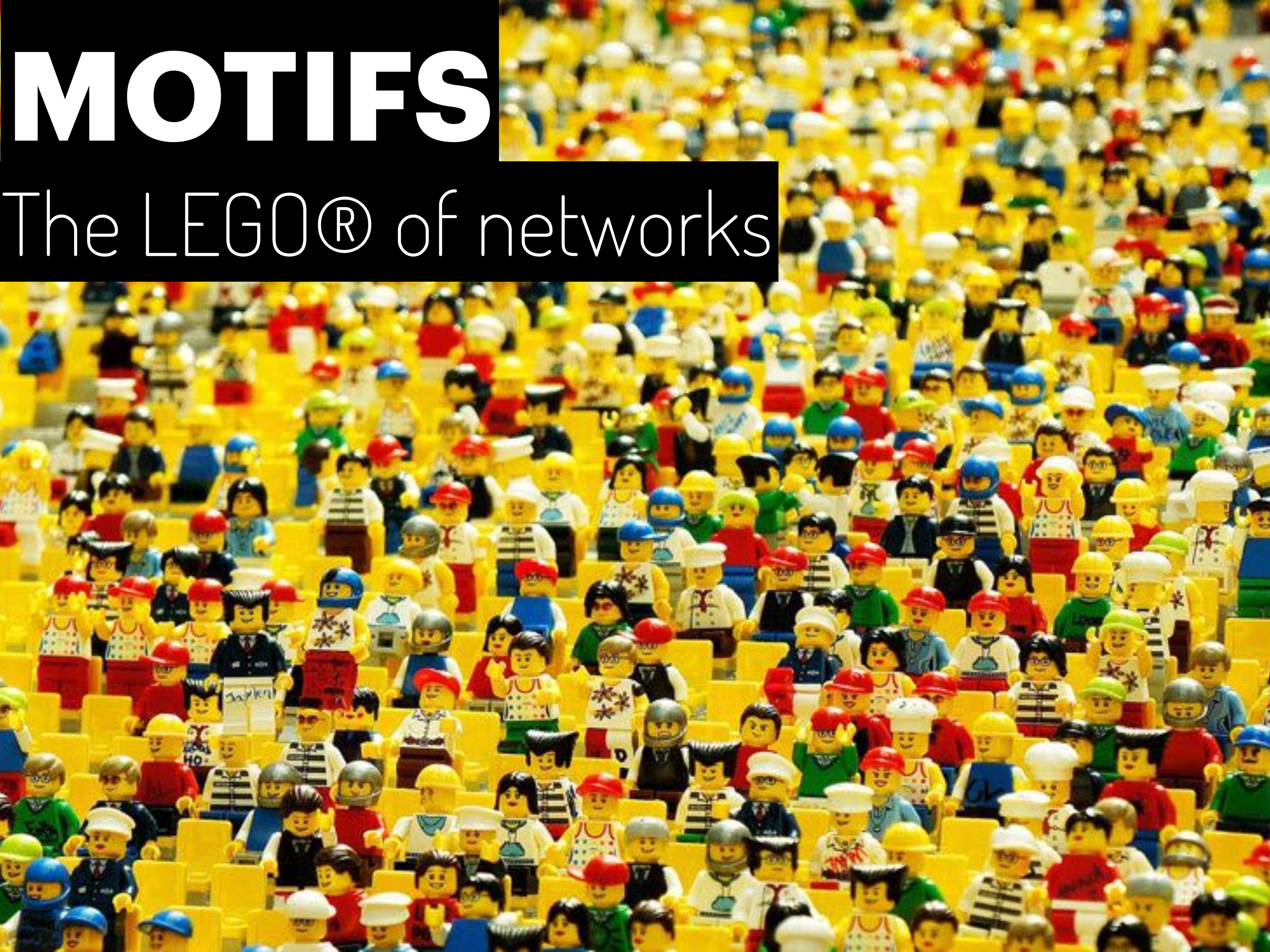
MOTIFS

The LEGO® of networks

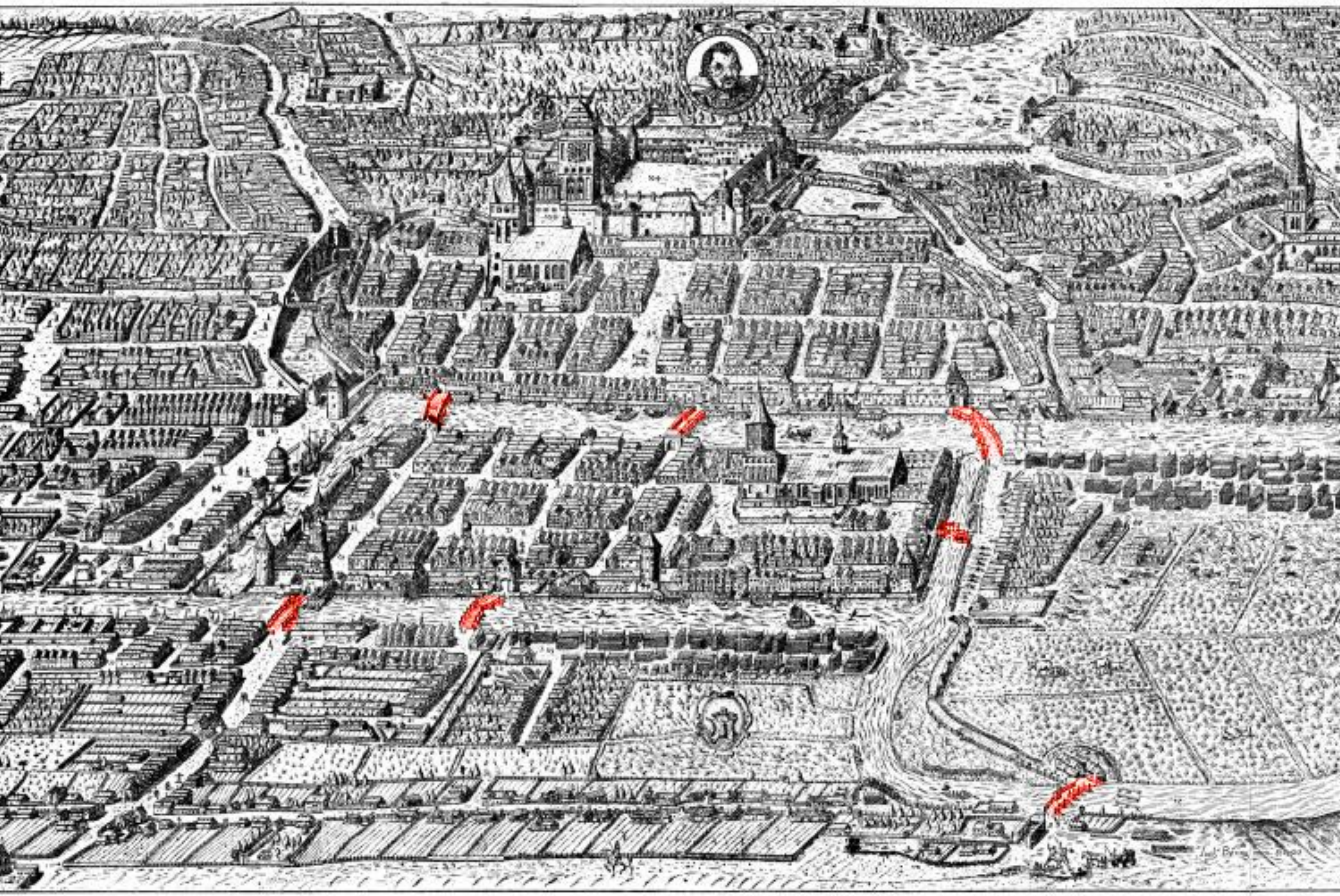


LEARNING OUTCOMES

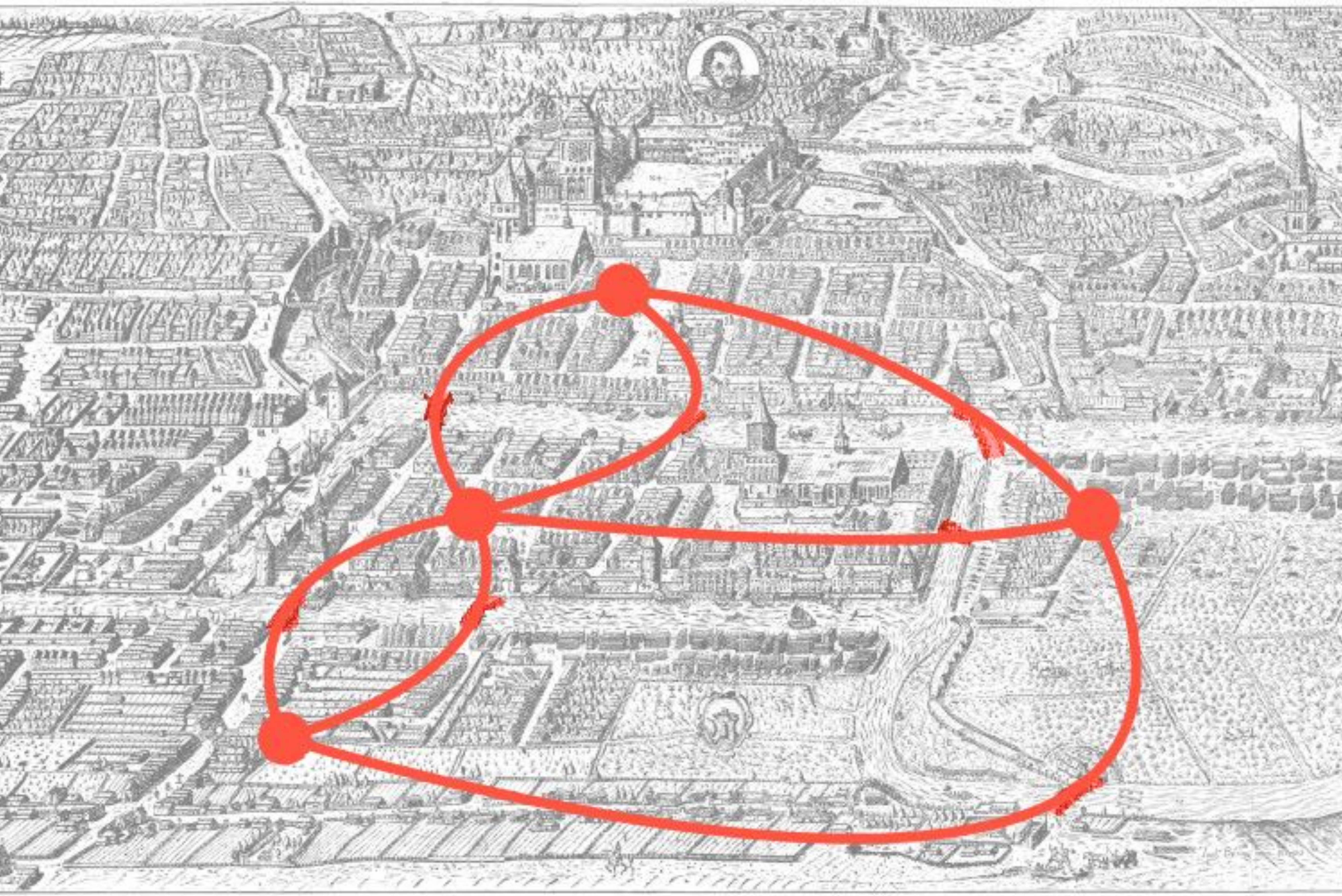
Understand **what motifs are**

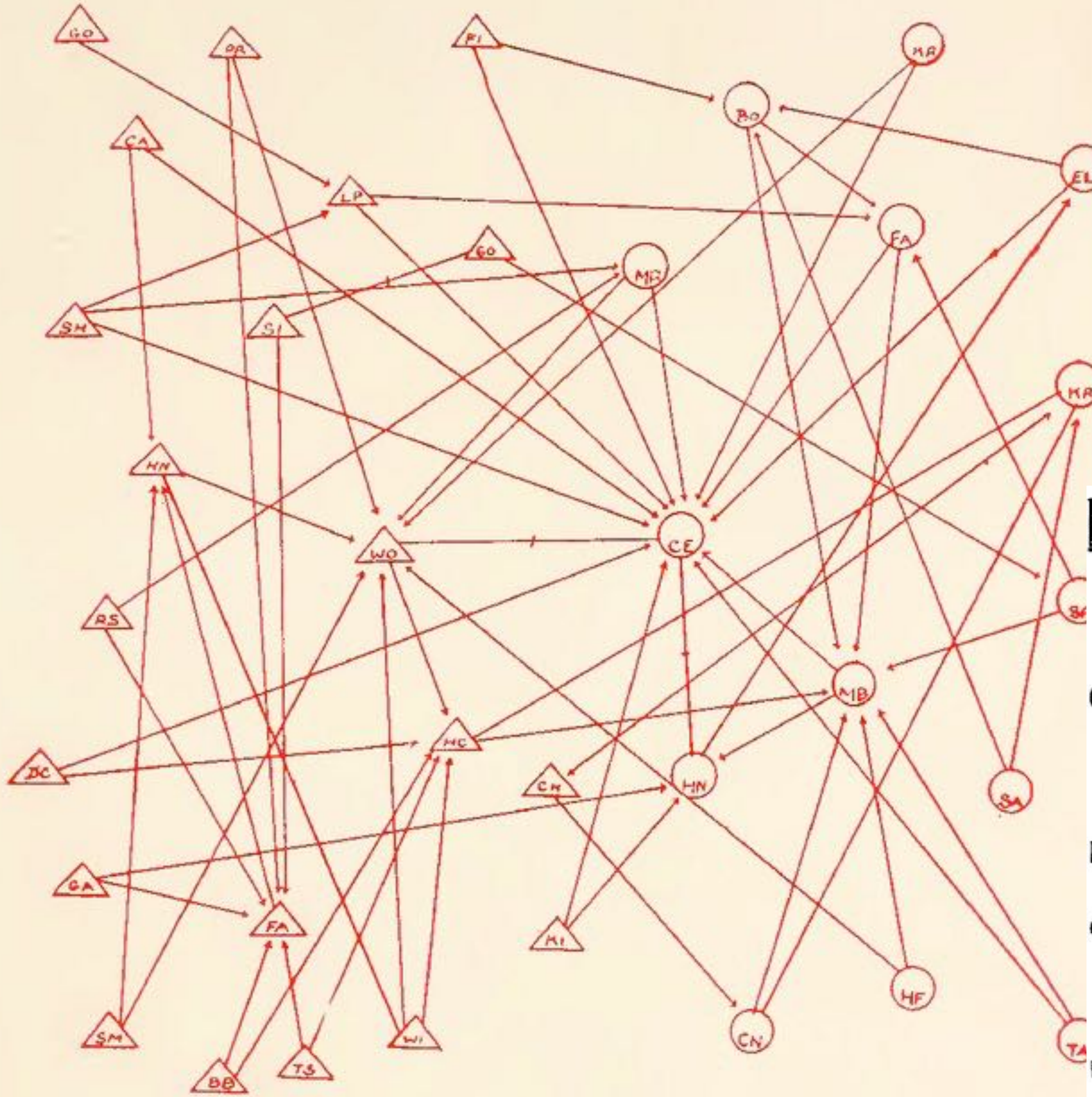
Analyse a network using motifs

Count motifs using different algorithms zur sechshundert jährigen Jubelfeier der Königlichen Haupt und Residenz=Stadt Königsberg



zur sechshundert jährigen Jubelfeier der Königlichen Haupt und Residenz-Stadt Königsberg





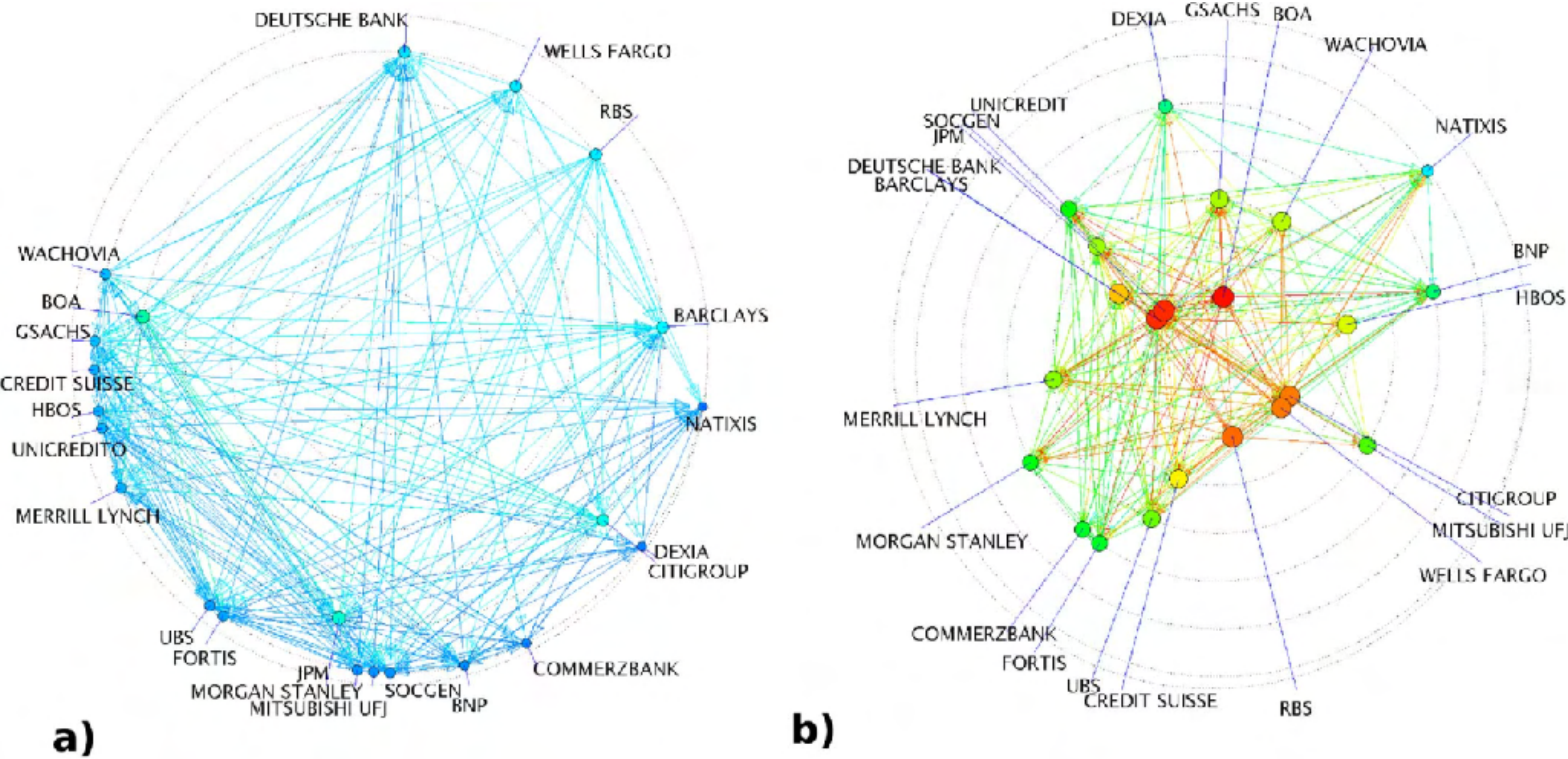
EMOTIONS MAPPED BY NEW GEOGRAPHY

Charts Seek to Portray the Psychological Currents of Human Relationships.

