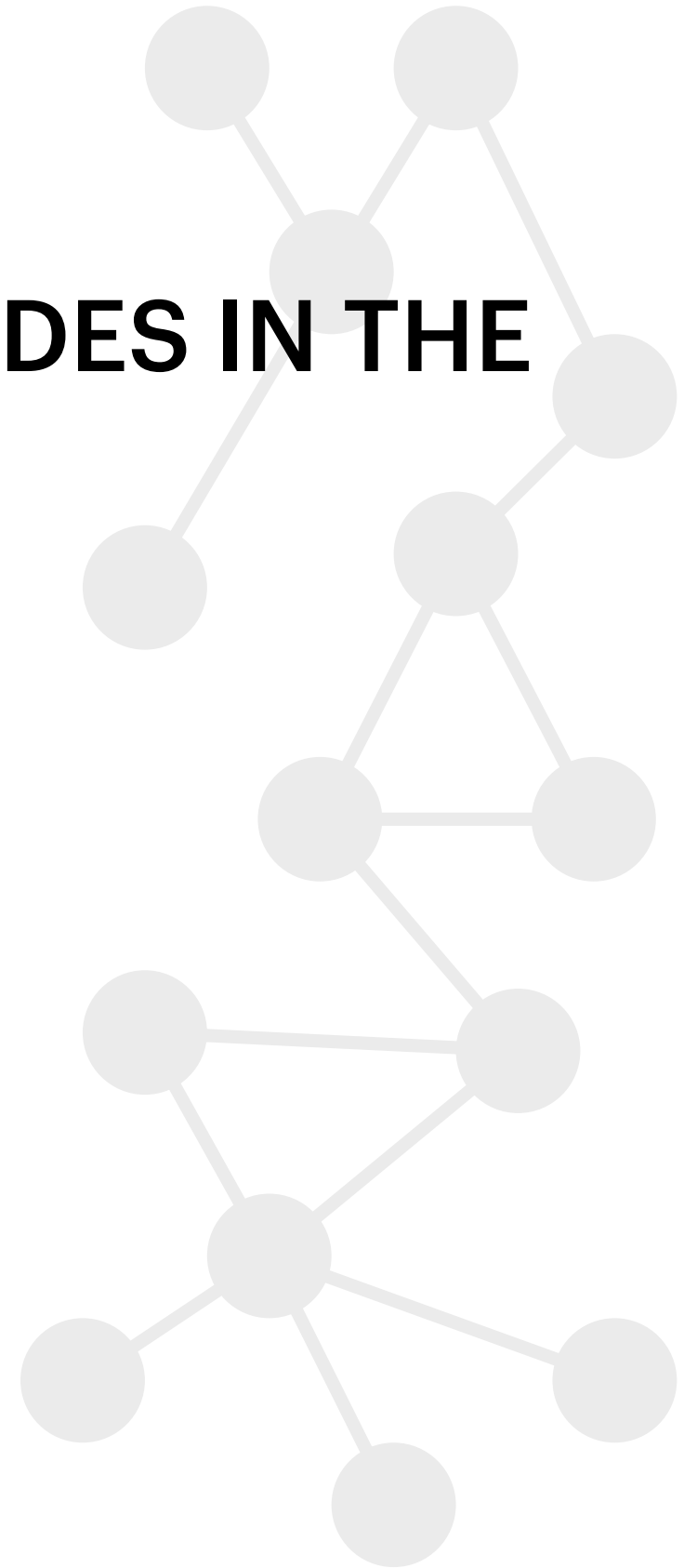


# DEFINITIONS

**COMMUNITY DEGREE:**

**THE SUM OF DEGREE OF ALL THE NODES IN THE  
COMMUNITY**

$$k_C = \sum_{i \in C} k_i$$



# DEFINITIONS

## **INTERNAL AND EXTERNAL DEGREE:**

THE NUMBER OF NEIGHBOURS INSIDE AND OUTSIDE THE COMMUNITY

## **NUMBER OF INTERNAL LINKS:**

THE NUMBER OF LINKS BETWEEN NODES WITHIN THE COMMUNITY

## **COMMUNITY DEGREE:**

THE SUM OF DEGREE OF ALL THE NODES IN THE COMMUNITY

## **INTERNAL LINK DENSITY:**

DENSITY THAT CONSIDERS ONLY LINKS BETWEEN MEMBERS OF THE COMMUNITY



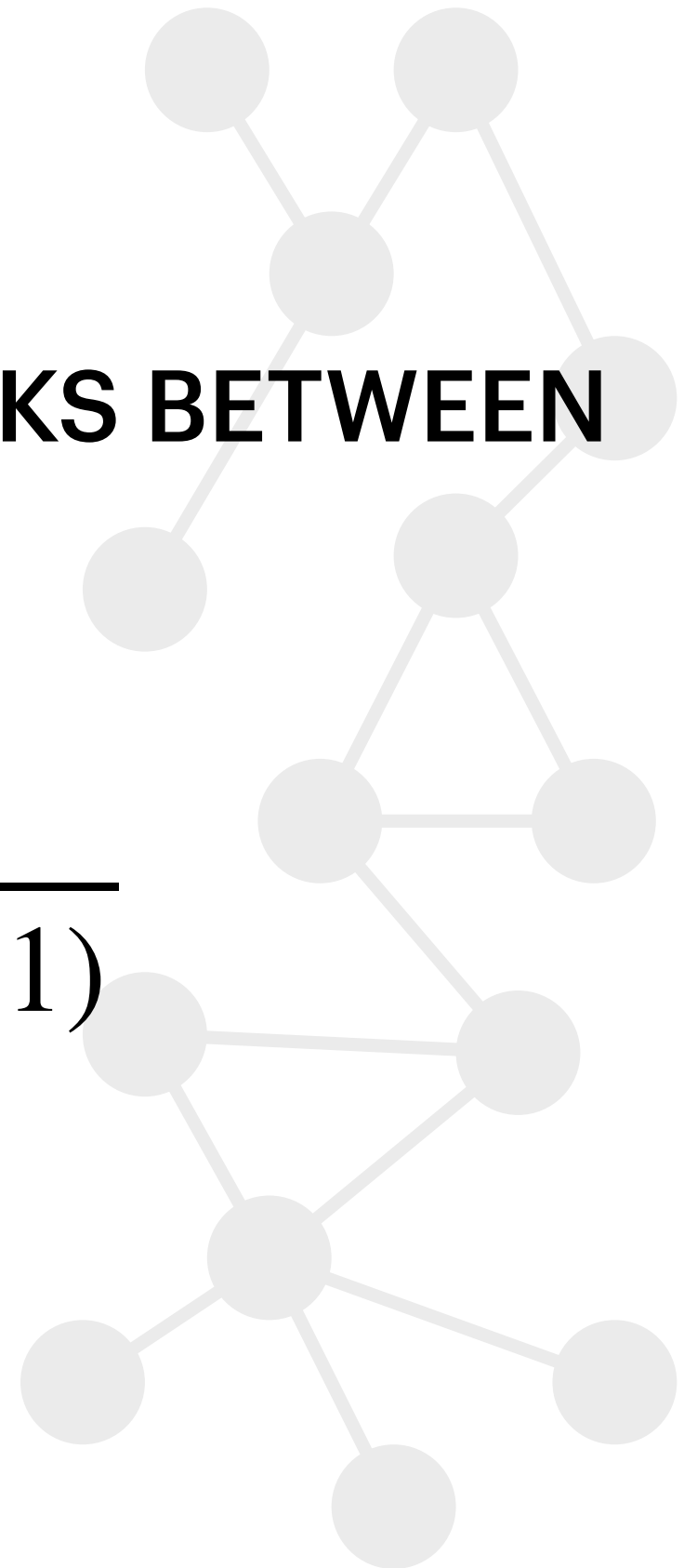


# DEFINITIONS

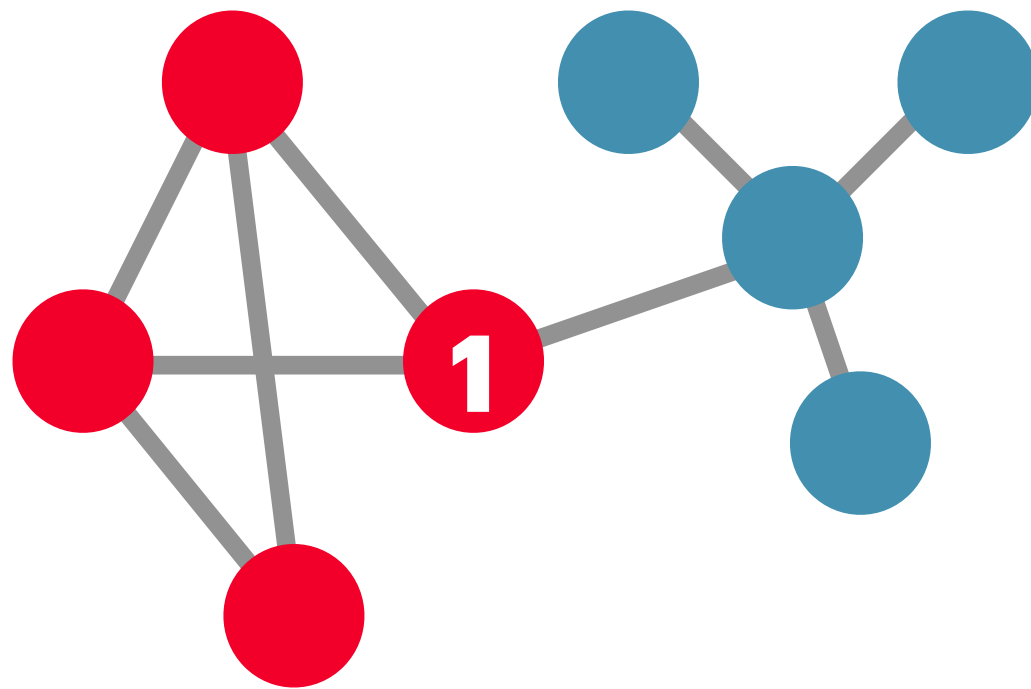
**INTERNAL LINK DENSITY:**

**DENSITY THAT CONSIDERS ONLY LINKS BETWEEN MEMBERS OF THE COMMUNITY**

$$\delta_C^{int} = \frac{L_C}{L_C^{max}} = \frac{2L_C}{N_C(N_C - 1)}$$

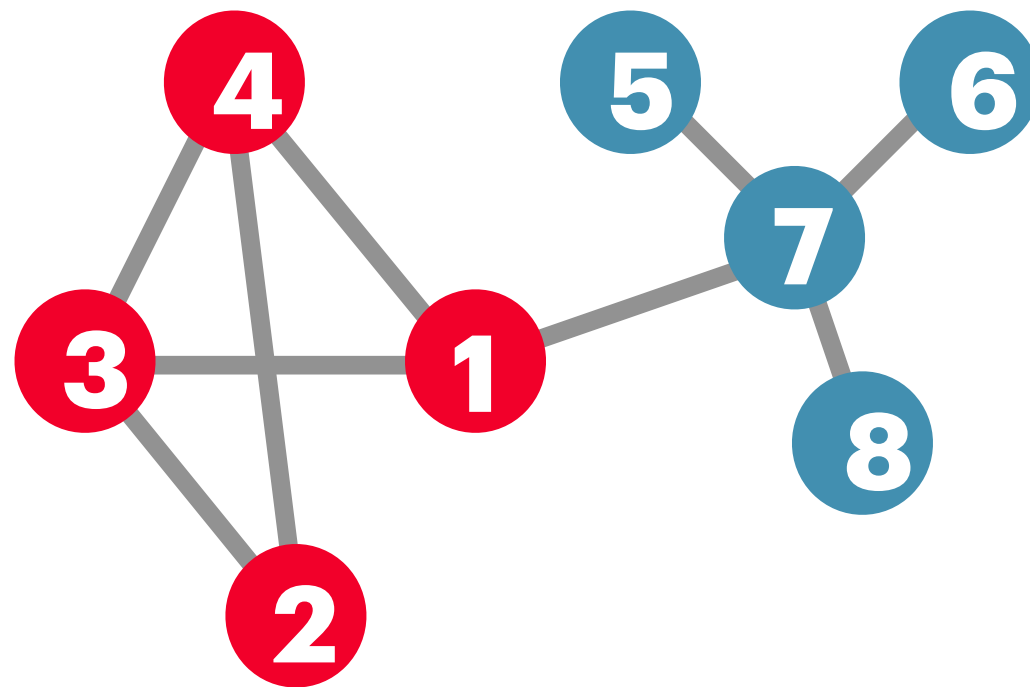


# DEFINITIONS



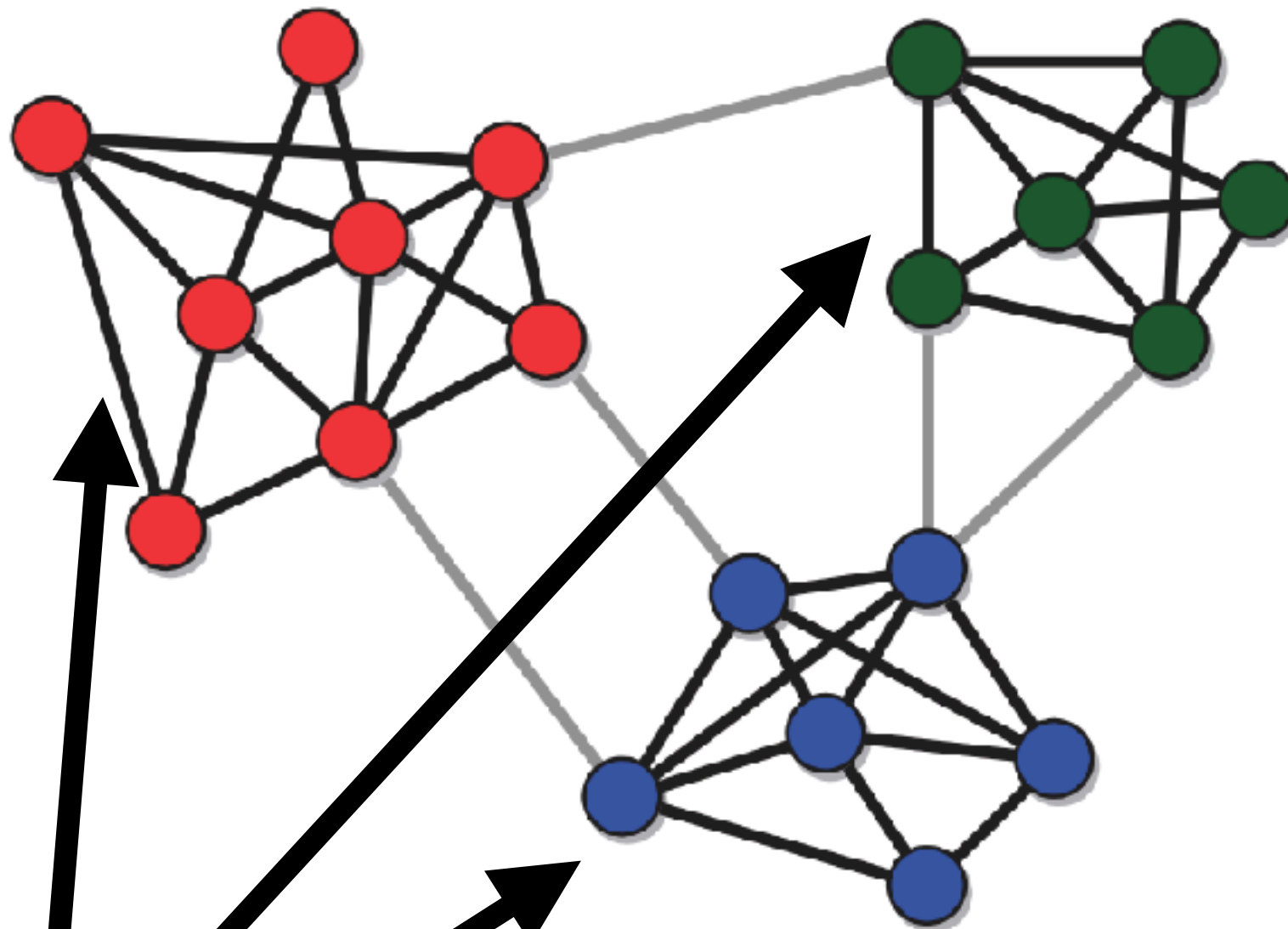
?  $k_1^{ext}$ ,  $k_1^{int}$ ,  $\delta_{red}^{int}$ ,  $k_{blue}$