FIRST STUDIES EXHIBITED

Colored Lines Show Likes and Dislikes of Individuals and of Groups.

MANY MISFITS REVEALED



DEUTSCHE BANK
WELLS FARGO
RBS
WACHOVIA
BOA
GSACHS
BARCLAYS
CREDIT SUISSE
HBOS
NATIXIS
UNICREDITO
MERRILL LYNCH
DEXIA
CITIGROUP
UBS
FORTIS
JPM
MORGAN STANLEY
MITSUBISHI UFJ
SOCGEN
BNP
COMMERZBANK
a)
DEXIA
GSACHS
BOA
WACHOVIA
UNICREDIT
SOCGEN
JPM
DEUTSCHE BANK
BARCLAYS
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MORGAN STANLEY
CITIGROUP
MITSUBISHI UFJ
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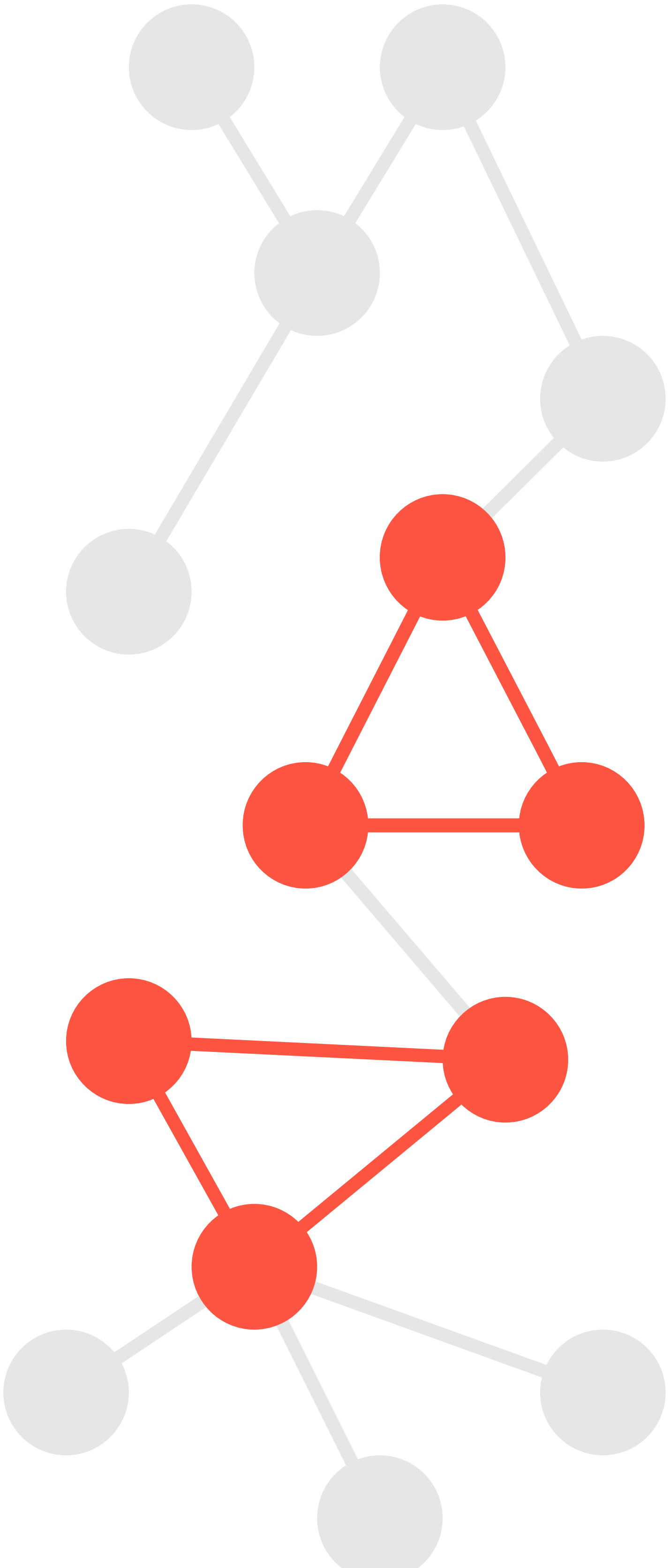
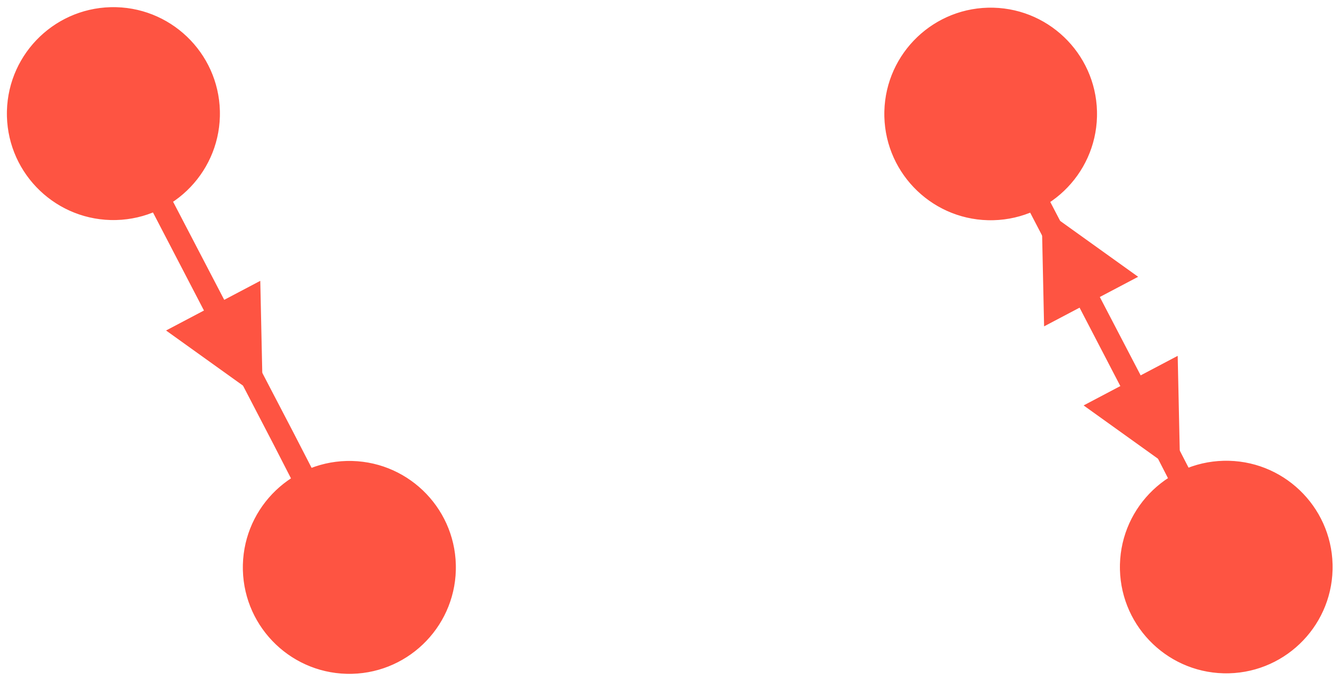
COMMERZBANK
FORTIS
UBS
CREDIT SUISSE
RBS
b)

What do these networks have in common?

What do these networks have in common?

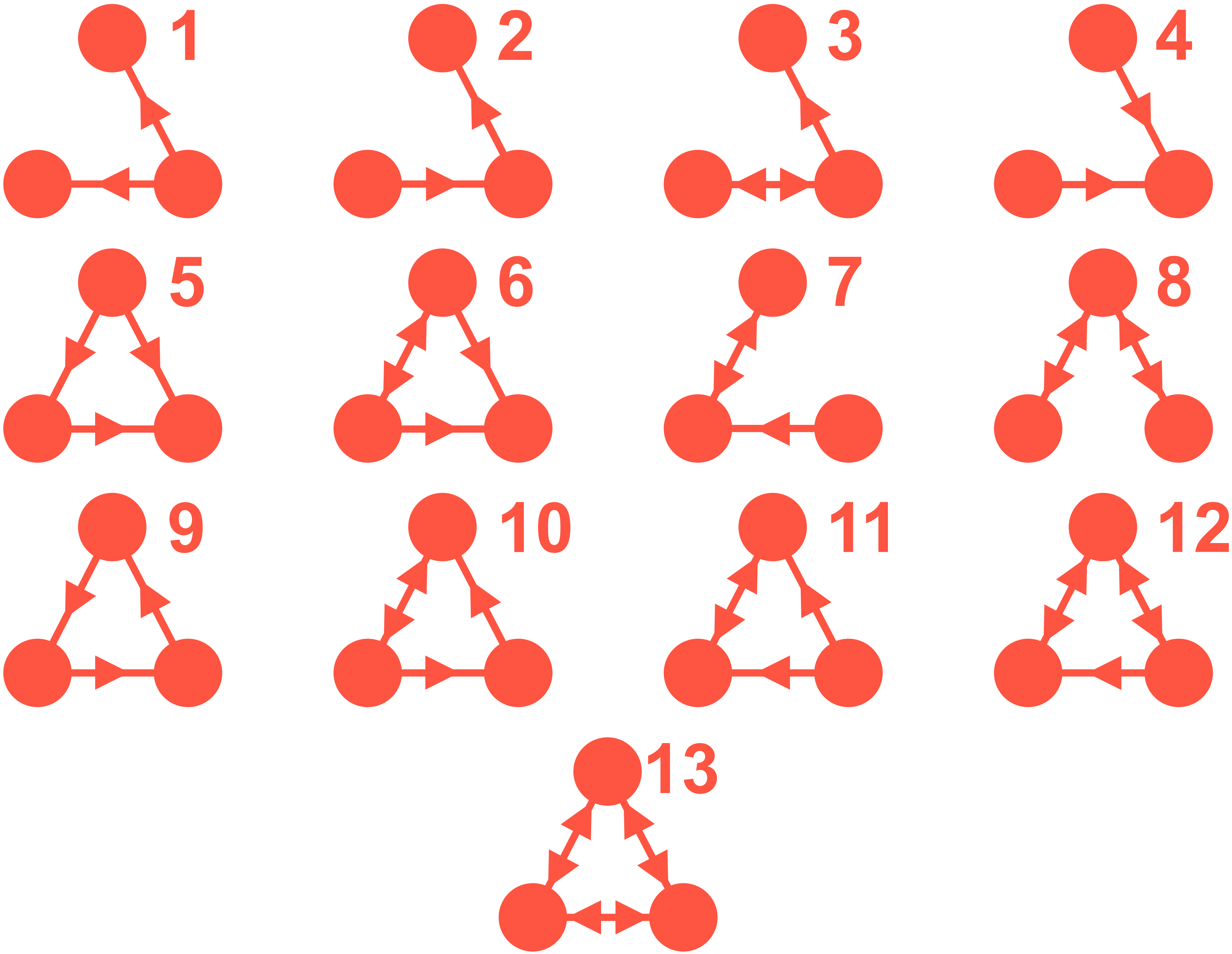


Dyads



Milo et al. **Network Motifs: Simple Building Blocks of Complex Networks** (2002) - Science

Triads



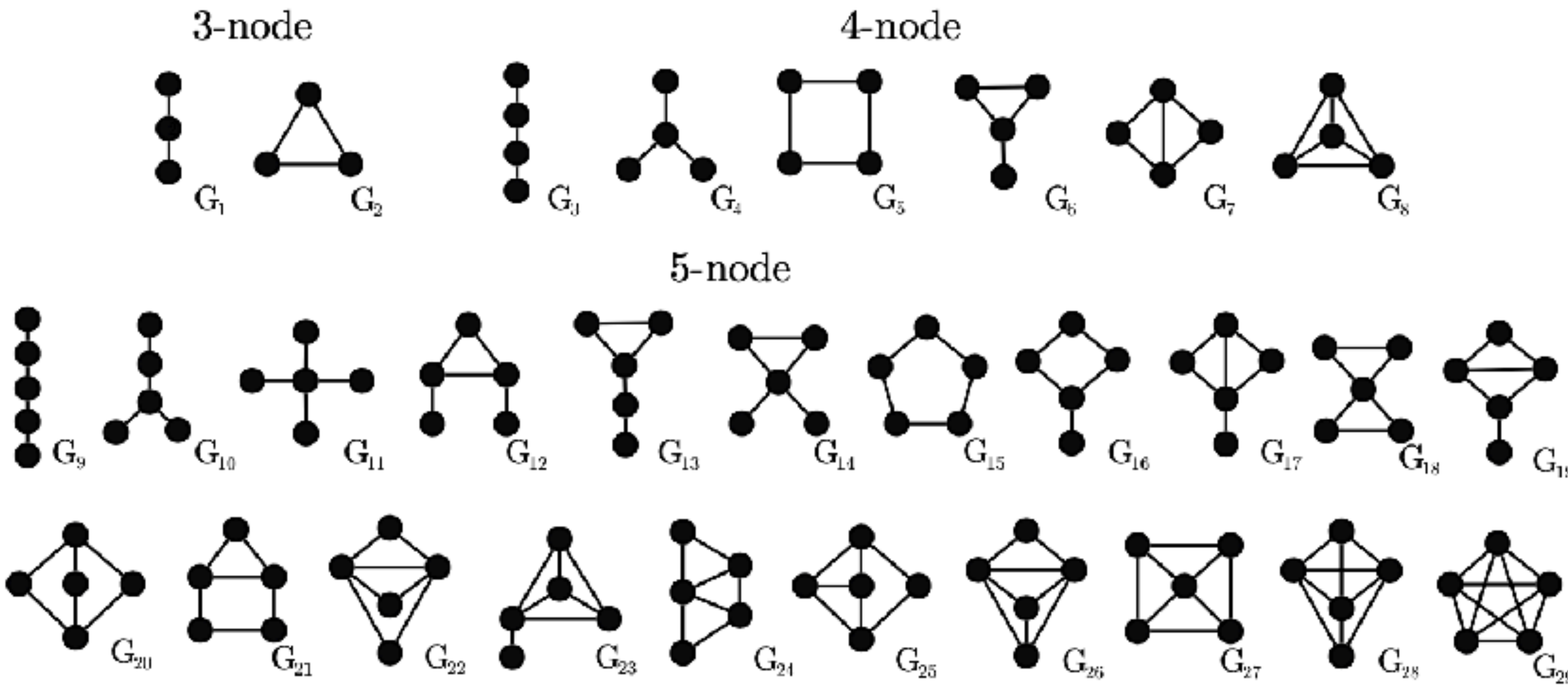
4 nodes and above (directed)

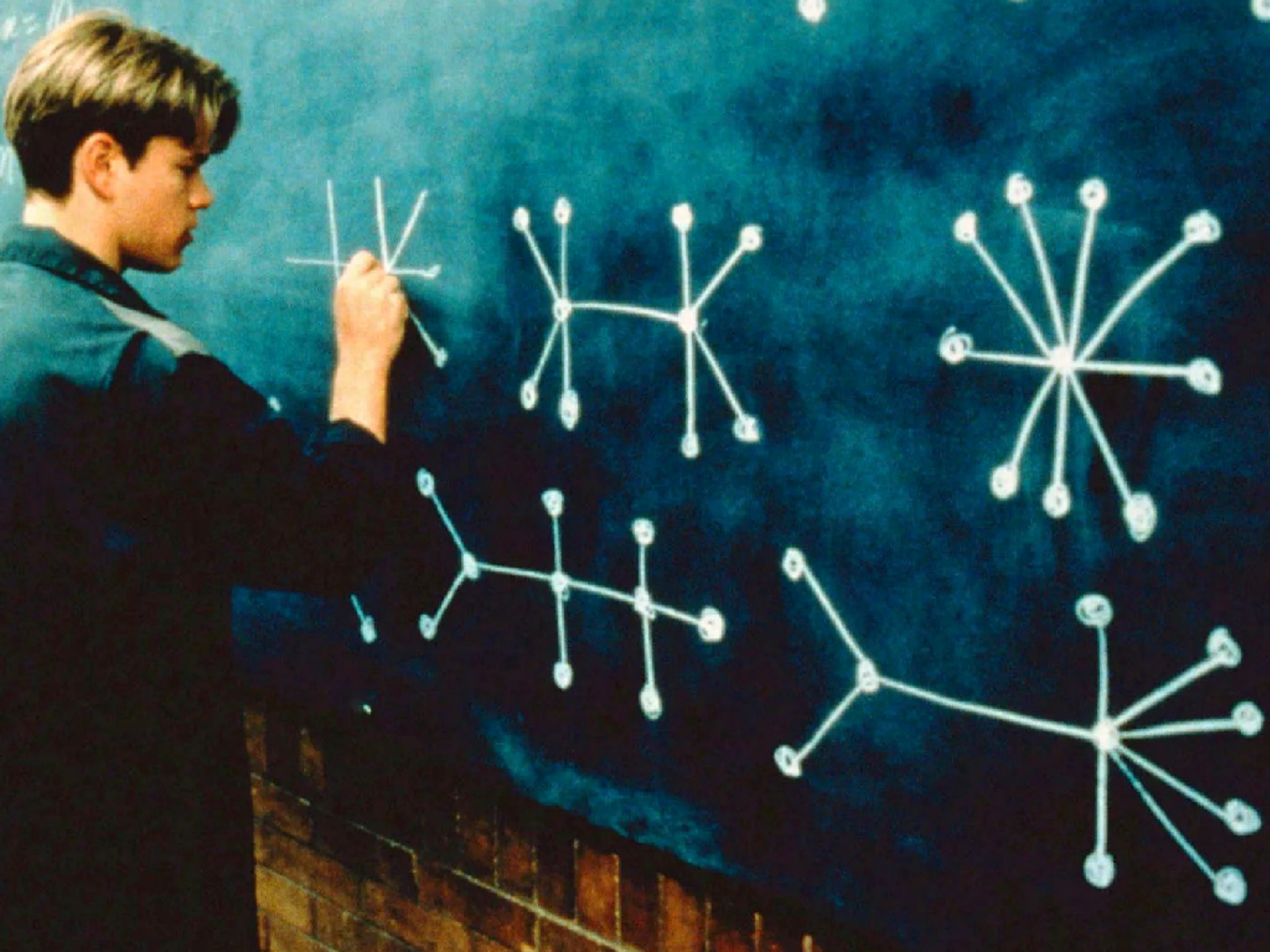
Quite **too many**

Difficult to compute

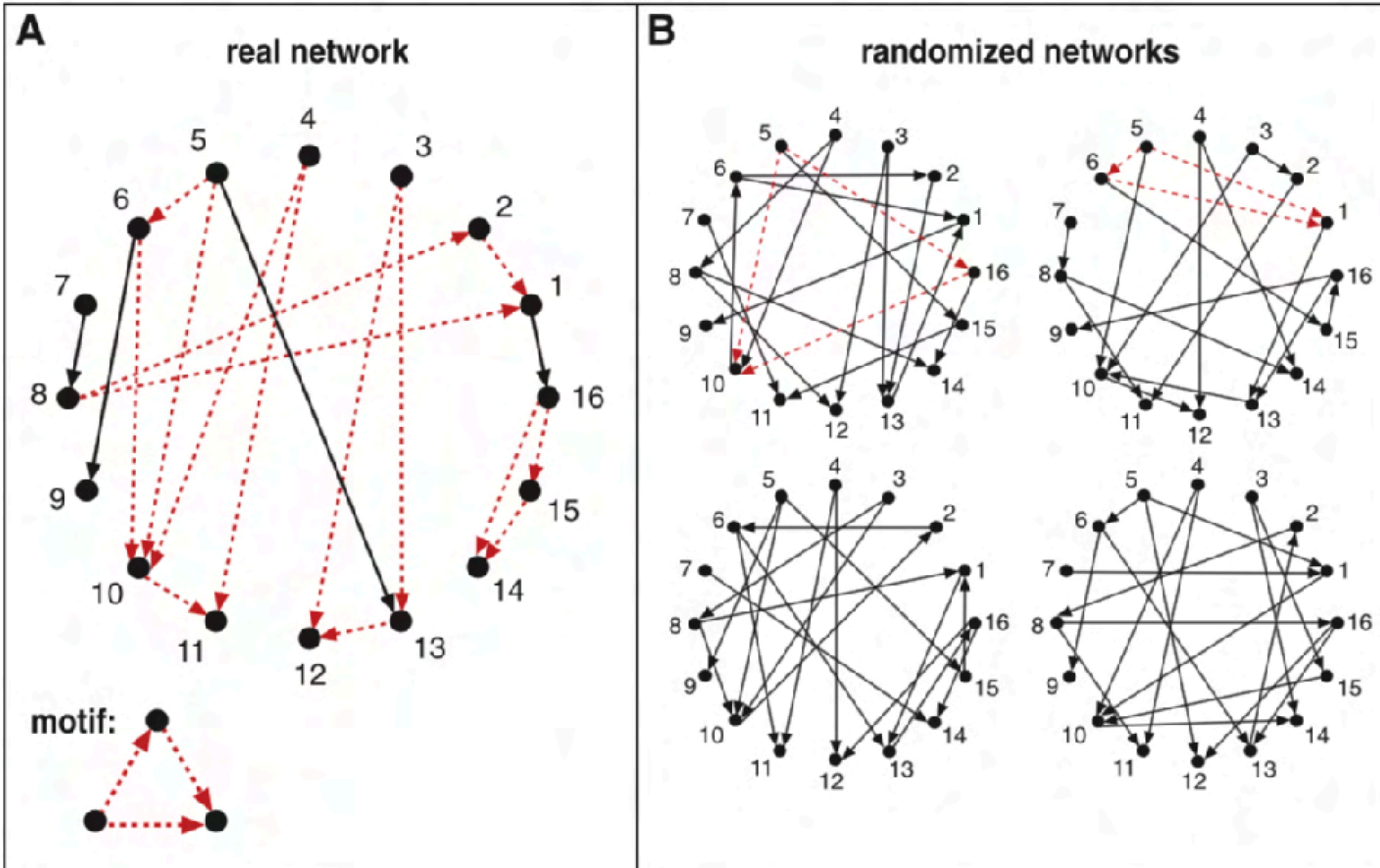
Often **No clear** explanation

4 nodes and above (undirected)

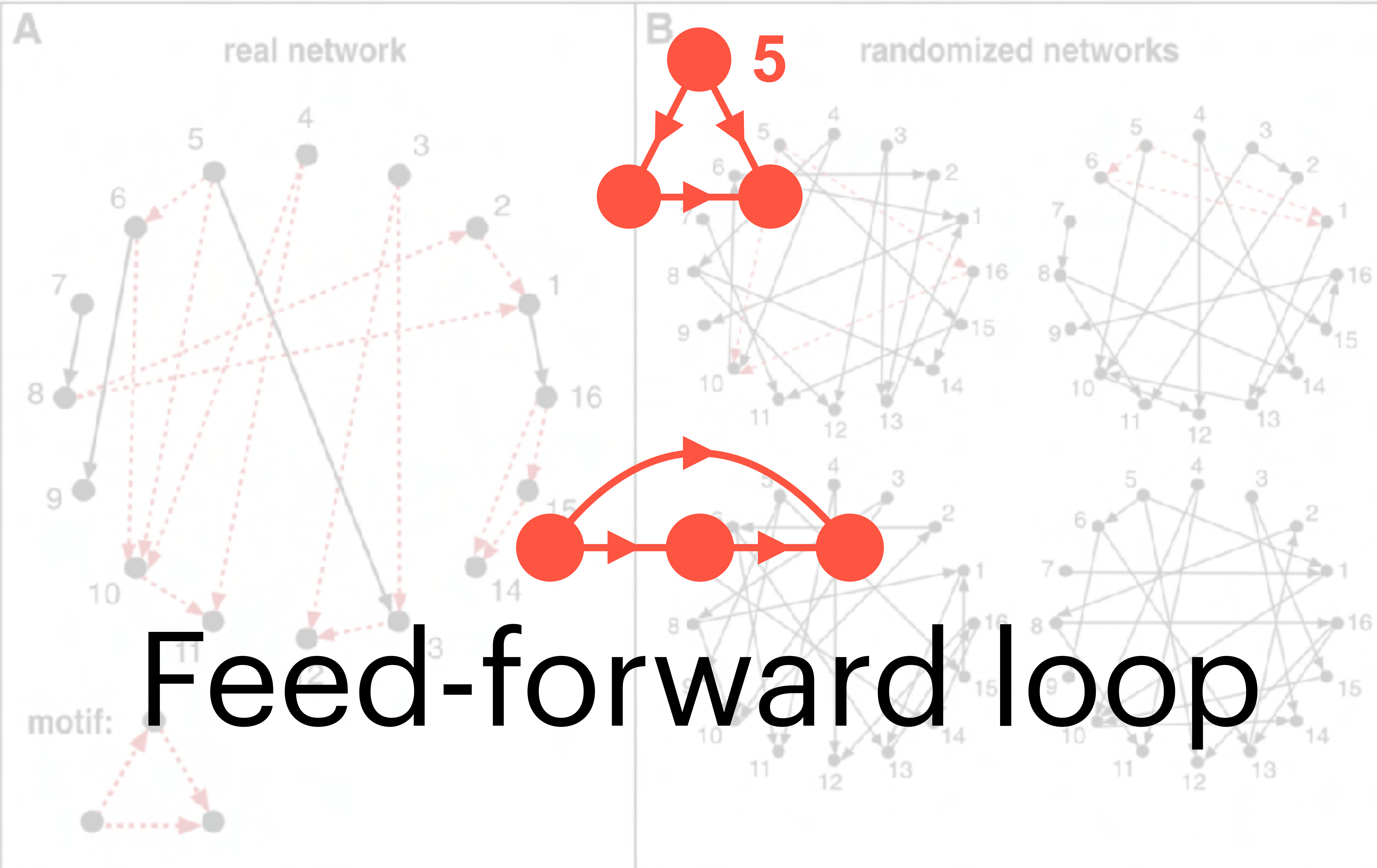





Matt Damon!



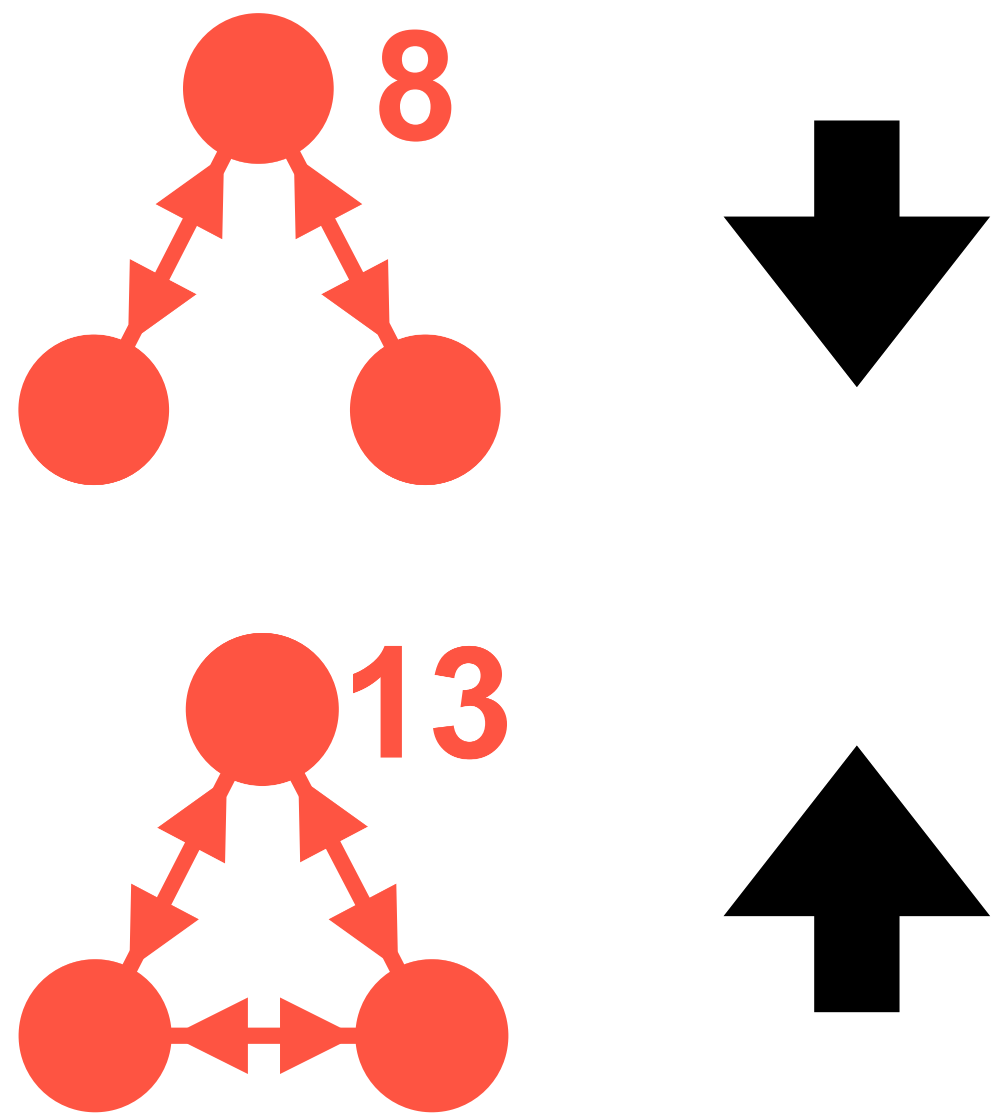
A
real network
B
randomized networks
motif:
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16



Feed-forward loop

↑	↓
Neurons	Food webs
Gene regulation	www
Electronic circuits	

Terrorism



Felmlee et al. **Social Network Motifs: A Comparison of Building Blocks across Multiple Social Networks** (2017) dais-ita

Motifs profiling

Phase I

Counting

Types of algorithms

Approach	**Type**
Enumeration	“Classical”
	Single subgraph
	Encapsulation
Analytic	Matrix-based
	Decomposition

Table 2. Overview of all major exact algorithms.