# DEFINITIONS



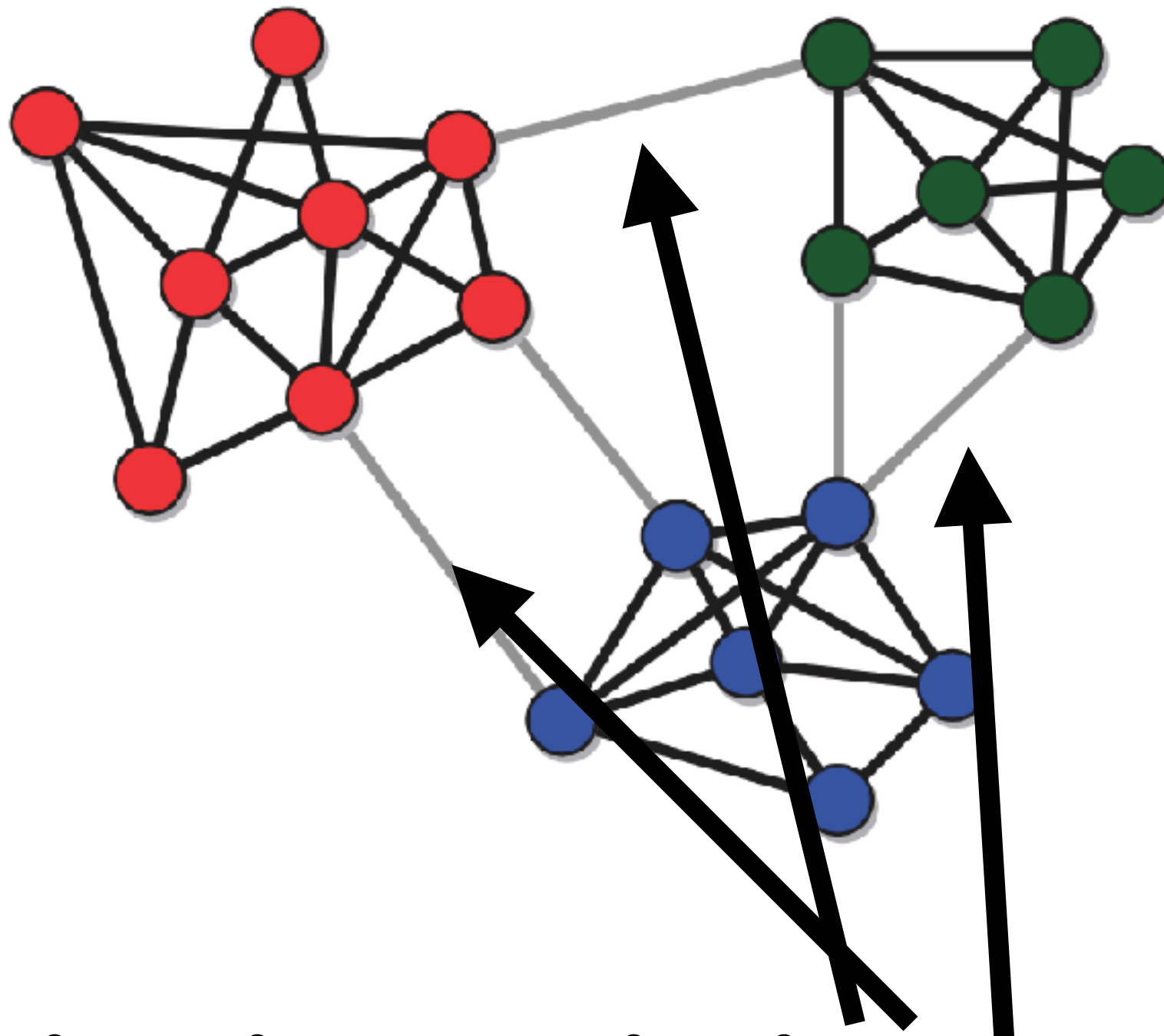
**Which nodes are the boundary nodes?**

# DEFINITIONS



**high cohesion, high separation**

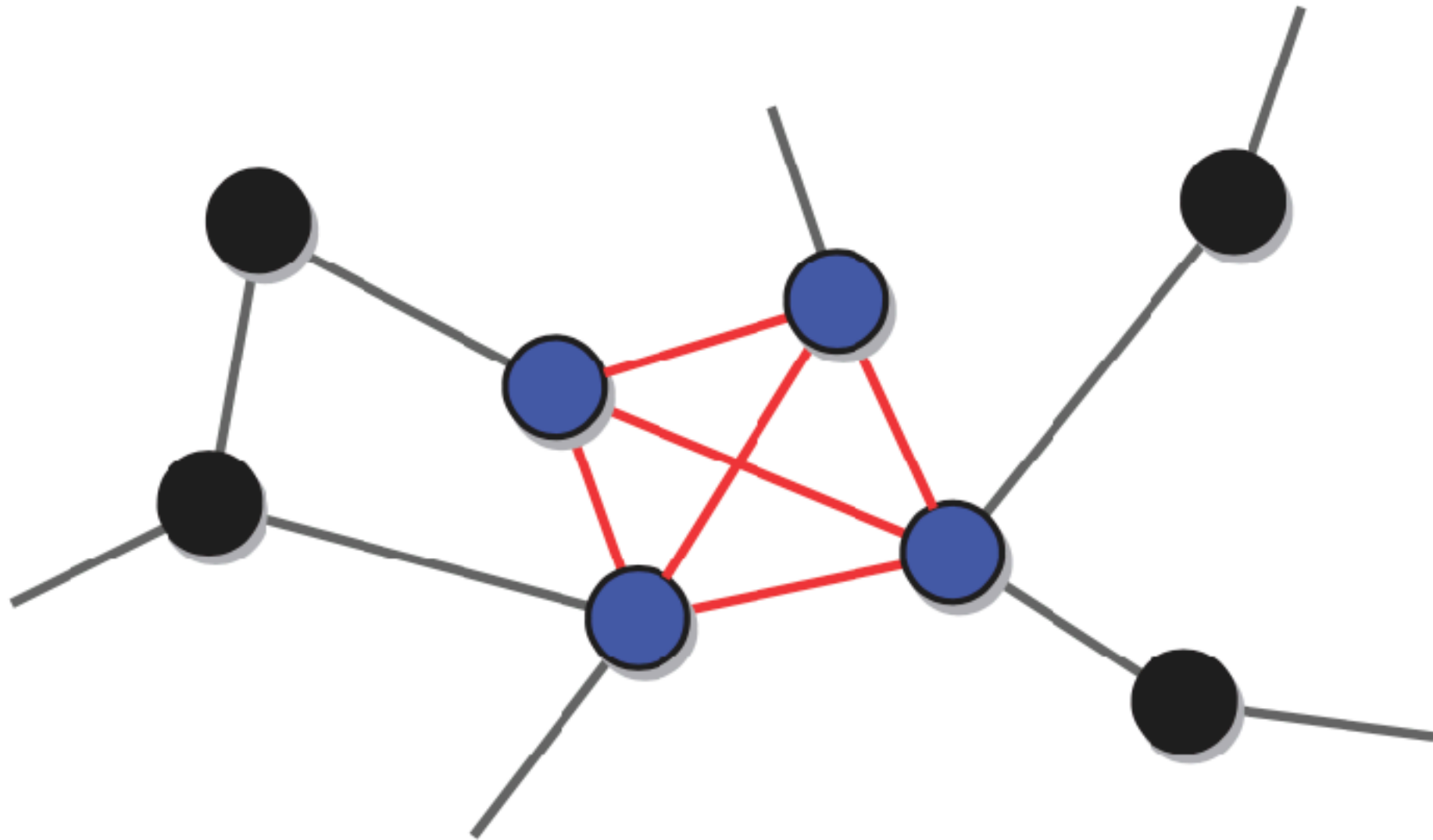
# DEFINITIONS



**high cohesion, high separation**



# DEFINITIONS



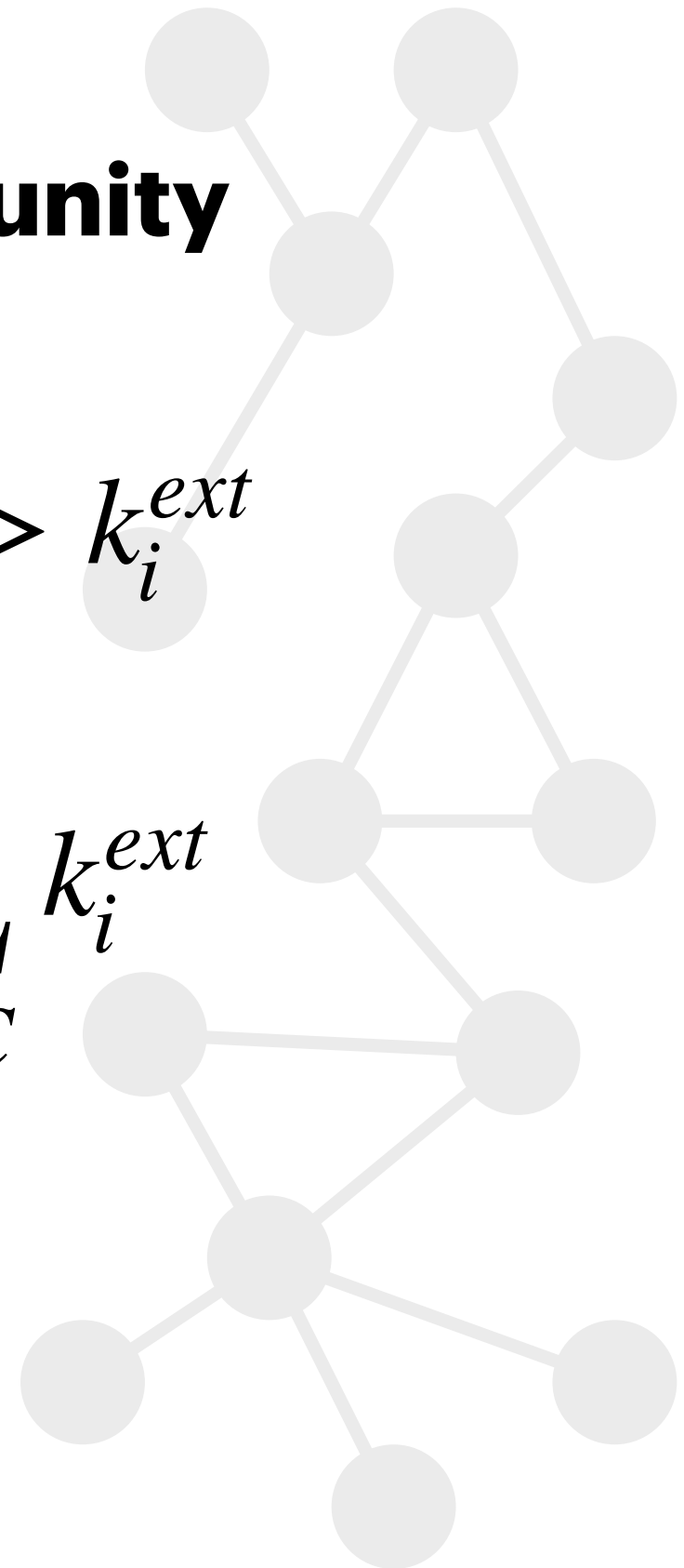
**clique** (a fully connected subgraph)