	Year	Approach	Type	k-restriction	Orbit	Directed	Code
Mfinder [121]	2002	Enum.	Classical	None	✗	✓	[9]
ESU [193, 196]	2005	Enum.	Classical	None	✗	✓	[194]
Itzhack [71]	2007	Enum.	Classical	≤ 5	✗	✓	✗
Grochow [56]	2007	Enum.	Single-subgraph	None	✗	✓	✗
Kavosh [78]	2009	Enum.	Classical	None	✗	✓	[122]
GTries [147, 149]	2010	Enum.	Encapsulation	None	✓	✓	[144]
Rage [102, 103]	2010	Analytic	Decomposition	≤ 5	✗	✓	[104]
NeMo [85]	2011	Enum.	Single-subgraph	None	✗	✓	[155]
Netmode [92]	2012	Enum.	Encapsulation	≤ 6	✗	✓	[93]
SCMD [185]	2012	Enum.	Encapsulation	None	✗	✗	✗
acc-Motif [110, 111]	2012	Analytic	Decomposition	≤ 6	✗	✓	[109]
ISMAGS [40, 68]	2013	Enum.	Single-subgraph	None	✗	✓	[133]
Quatexelero [80]	2013	Enum.	Encapsulation	None	✗	✓	[81]
FaSE [130]	2013	Enum.	Encapsulation	None	✗	✓	[145]
ENSA [205]	2014	Enum.	Encapsulation	None	✗	✓	✗
Orca [62, 63]	2014	Analytic	Matrix-based	≤ 5	✓	✗	[64]
Hash-ESU [75]	2015	Enum.	Encapsulation	None	✗	✓	✗
Song [176]	2015	Enum.	Encapsulation	None	✗	✓	✗
Ortmann [127, 128]	2016	Analytic	Matrix-based	≤ 4	✓	✓	✗
PGD [3, 5]	2016	Analytic	Decomposition	≤ 4	✓	✗	[2]
Patcomp [61]	2017	Enum.	Encapsulation	None	✗	✓	✗
Escape [136]	2017	Analytic	Decomposition	≤ 5	✓	✗	[168]
Jesse [112, 114]	2017	Analytic	Matrix-based	None	✓	✗	[113]

Classical enumeration algorithms

Main idea:

1) randomly **pick** a node

Classical enumeration algorithms

Main idea:

1) randomly **pick** a node

2) **count** all motifs this node forms

Classical enumeration algorithms

Main idea:

1) randomly **pick** a node

2) **count** all motifs this node forms

3) **remove** the node

Classical enumeration algorithms

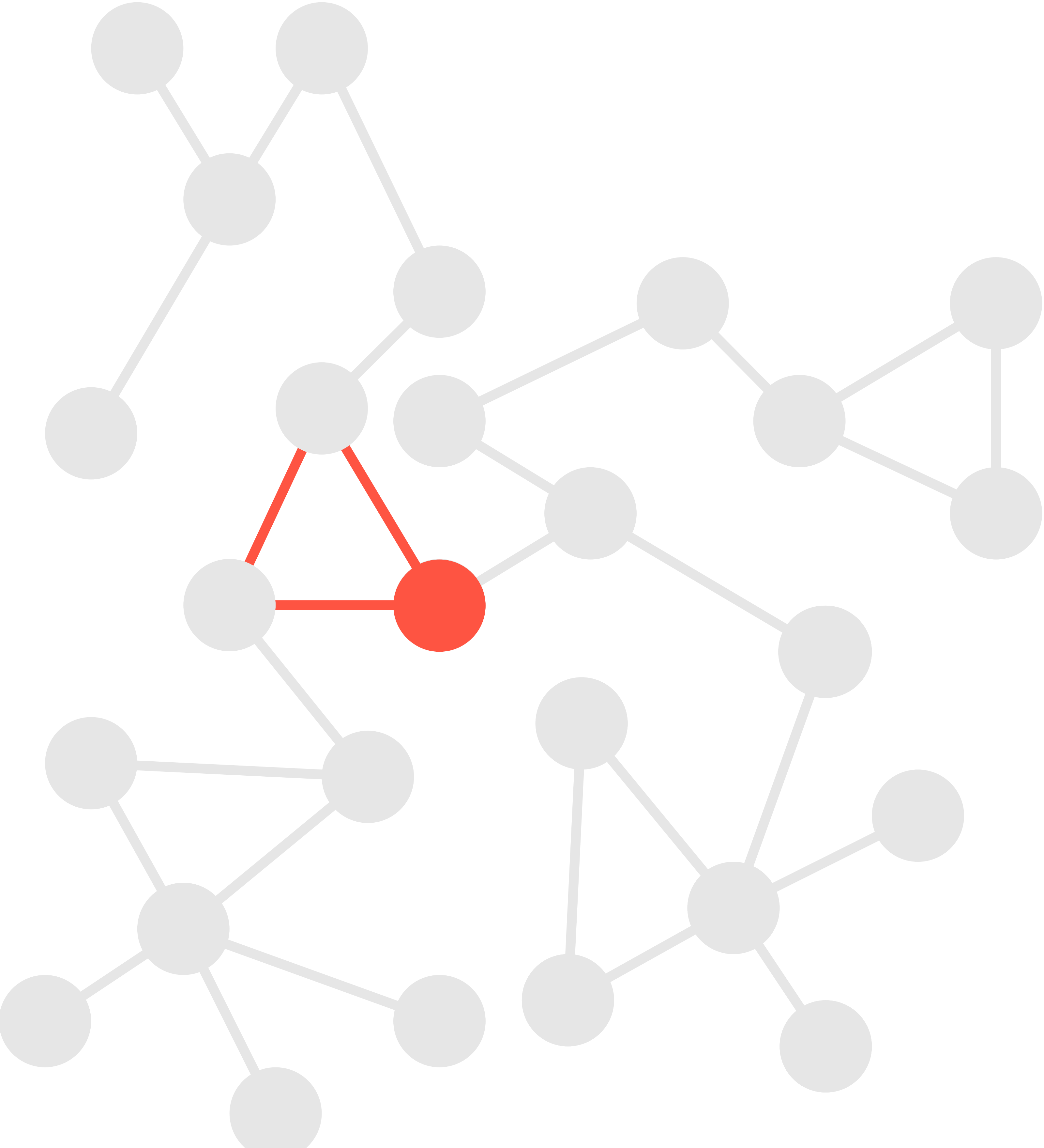
Main idea:

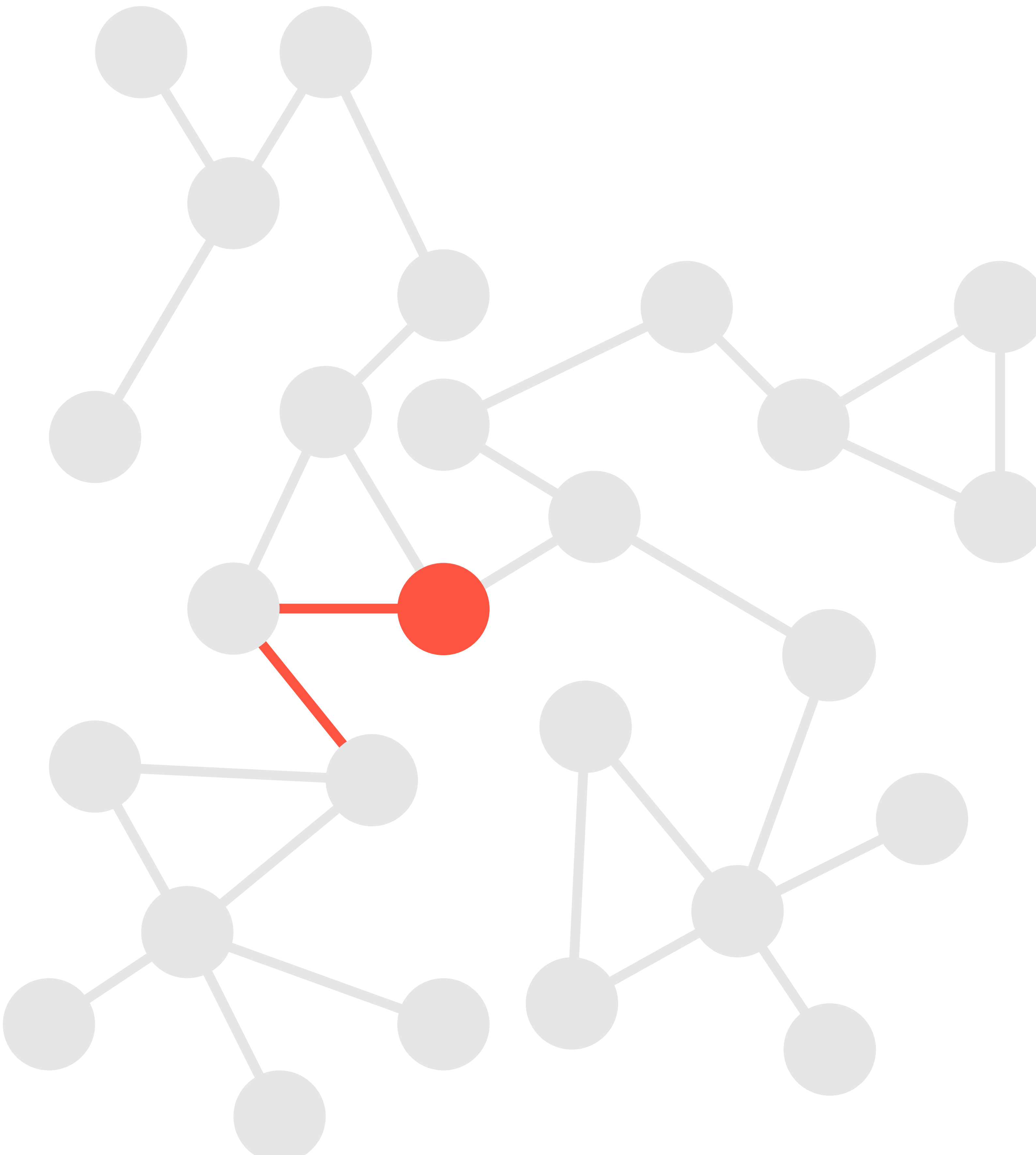
1) randomly **pick** a node

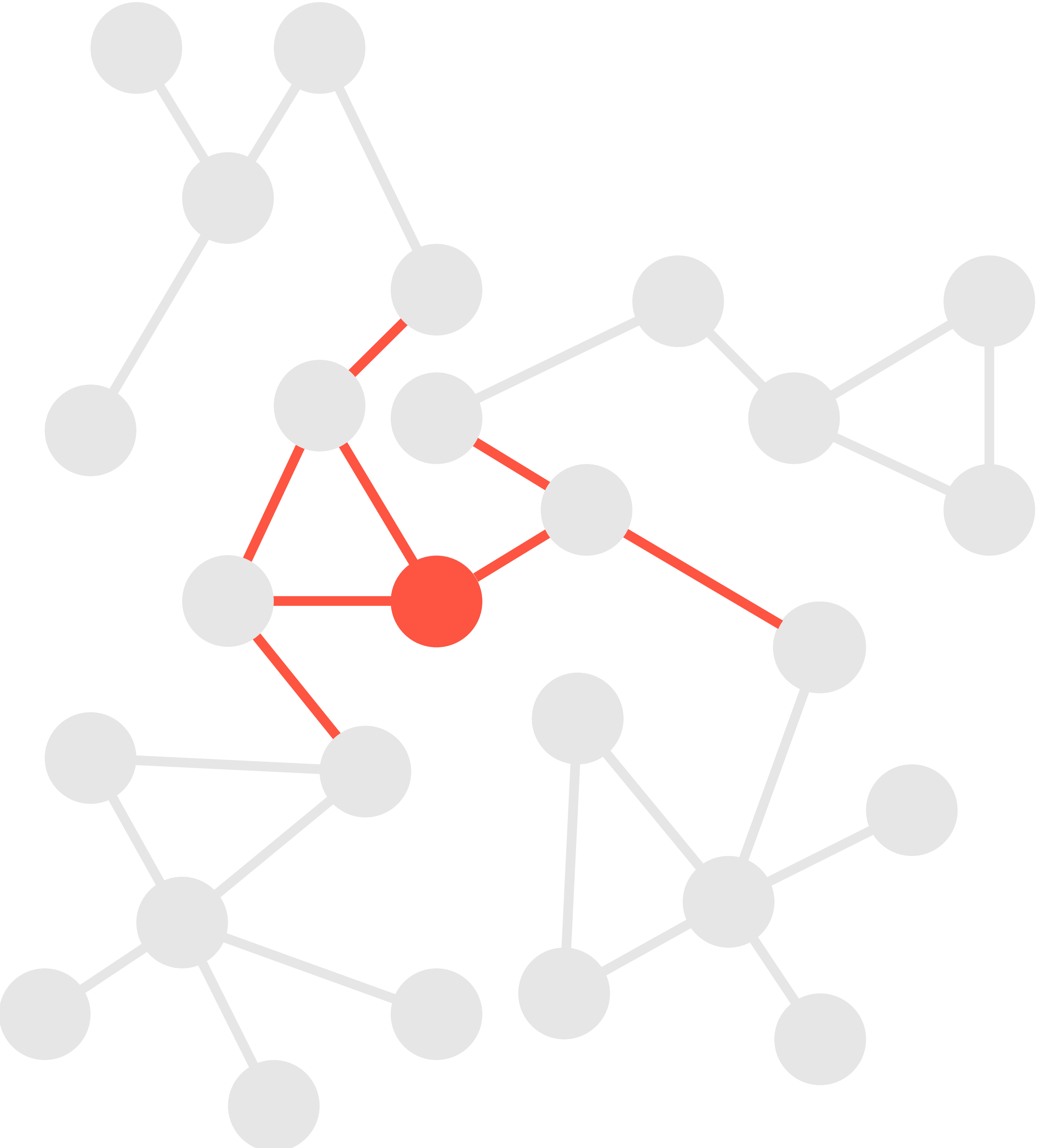
2) **count** all motifs this node forms

3) **remove** the node

4) **repeat** until no nodes left







Motifs profiling

Phase II

Motifs profiling

Phase II

Benchmarking

Random network

Simple to generate

Simple to count motifs on

Not a **good** representation of real networks

Z-score

Quantity that compares the **difference** between the observed and the expected value in **units of standard deviation**

Z-score

$$z_x = \frac{x - \langle x \rangle}{\sigma[x]}$$

Z-score

Motifs count

$$z_x = \frac{x - \langle x \rangle}{\sigma[x]}$$

Z-score

Motifs count

Benchmark average count

$$z_x = \frac{x - \langle x \rangle}{\sigma[x]}$$

Z-score

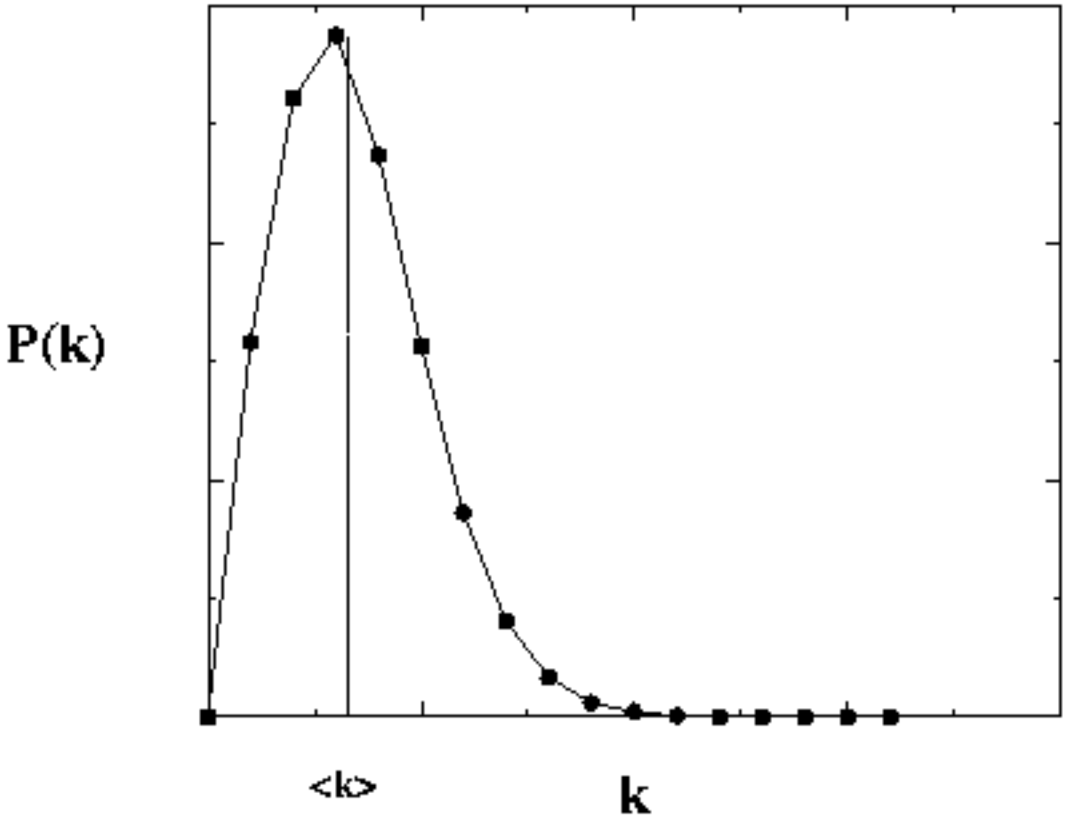
Motifs count

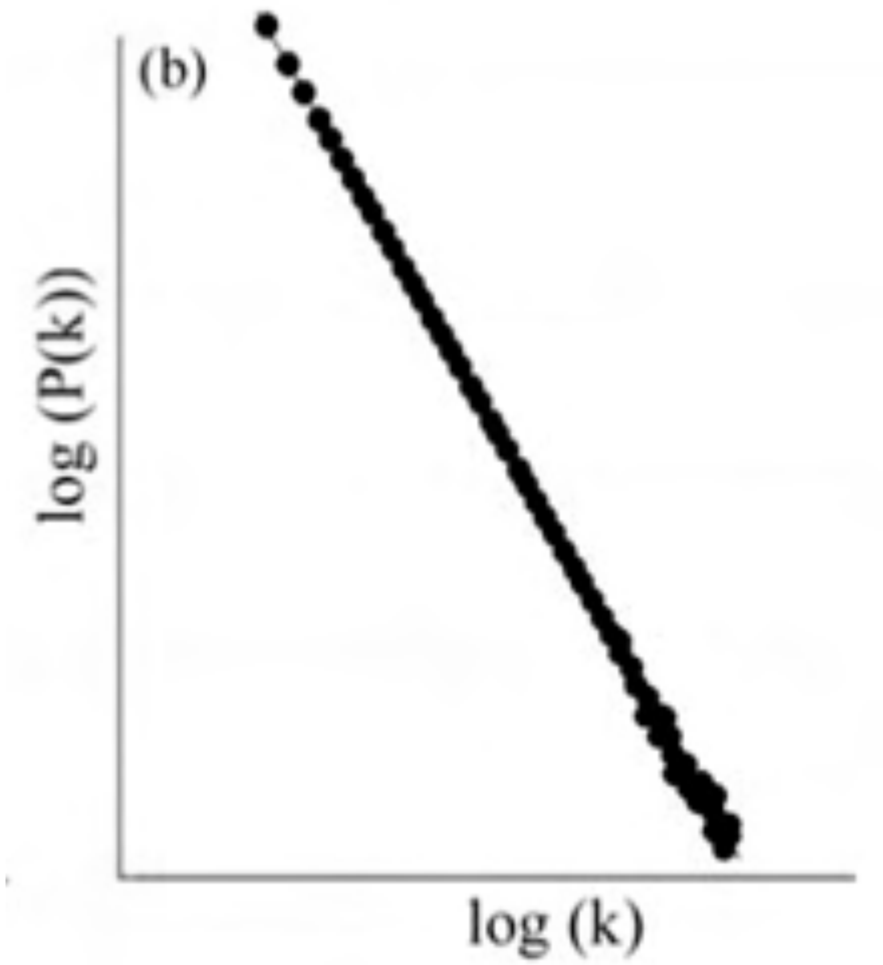
Benchmark average count

$$z_x = \frac{x - \langle x \rangle}{\sigma[x]}$$

Benchmark st dev

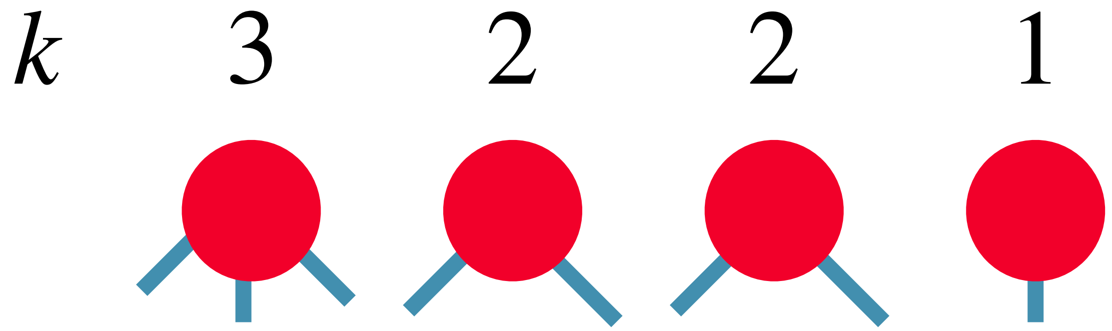
Benchmark



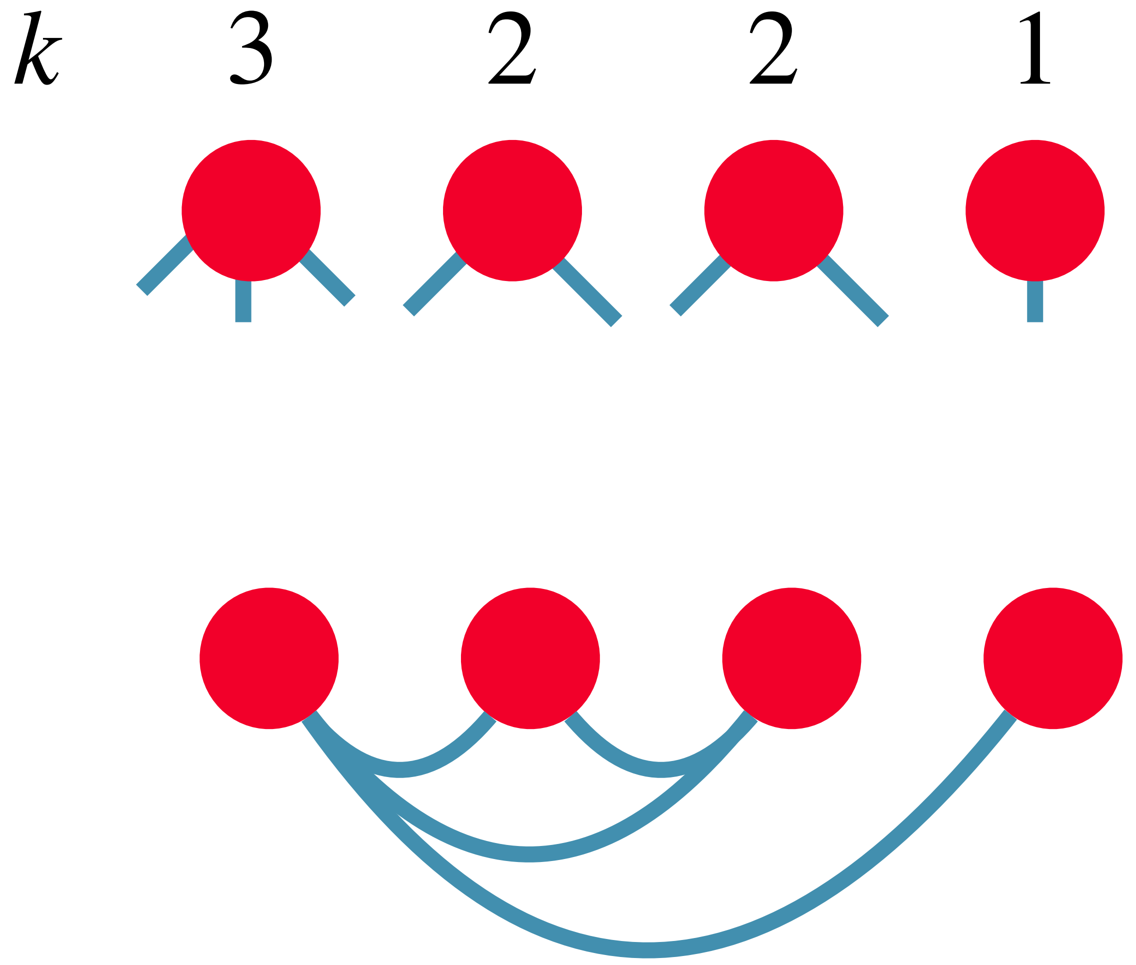


Erdos-renyi Barabási-albert

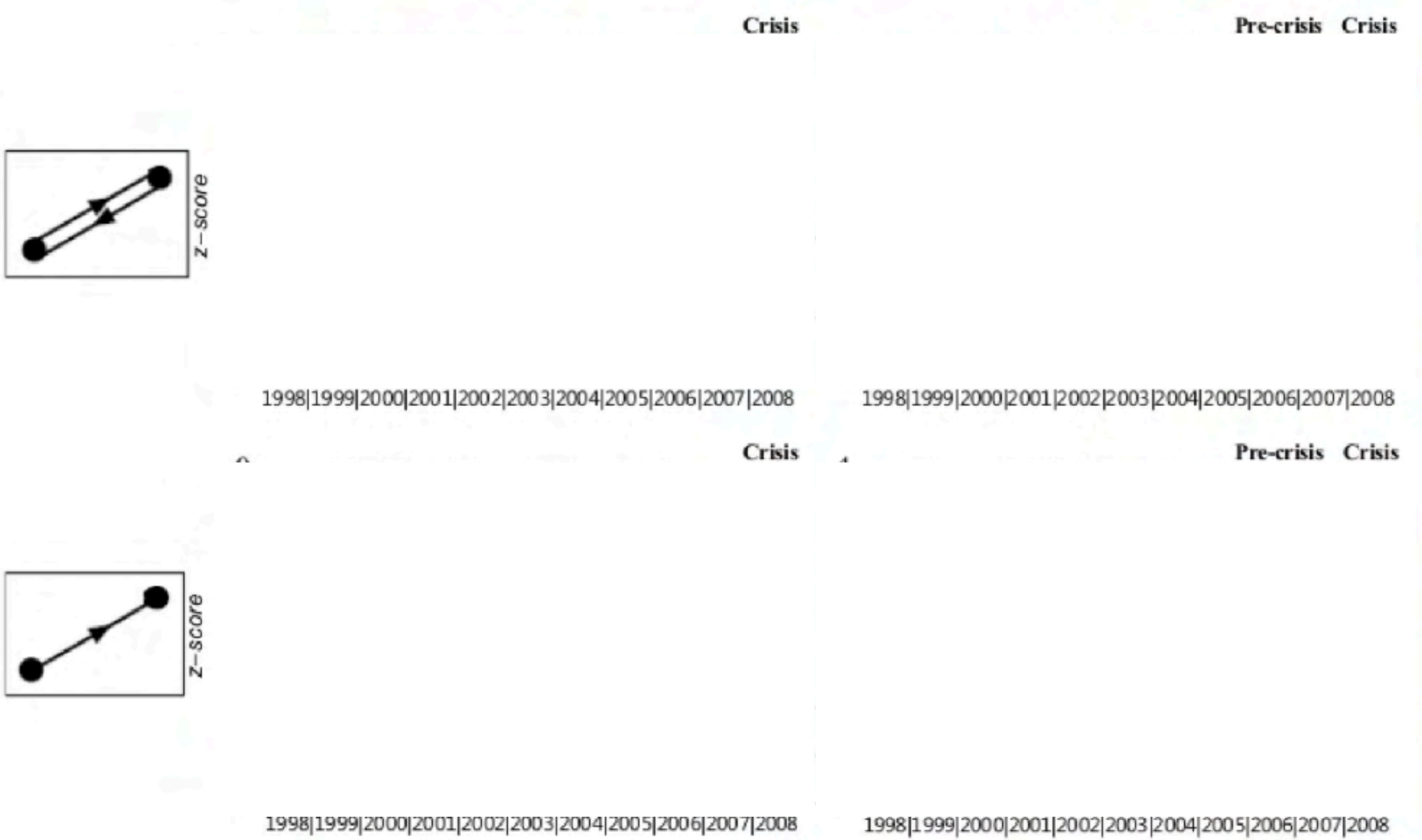
CONFIGURATION MODEL



CONFIGURATION MODEL



Interbank loans

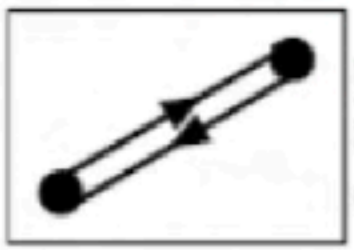


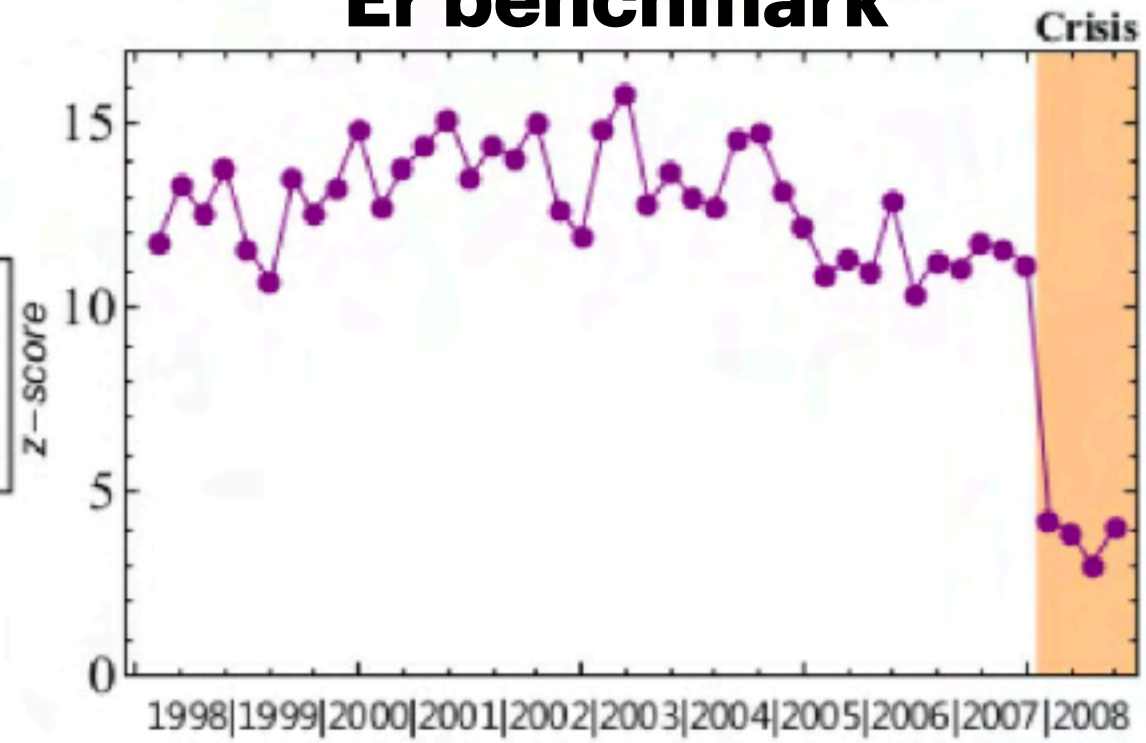
Squartini et al. **Early-warning signals of topological collapse in interbank networks** (2013) Sci. Rep.