# DEFINITIONS

## Stricter definition of community

**Strong community:**  $\forall i \in C : k_i^{int} > k_i^{ext}$

**Weak community:**  $\sum_{i \in C} k_i^{int} > \sum_{i \in C} k_i^{ext}$

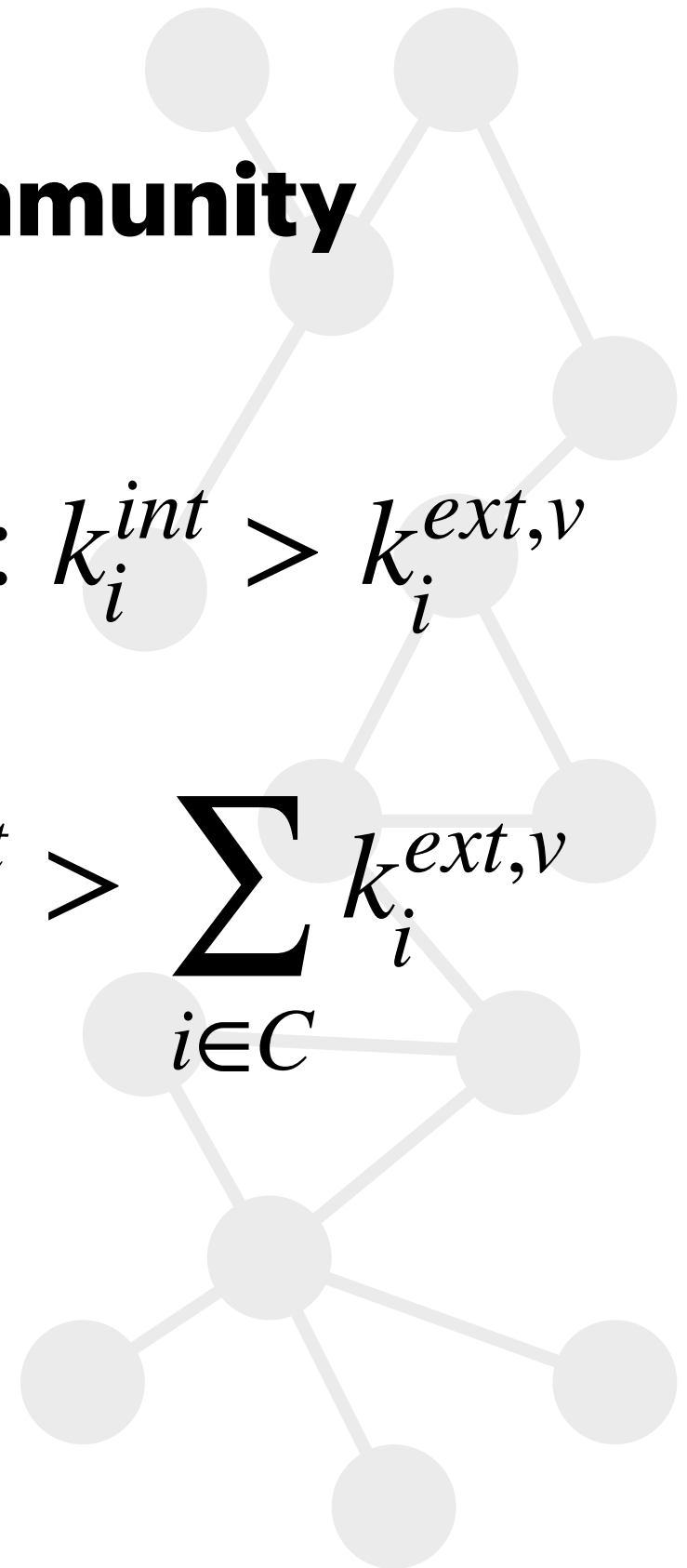


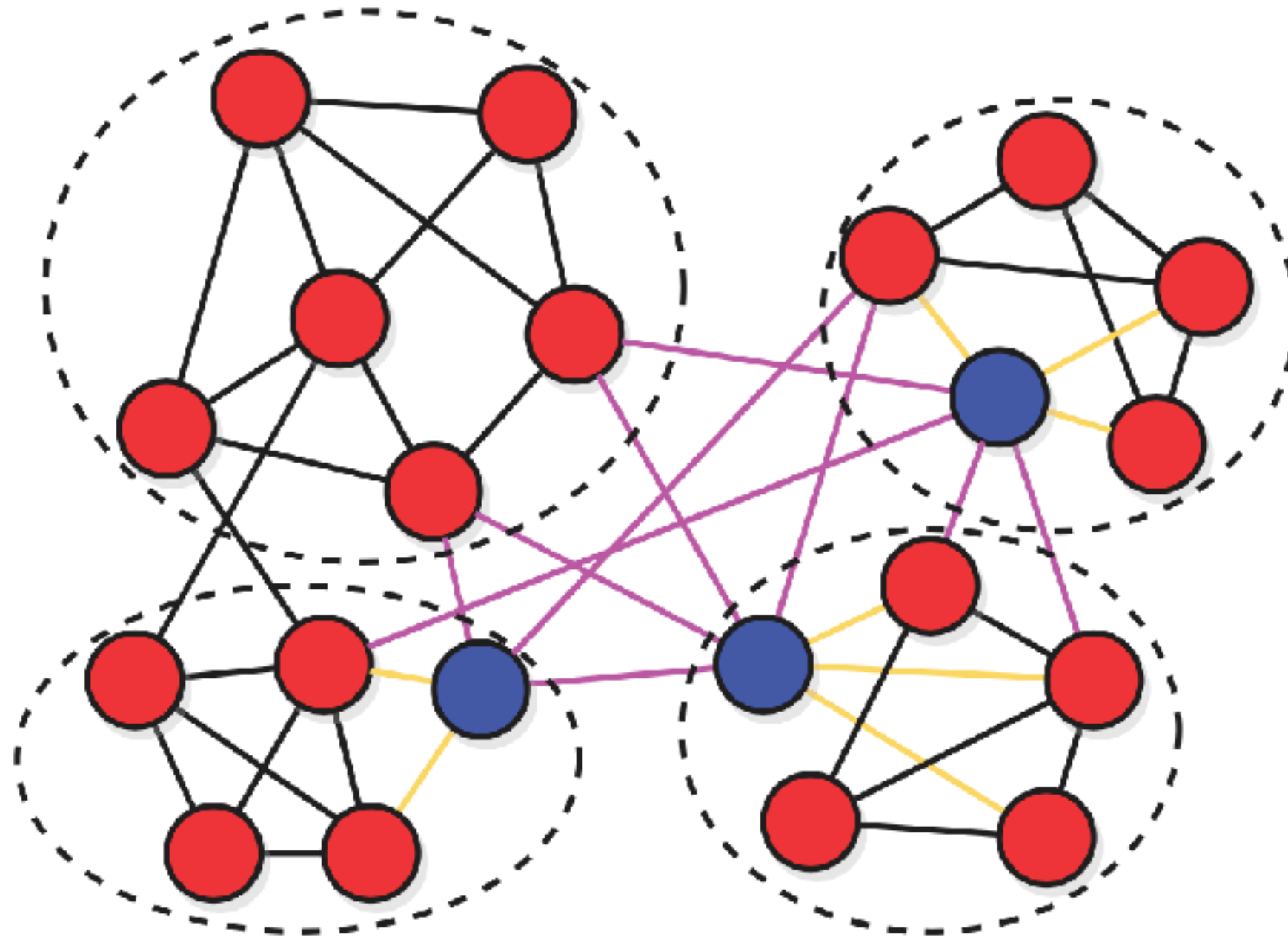
# DEFINITIONS

## Less strict definition of community

**Strong community:**  $\forall i \in C, \forall v \in V : k_i^{int} > k_i^{ext,v}$

**Weak community:**  $\forall v \in V : \sum_{i \in C} k_i^{int} > \sum_{i \in C} k_i^{ext,v}$





Strong and weak communities. The four subnetworks enclosed in the dashed contours are weak communities according to both definitions we have given. They are also strong communities according to the less stringent definition, as the internal degree of each node exceeds the number of links joining the node with those of every other community. However, three of the subnetworks are not strong communities in the more stringent sense, because some nodes (in blue) have external degree larger than their internal degree (the internal and external links of these nodes are colored in yellow and magenta, respectively). Adapted from Fortunato and Hric (2016).