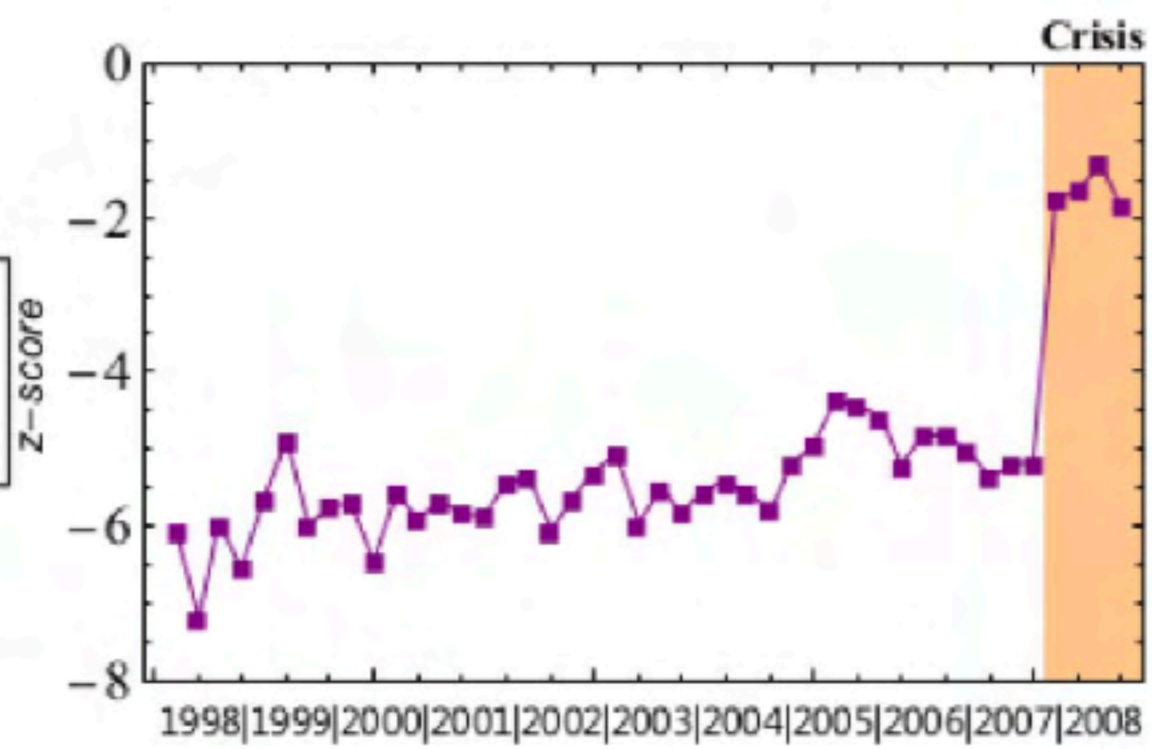
Interbank loans

Er benchmark

CM benchmark

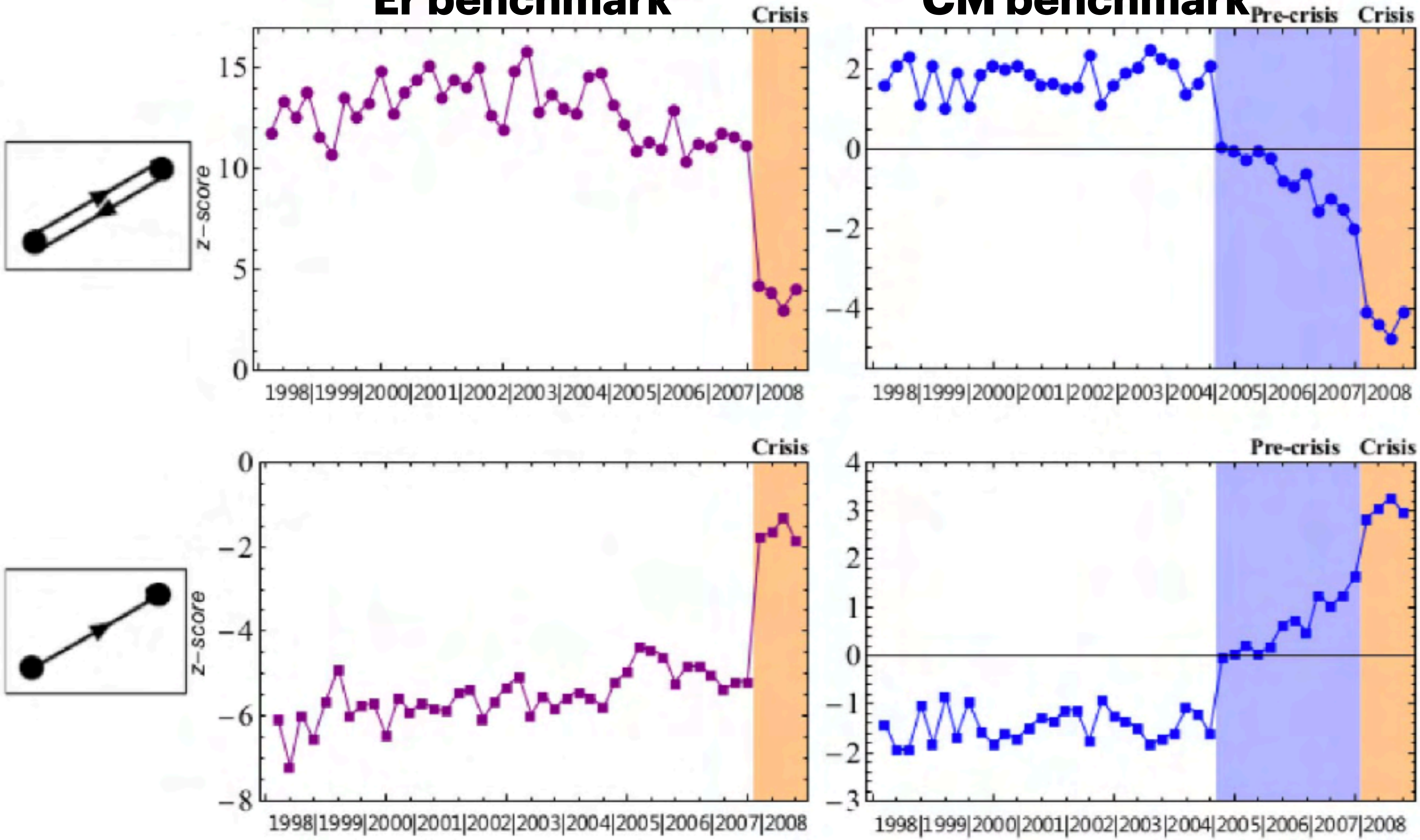




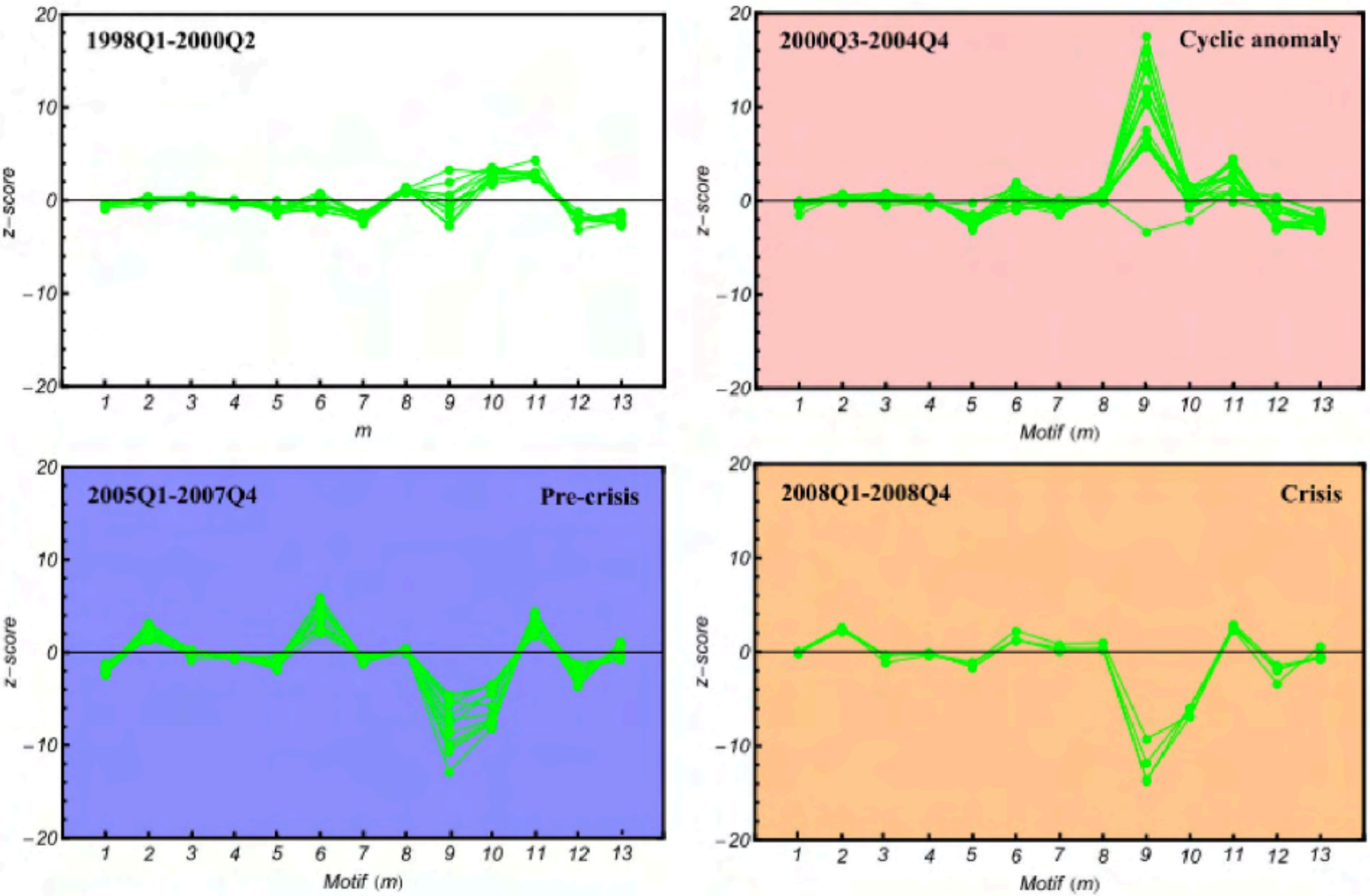


Interbank loans

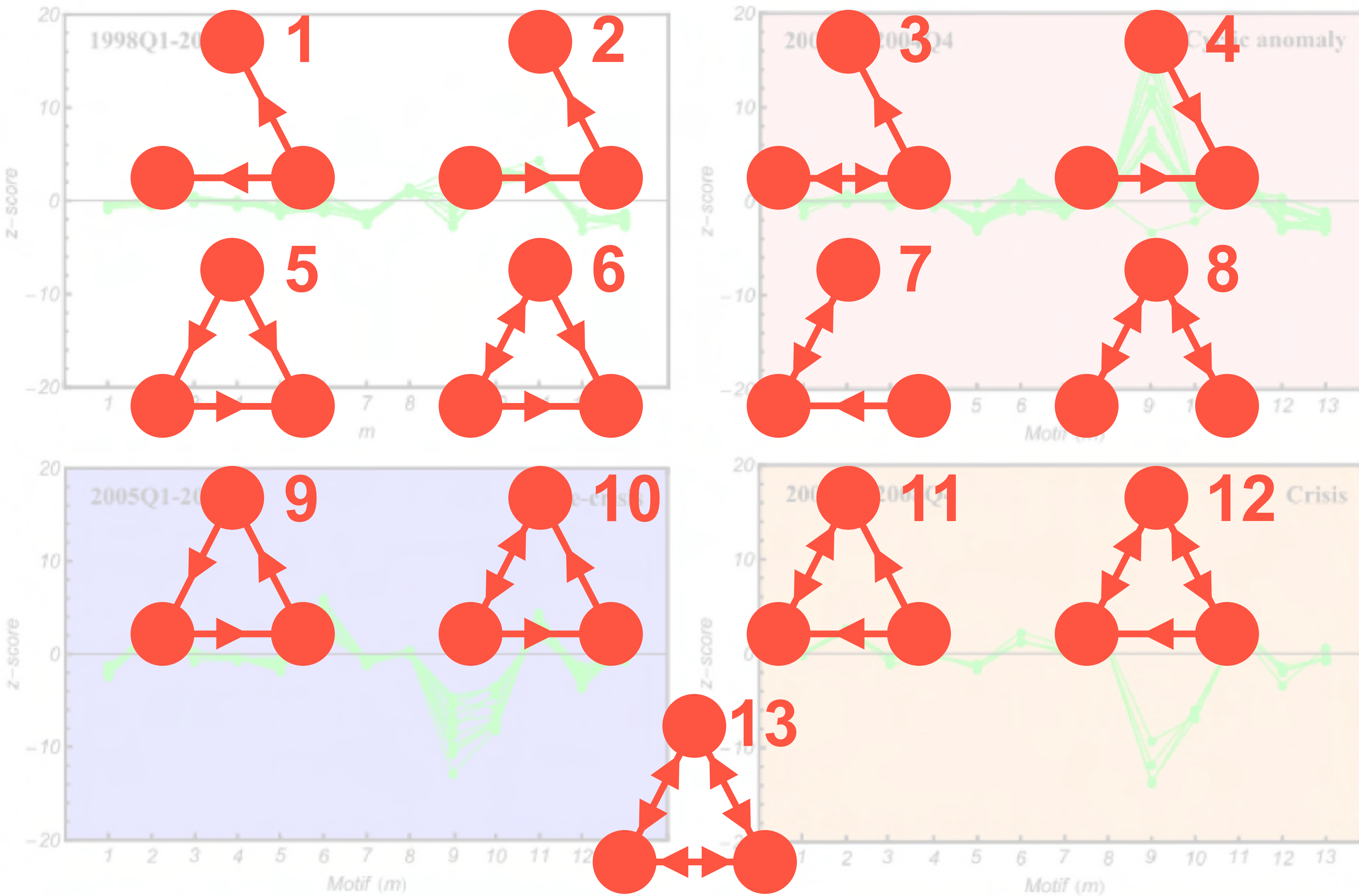
Er benchmark **CM benchmark**

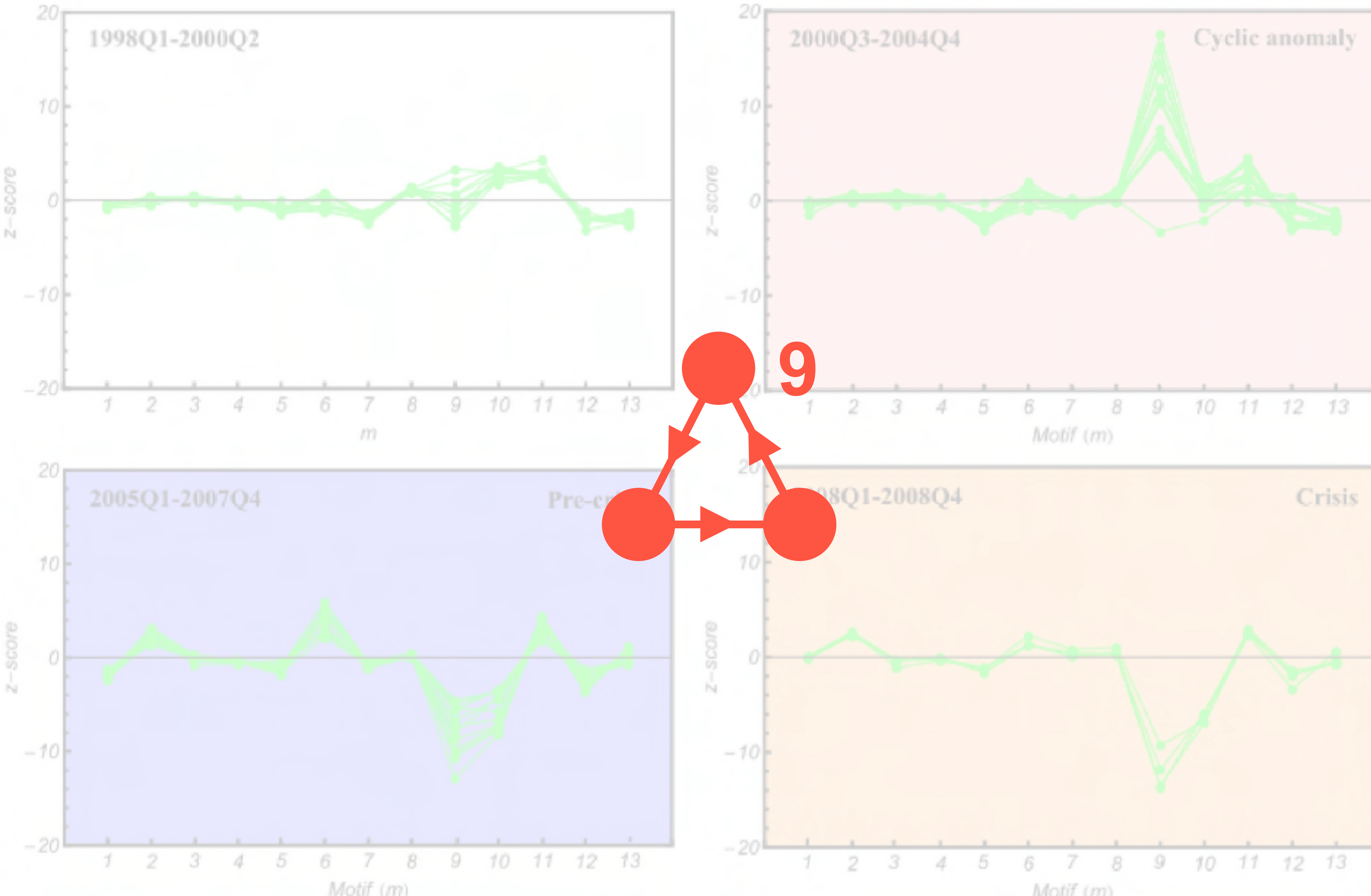


Interbank loans



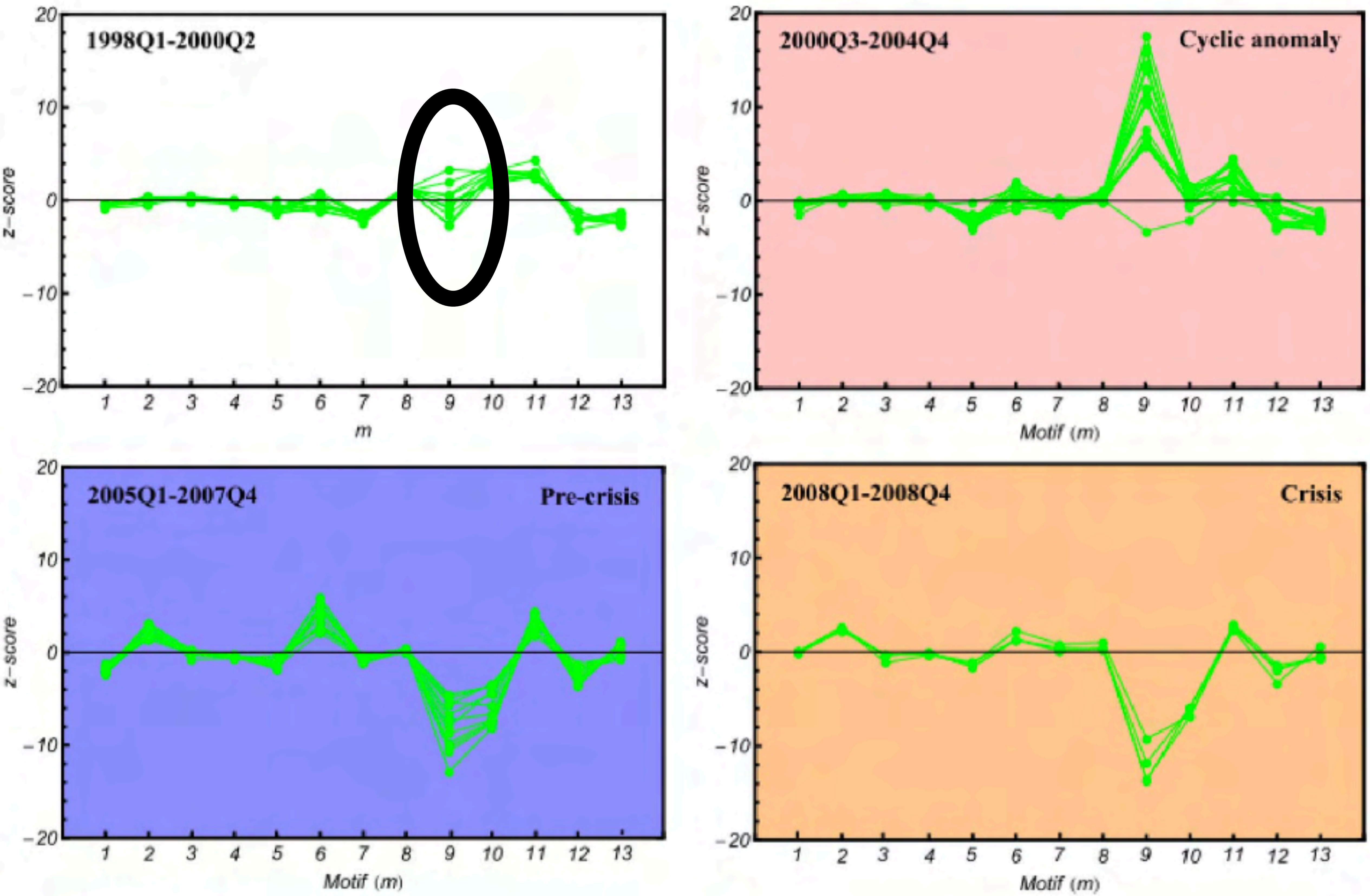
Interbank loans



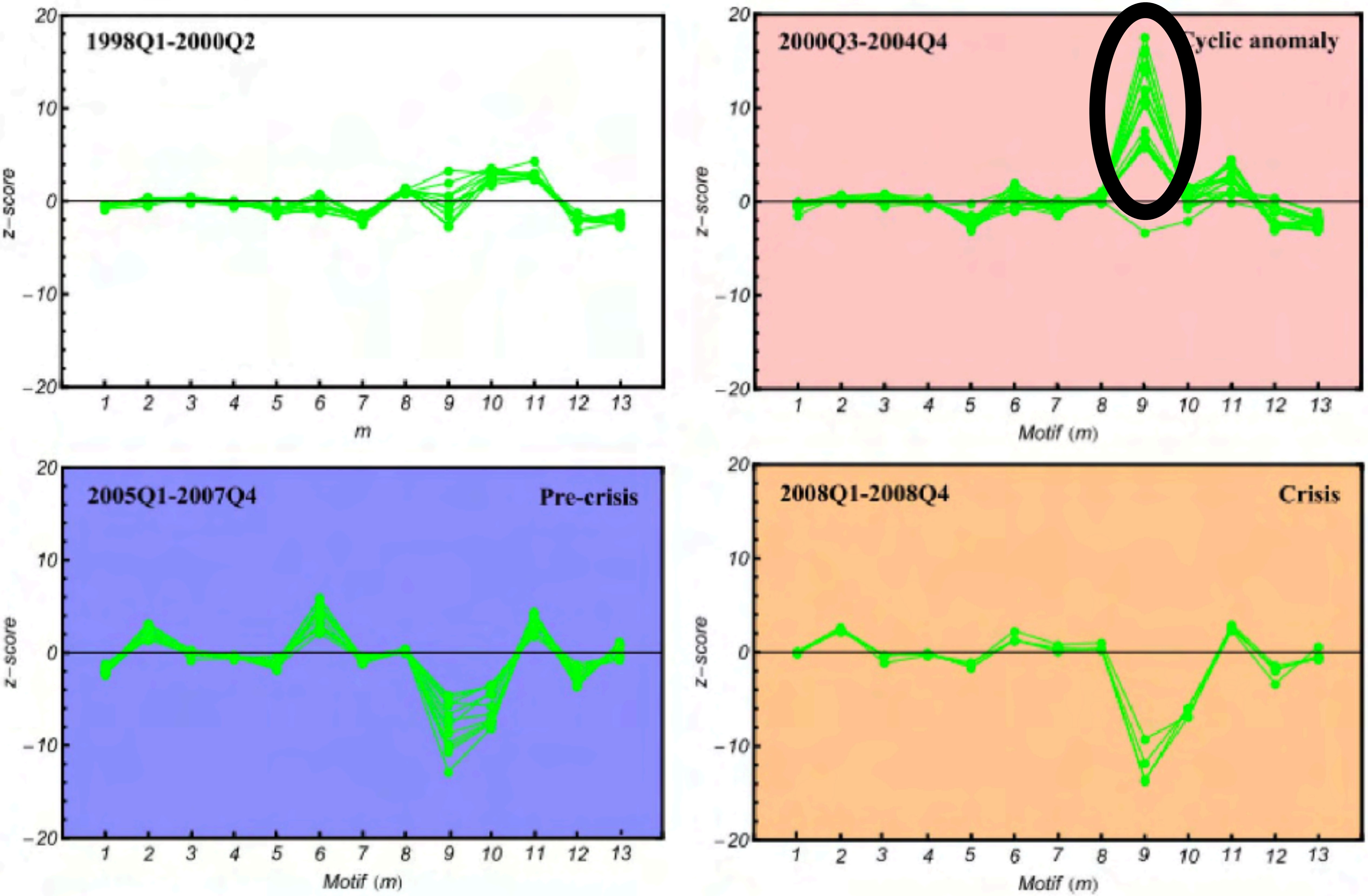


1998Q1-2000Q2
2000Q3-2004Q4
Cyclic anomaly
2005Q1-2007Q4
Crisis
z-score
Motif (m)
m
9

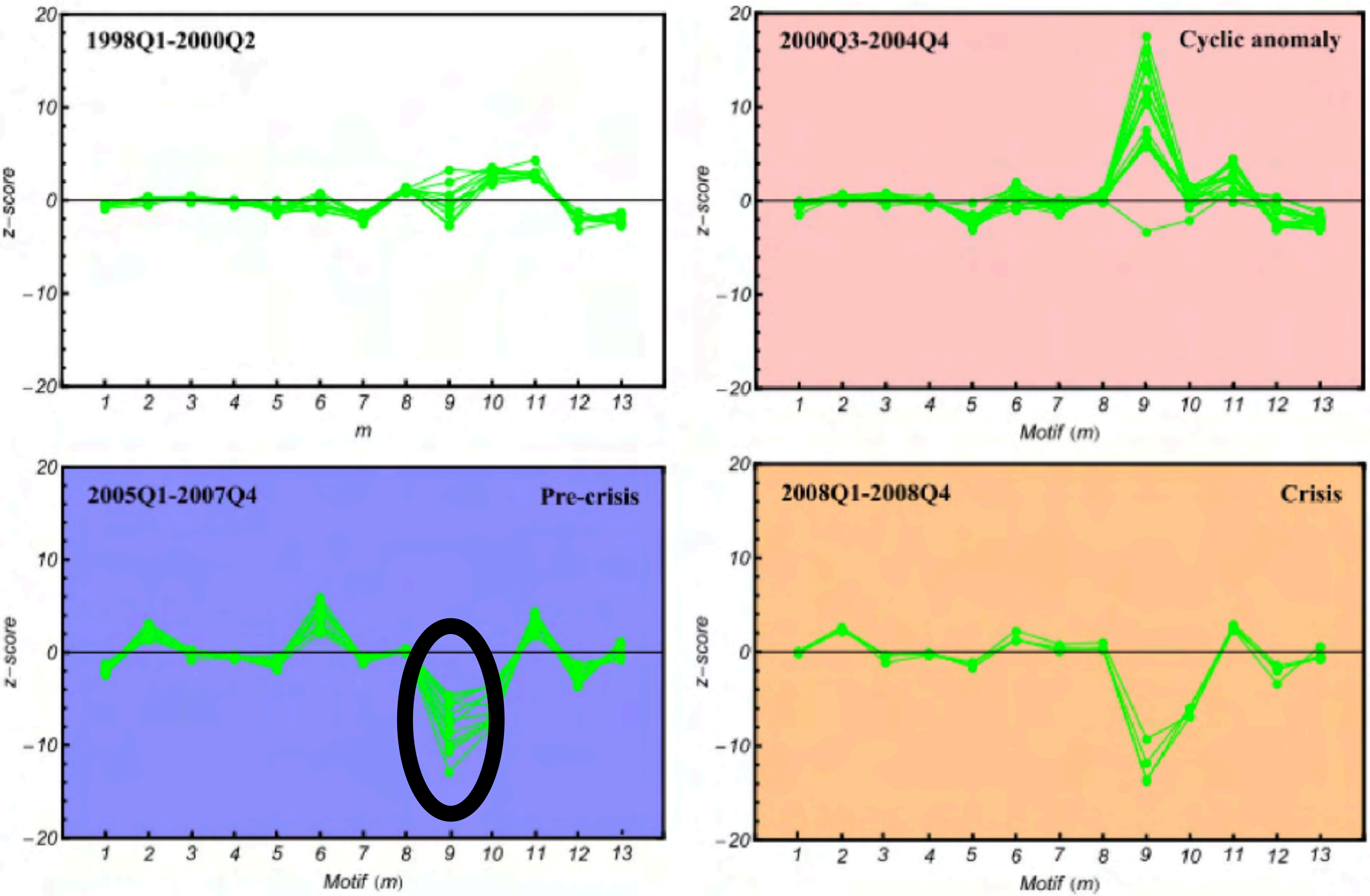
Interbank loans



Interbank loans



Interbank loans



Interbank loans

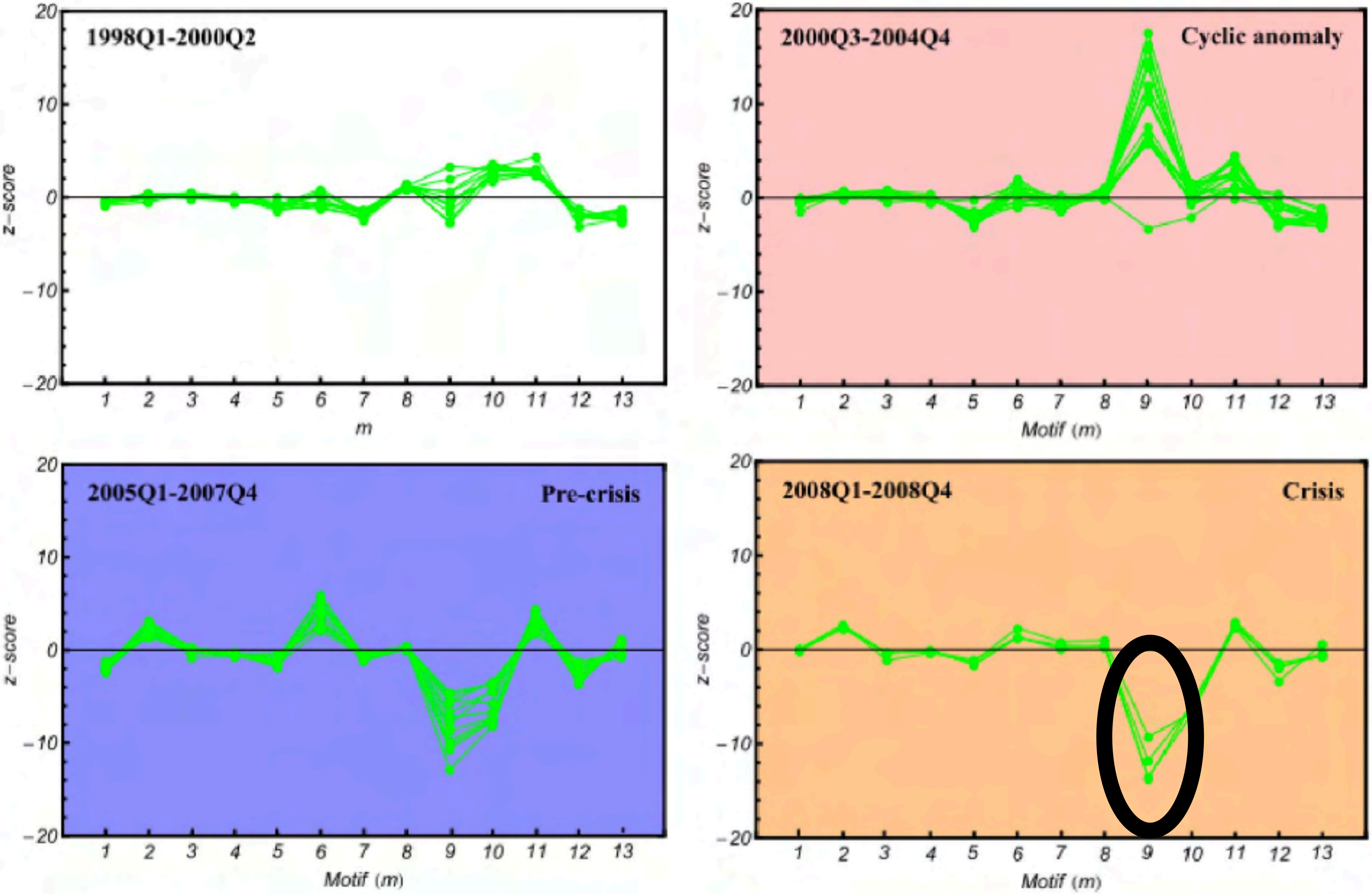
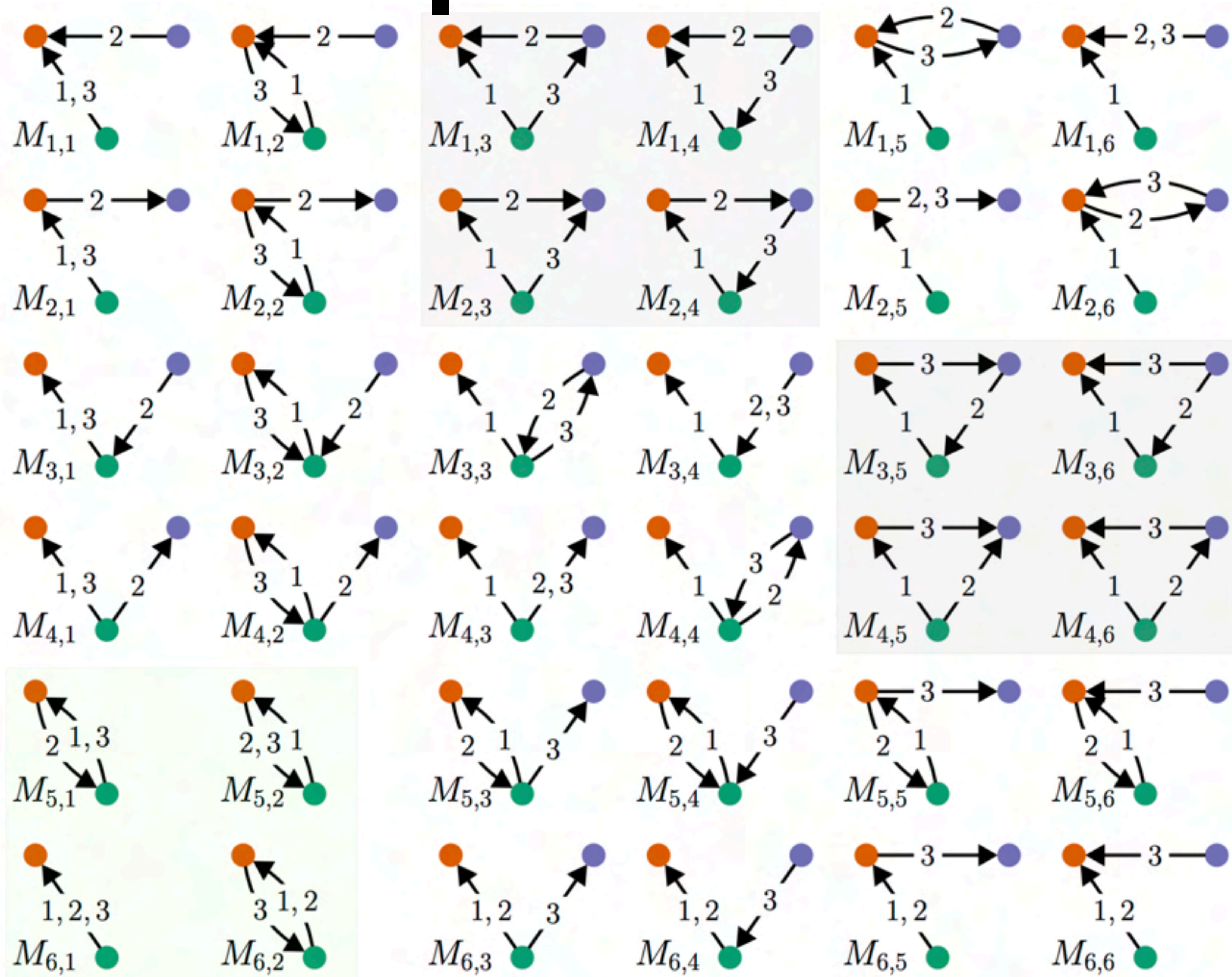


Table 3. Summary statistics on Z-scores for money flow networks, calculated using Directed Random Graph (DRG) and Directed Configuration Model (DCM).