# PARTITIONS

**A PARTITION IS A DIVISION OF THE NETWORK IN COMMUNITIES**





# PARTITIONS

SUPPOSE YOU HAVE A NETWORK **G** WITH **10 NODES**  
1,2,...,10

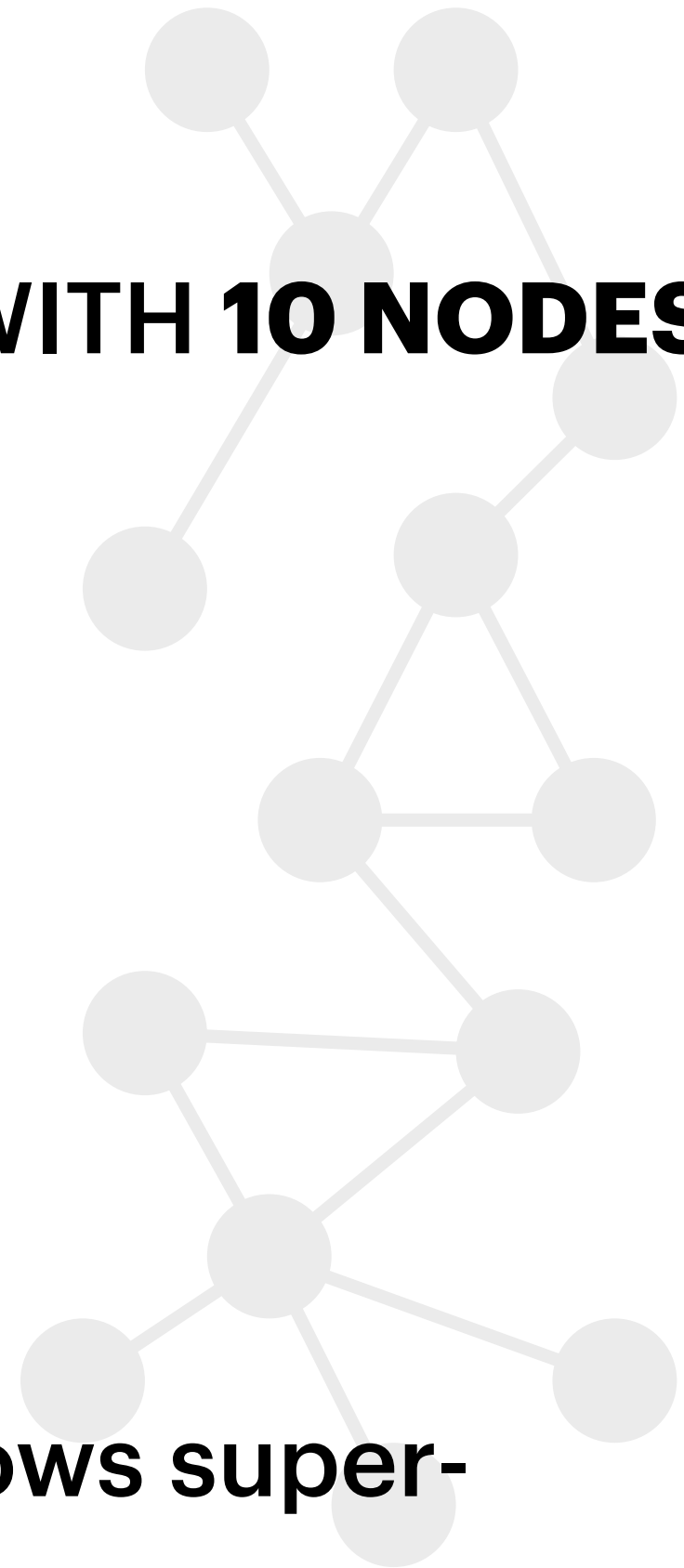
{1,2,...,10}

{1} {2} {3} ... {10}

{1,2} {3,6,9} {5,8,10} {7,4}

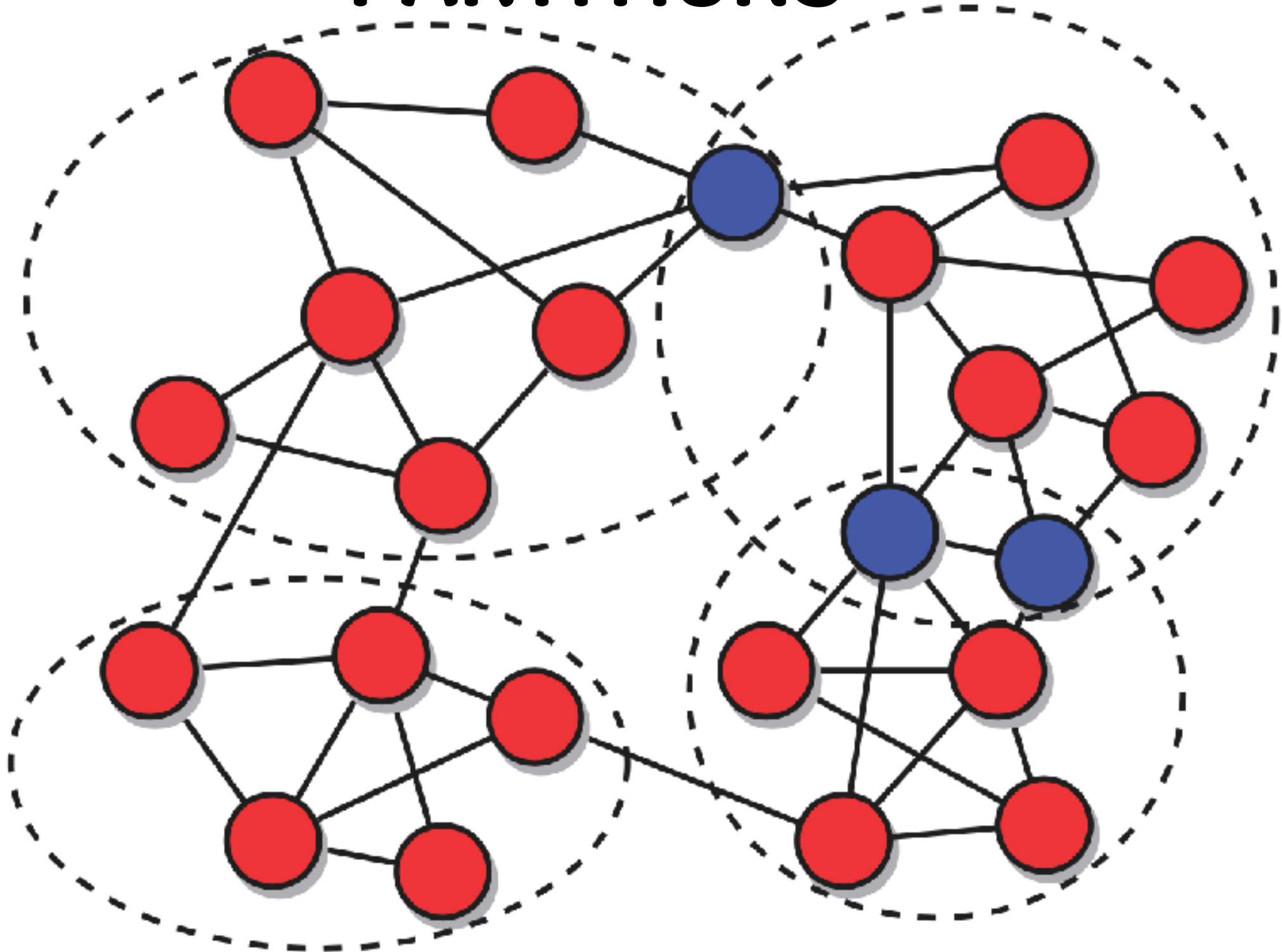
**THESE ARE ALL VALID PARTITIONS**

The number of possible partitions grows super-exponentially





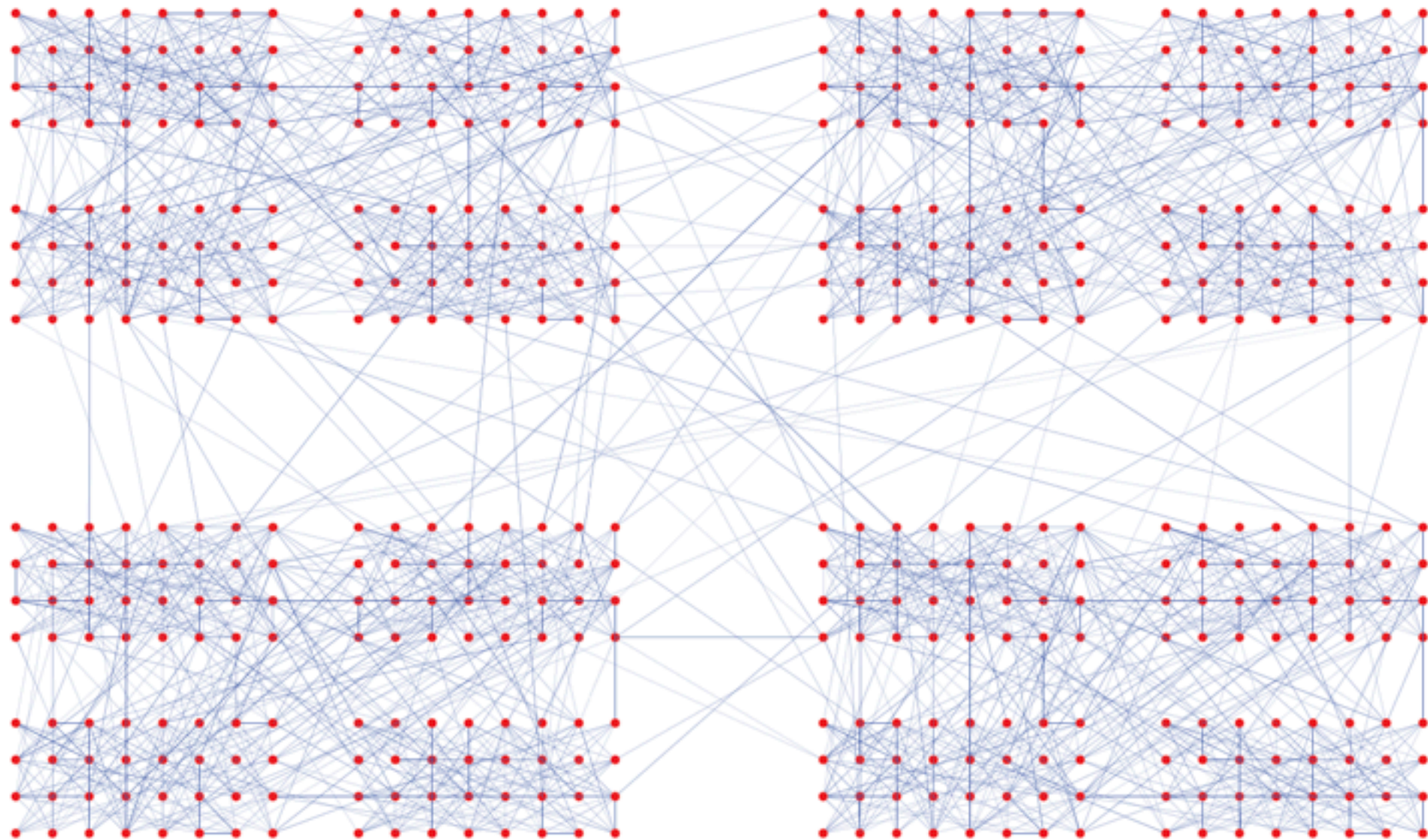
# PARTITIONS



**COMMUNITIES CAN OVERLAP**

(You are part of different communities, think about it)

# PARTITIONS

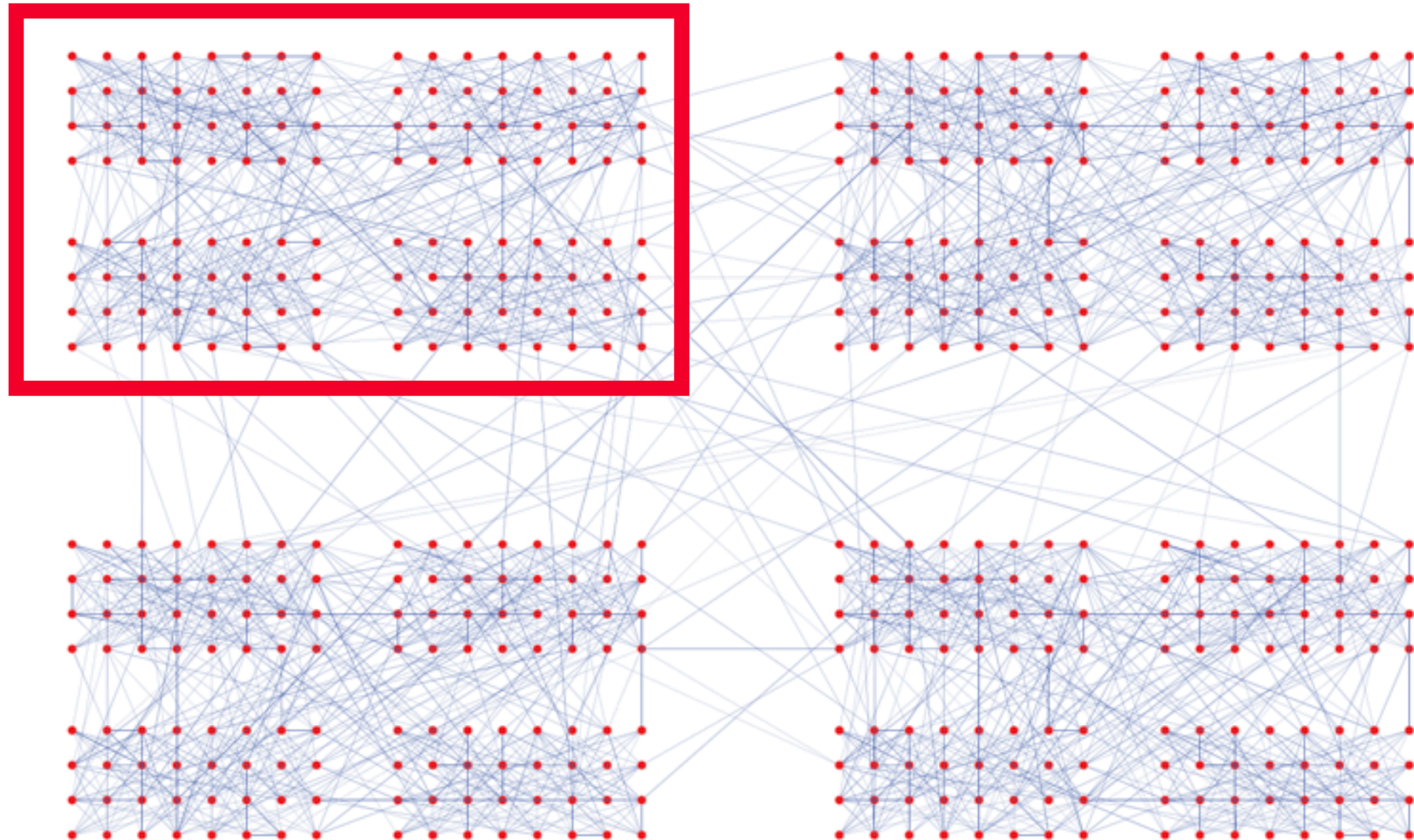


**COMMUNITIES CAN BE HIERARCHICAL**

**(There might be communities within communities)**



# PARTITIONS

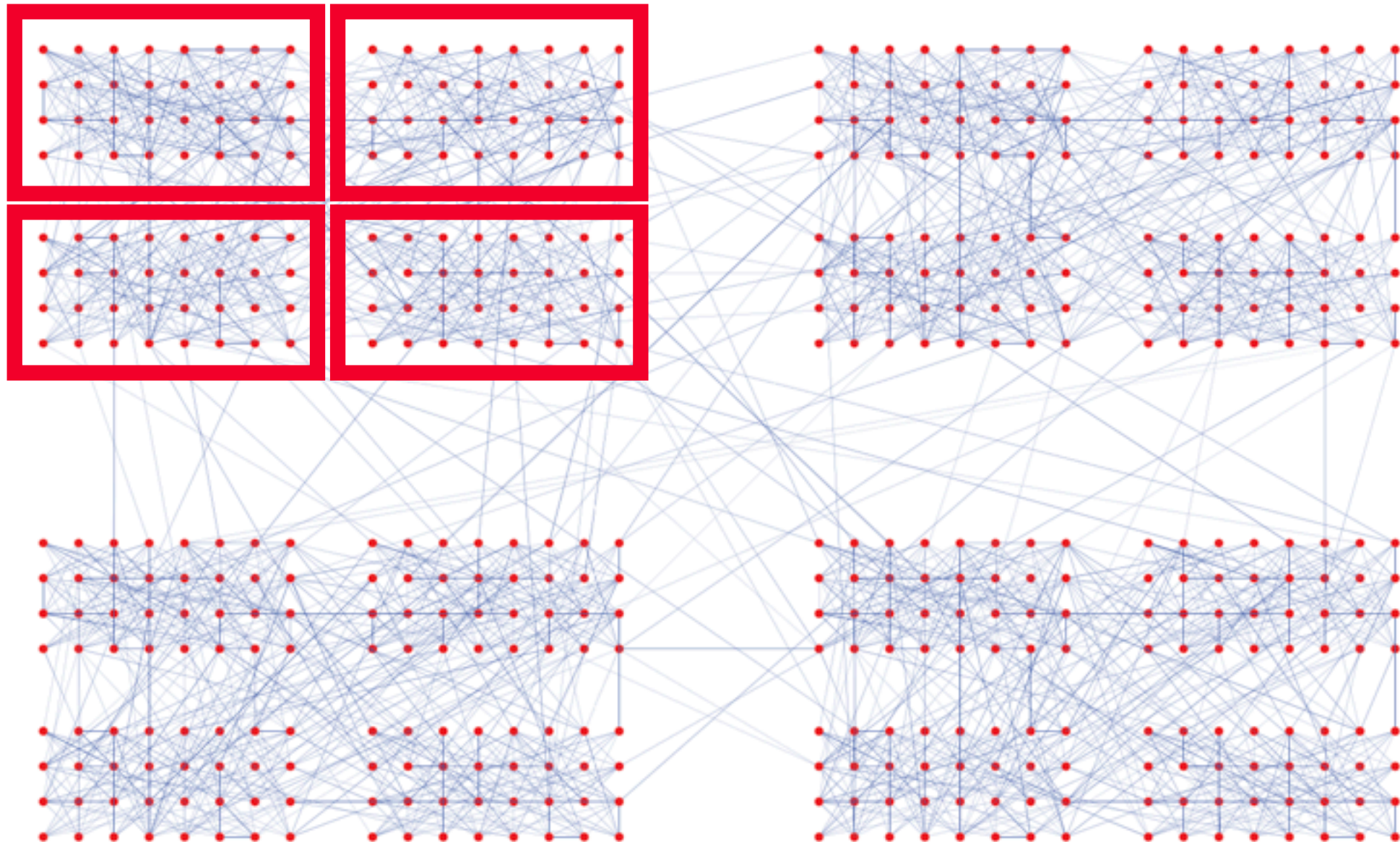


**COMMUNITIES CAN BE HIERARCHICAL**

**(There might be communities within communities)**



# PARTITIONS

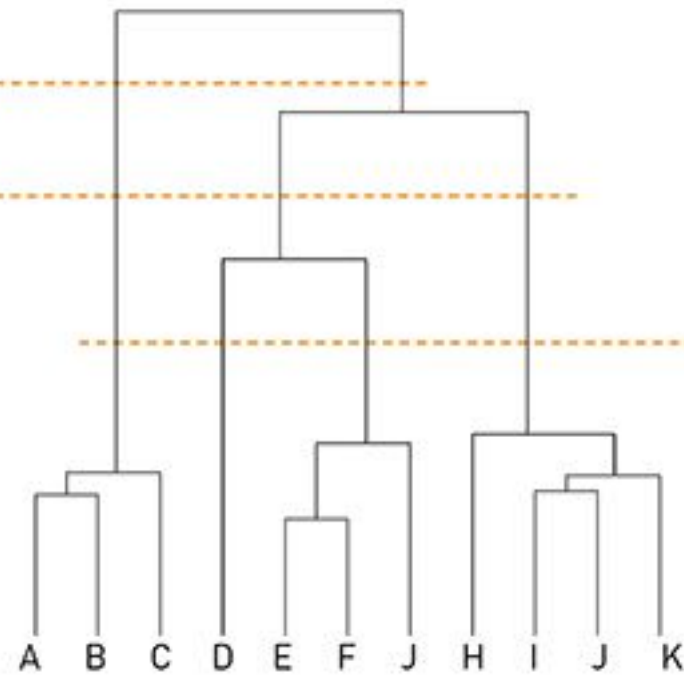


**COMMUNITIES CAN BE HIERARCHICAL**

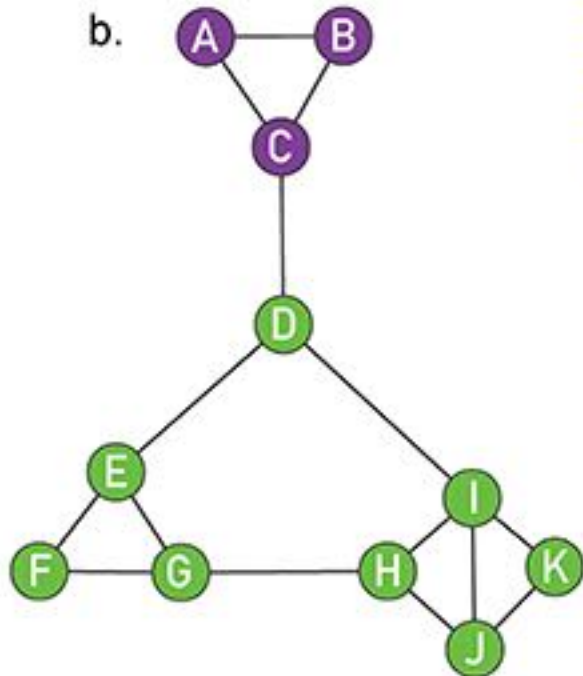
**(There might be communities within communities)**

# PARTITIONS

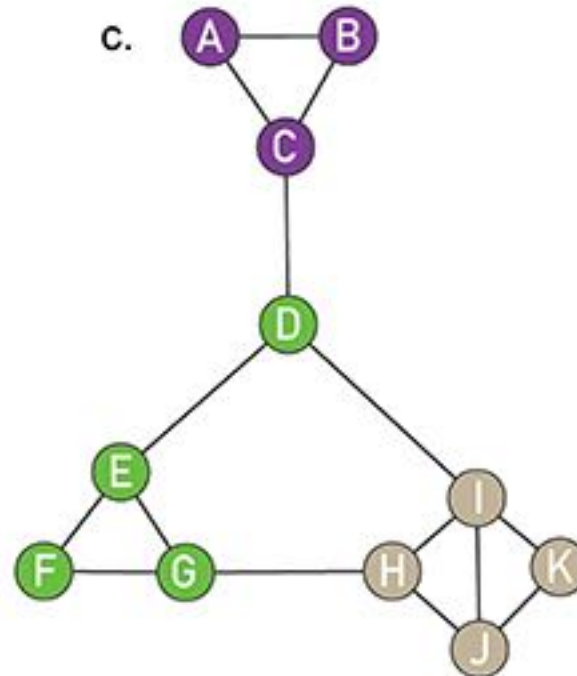
a.



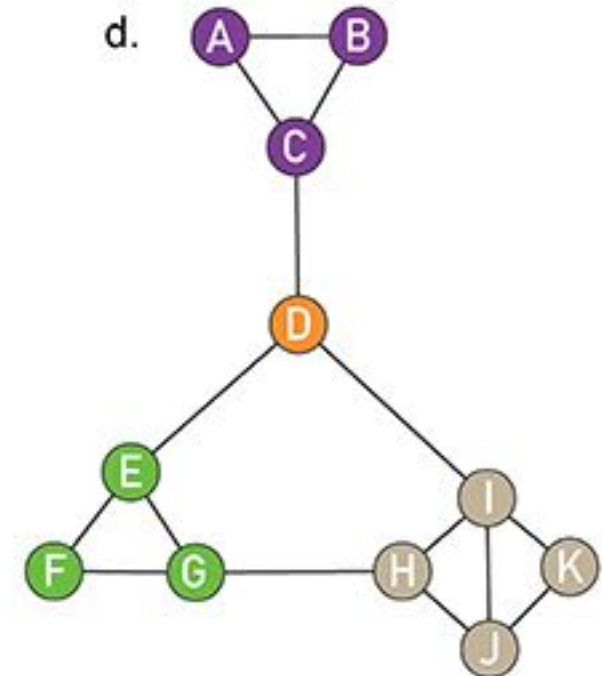
b.



c.