		Motifs				
		1	2	4	5	9
		Panel A: Z-scores				
Financial institutions						
DRG	Mean	18.69	−3.38	20.20	49.35	3.95
	StDev	(9.06)	(2.37)	(9.05)	(8.27)	(4.48)
	Min	−4.15	−19.13	−0.50	7.12	−8.80
	Max	93.11	6.98	97.09	78.78	21.39
DCM	Mean	6.08	8.15	6.29	8.94	8.73
	StDev	(2.18)	(1.71)	(2.15)	(2.35)	(2.61)
	Min	−0.27	3.25	−1.01	−0.14	0.37
	Max	19.06	14.62	17.43	20.02	20.52

Retail investors						
DRG	Mean	16.32	9.67	21.92	104.27	37.61
	StDev	(5.83)	(3.44)	(6.96)	(18.03)	(10.10)
	Min	2.31	– 0.57	– 3.00	– 1.22	– 1.22
	Max	36.02	25.55	43.26	159.99	73.20
DCM	Mean	10.43	10.76	10.14	7.49	5.27
	StDev	(1.65)	(1.41)	(1.82)	(2.39)	(2.23)
	Min	1.36	1.36	1.14	– 0.47	0.83
	Max	15.62	15.74	15.37	16.81	17.33

Temporal motifs



Temporal motifs

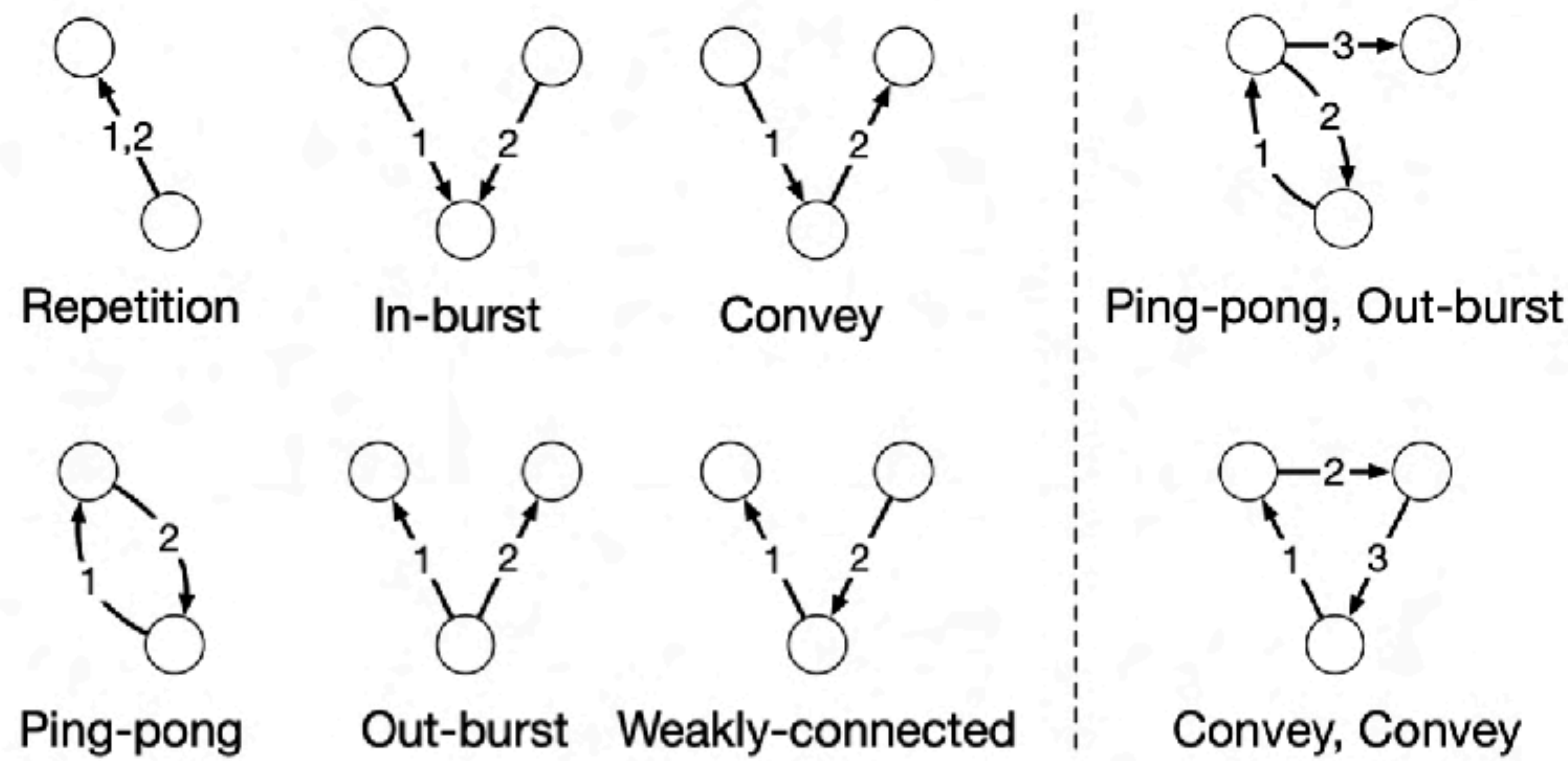
Fraud detection

Friendship prediction

Vendor identification

Liu et al. **Temporal Motifs for Financial Networks: A Study on Mercari, JPMC, and Venmo Platforms** (2025)

Temporal motifs



Temporal motifs

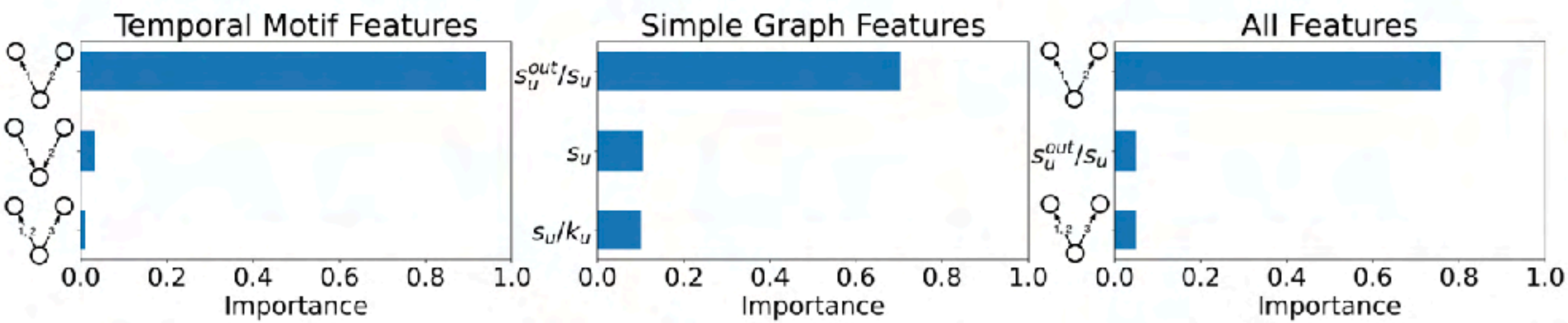


Fig. 2: Importance of features in the online marketplace transactions in Mercari network. Each bar shows the importance of the feature determined by random forests classifiers. For each feature set, shown in each panel, we only show the top three features.

Temporal motifs

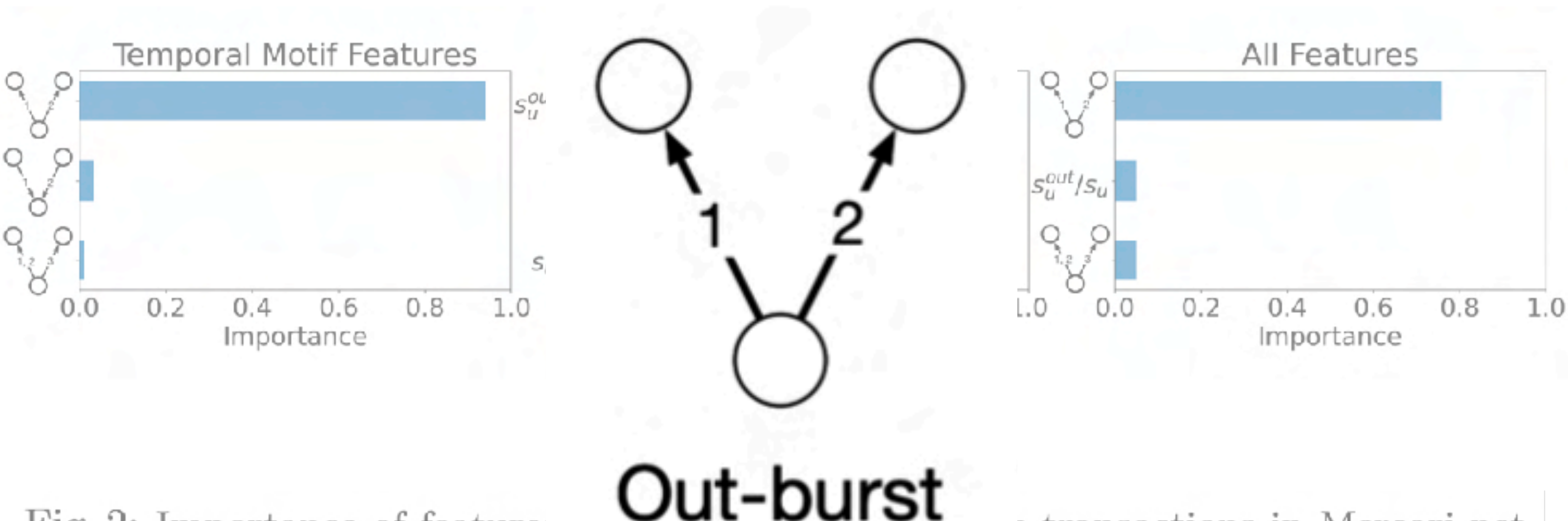


Fig. 2: Importance of feature … transactions in Mercari network. Each bar shows the in… etermined by random forests classifiers. For each feature set, shown in each panel, we only show the top three features.

Temporal motifs

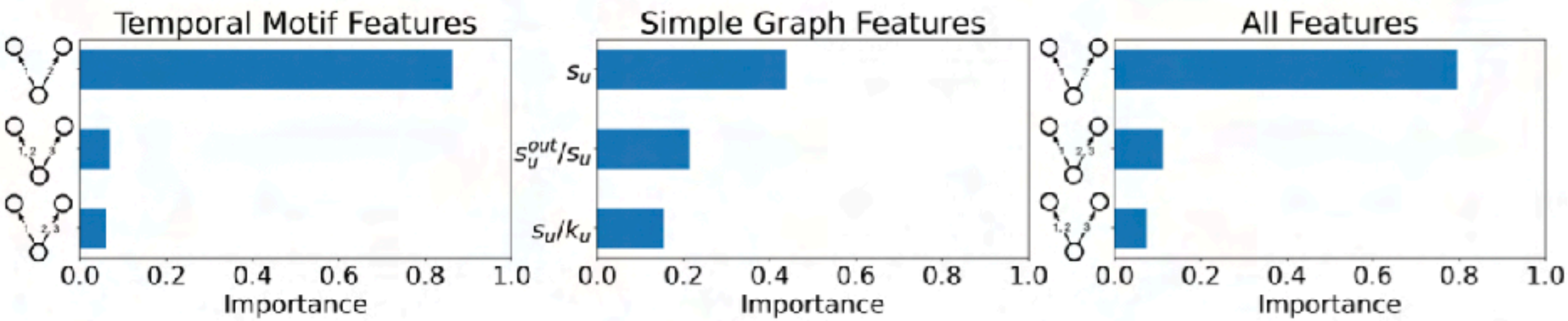


Fig. 3: Importance of the features in the synthetic payment transactions in JPMC network. Each bar shows the importance of the feature determined by random forests classifiers. For each feature set, shown in each panel, we only show the top three features.

Temporal motifs

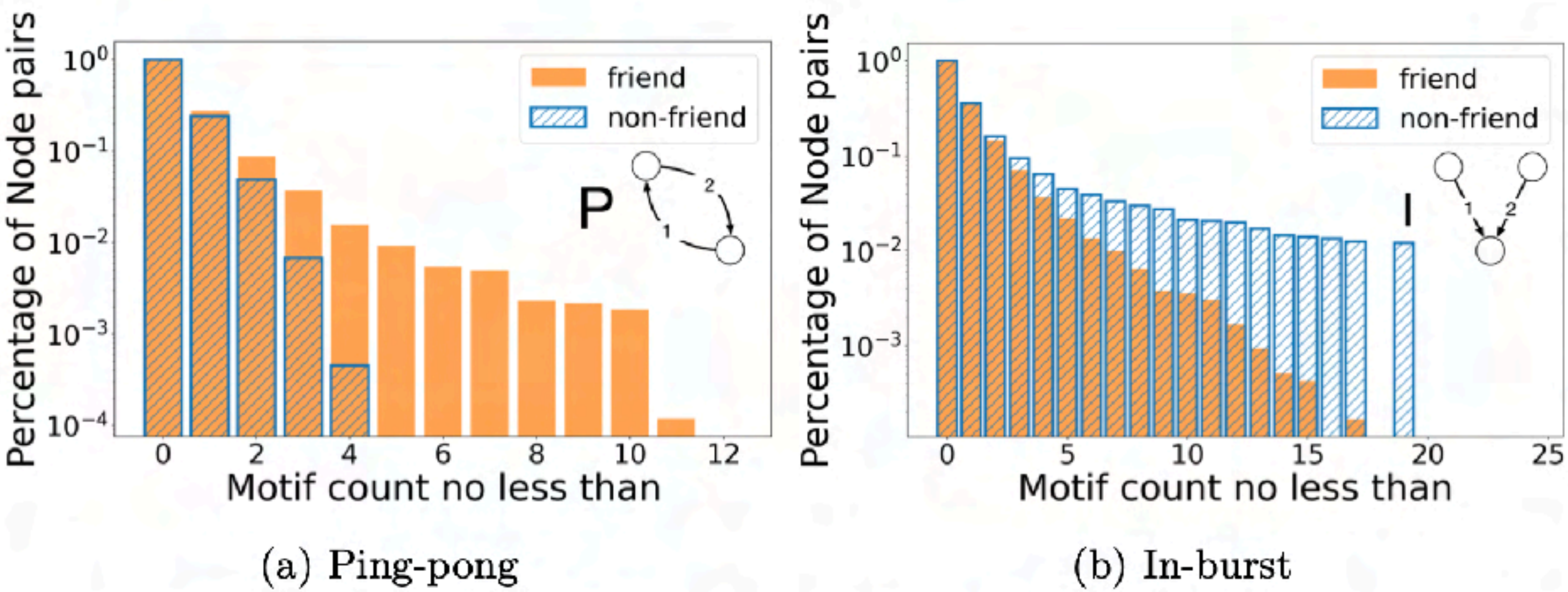
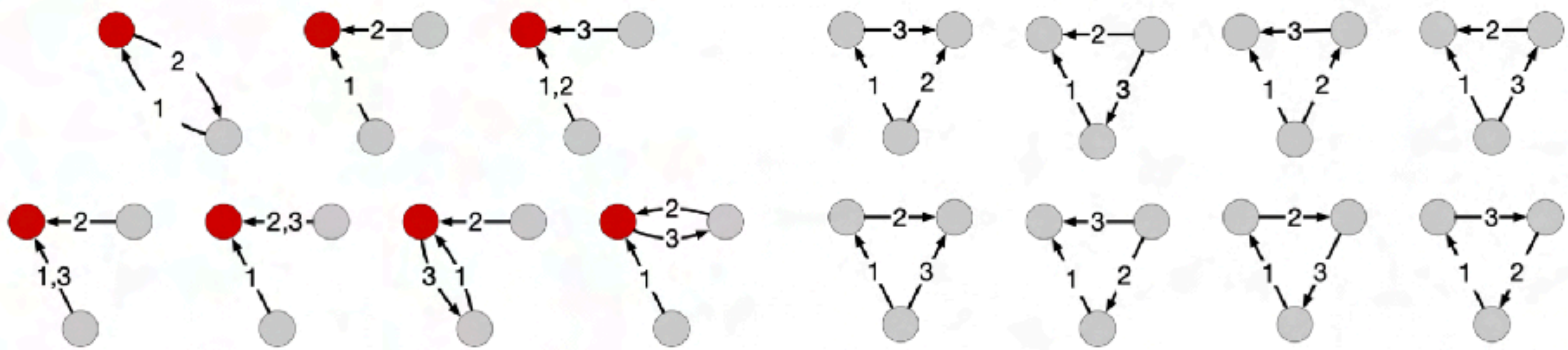


Fig. 6: The percentage of node pairs with motif count no less than the given amount. The horizontal axis represents the threshold amount, and the vertical axis shows the percentage of node pairs with the motif count no less than the threshold shown on the horizontal axis. No non-friend node pair has more than five ping-pong motifs, while half of the node pairs between friends are involved in more than five ping-pong motifs. On the other hand, we observe more than half of the non-friend node pairs are involved in more than 15 in-burst motifs.

Temporal motifs



(a) Positive patterns

(b) Negative patterns

Fig. 8: Vendor motif patterns. We select seven positive patterns in which the target node (red) is likely to be a vendor user (Figure 8a), and eight negative patterns that are unlikely to contain vendor users (Figure 8b).

Past project

Network of passes

Do pass networks change based on performance?

Do different networks lead to goals?

Past project

When there’s a shot on goal, reciprocity was lower

Final discussion

Motifs can tell us more about the system

Expensive to count

Sometimes difficult interpretation