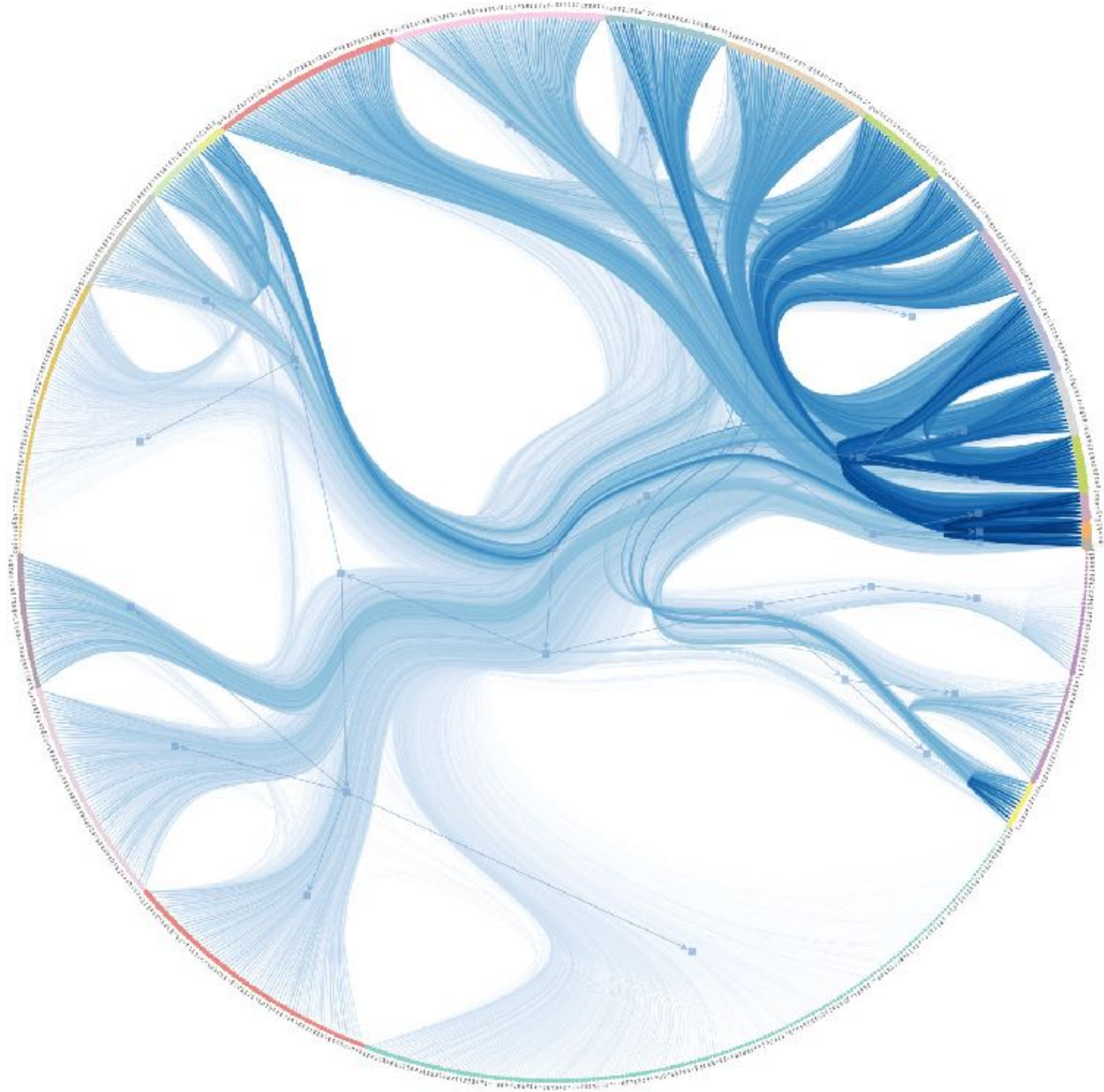


d.



# Dendrogram



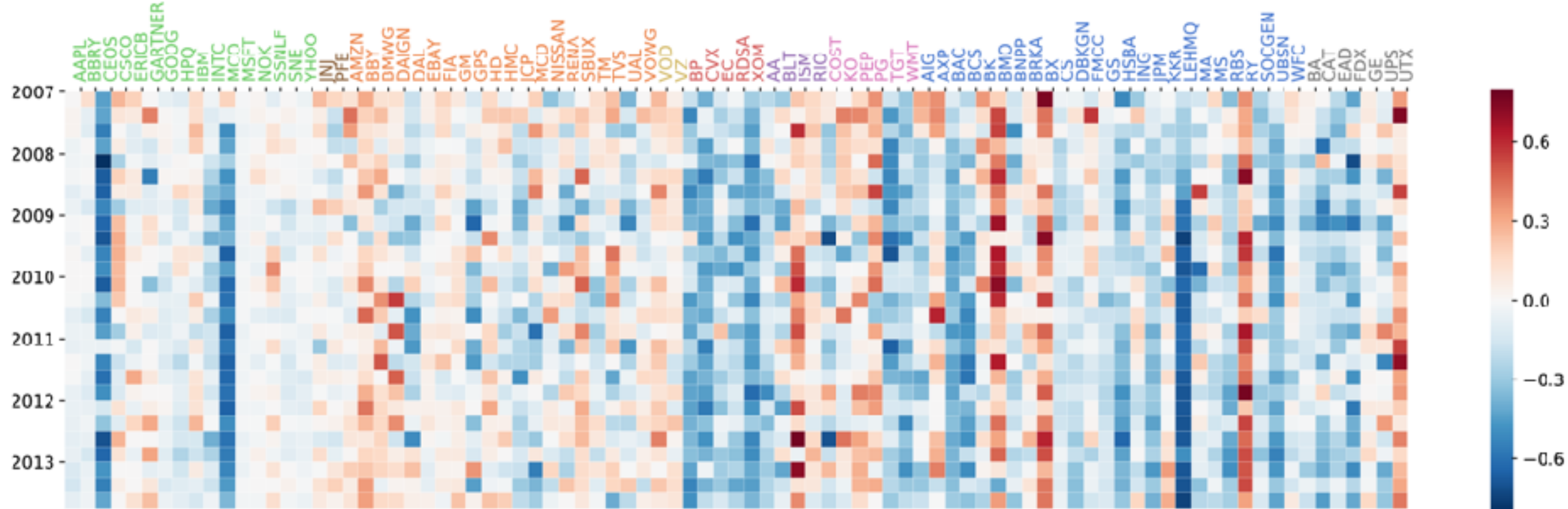


# **EXERCISE**

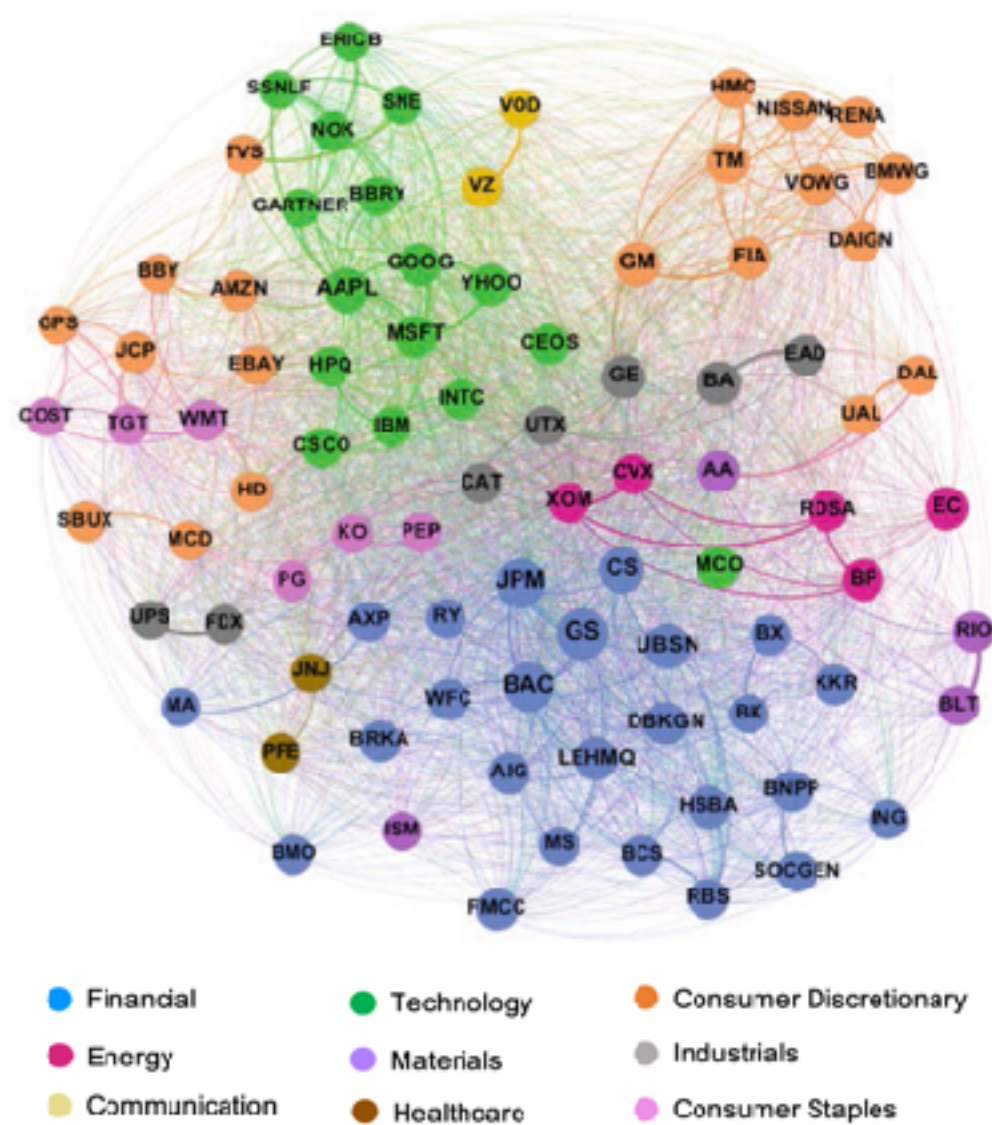
**MAKE SOME EXAMPLES  
OF SOCIAL AND FINANCIAL  
NETWORKS WITH COMMUNITIES**



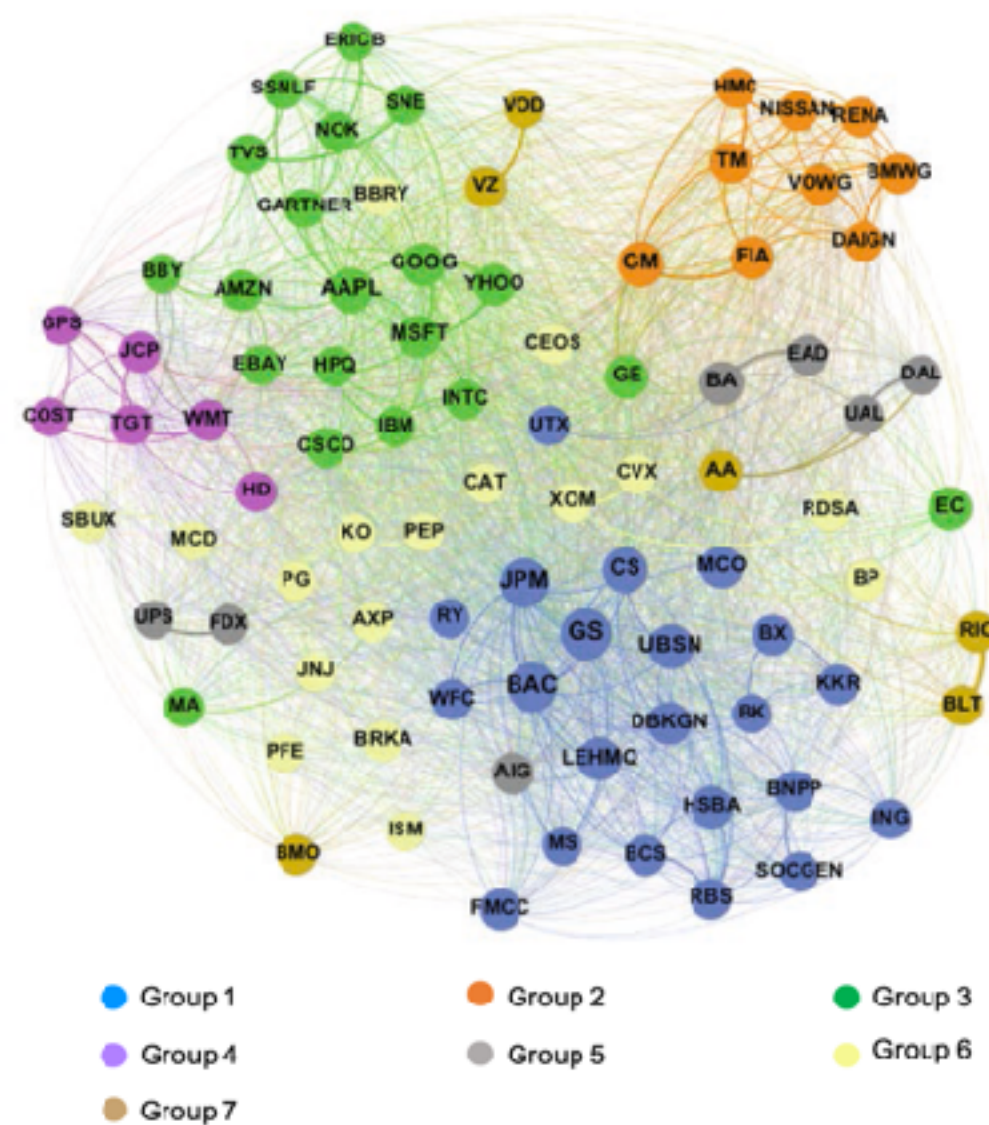
(a)



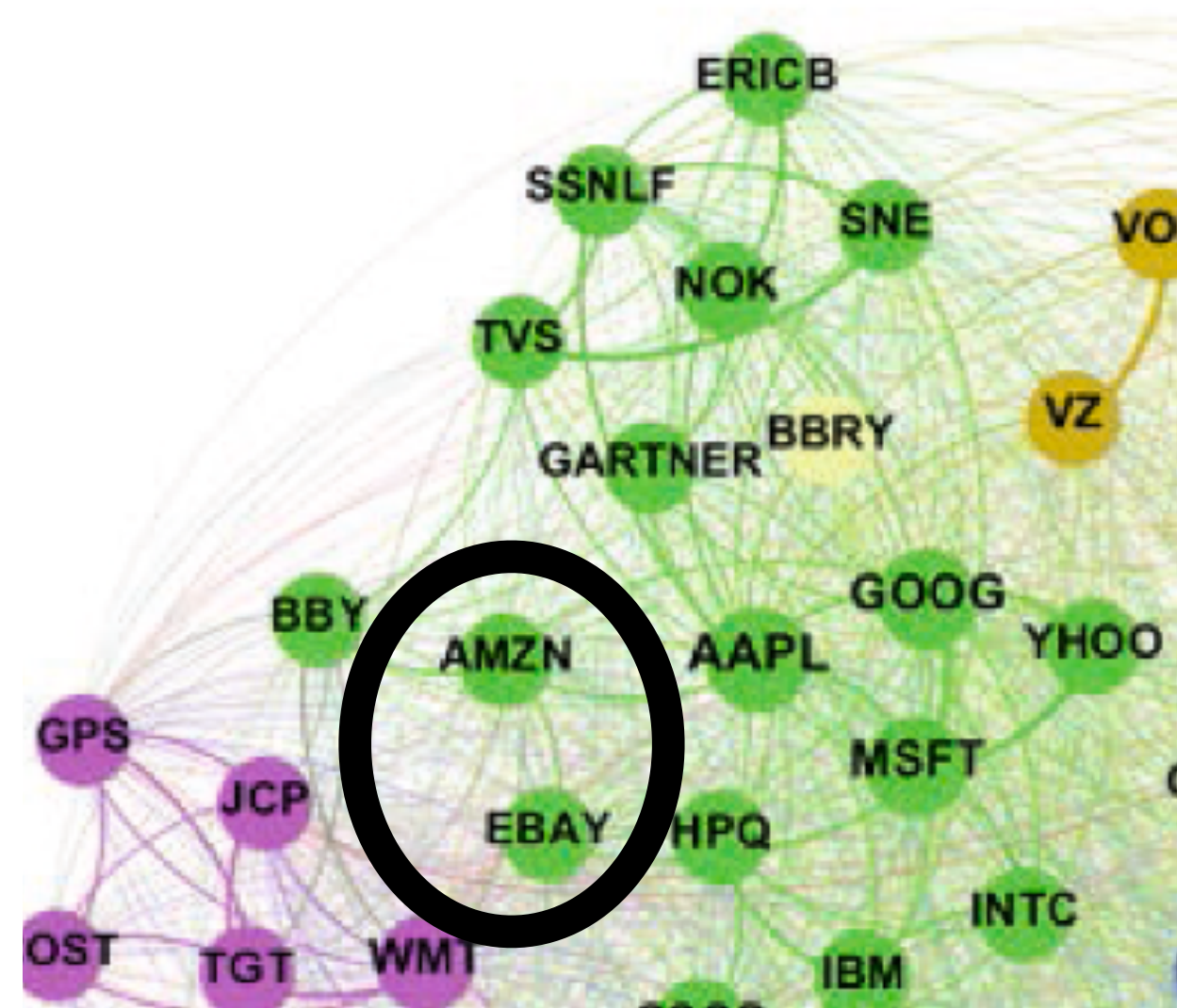
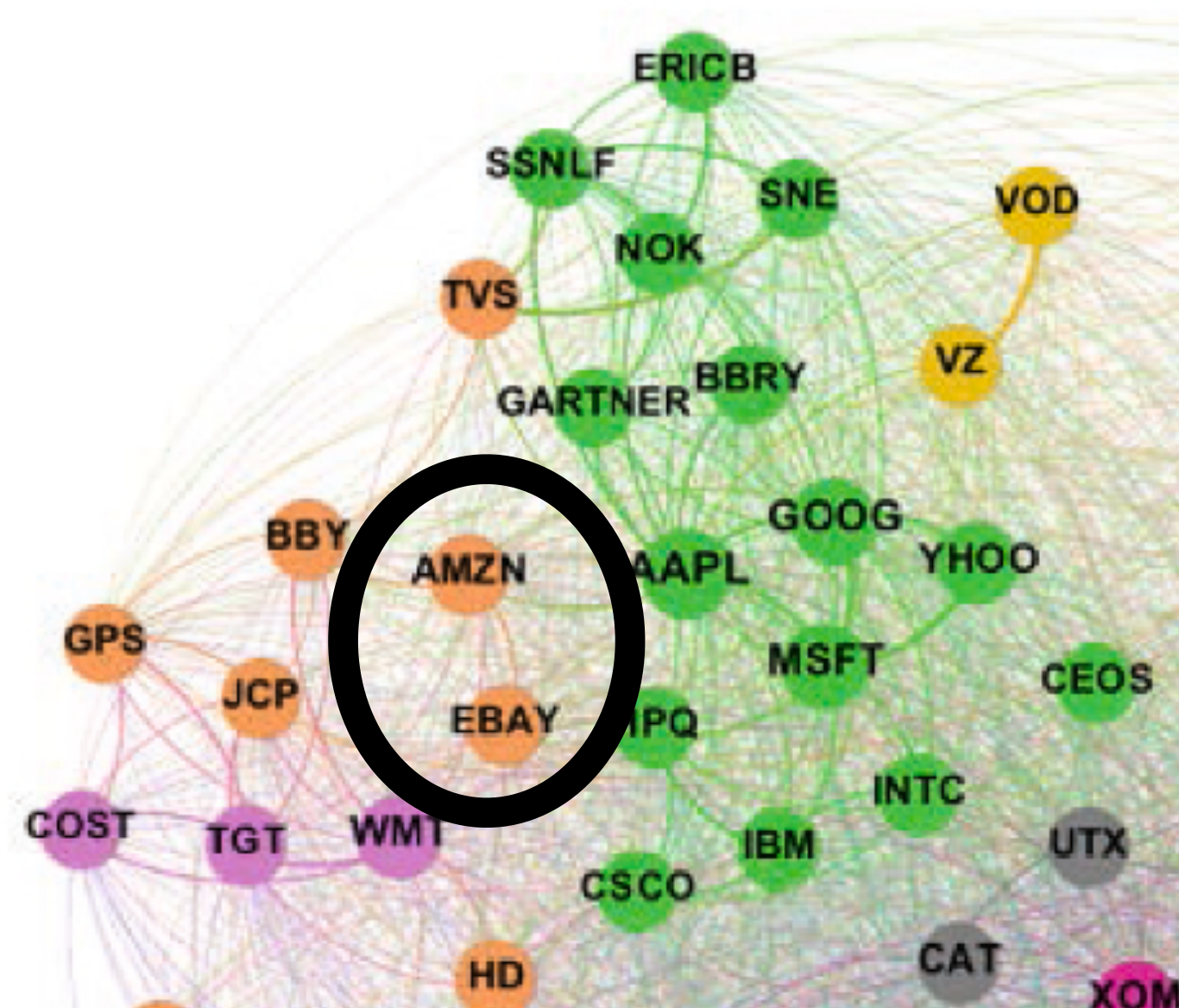
(b)



(c)







- |   |   |   |
|---|---|---|
| <span style="color: blue;">●</span> Financial       | <span style="color: green;">●</span> Technology | <span style="color: orange;">●</span> <u>Consumer Discretionary</u> |
| <span style="color: pink;">●</span> Energy          | <span style="color: purple;">●</span> Materials | <span style="color: grey;">●</span> Industrials                     |
| <span style="color: yellow;">●</span> Communication | <span style="color: brown;">●</span> Healthcare | <span style="color: lightpink;">●</span> Consumer Staples           |



