Recap

Independent cascade: irreversible, simple contagion
Linear threshold: a fraction of neighbours must agree
Voter model: flipping states
Deffuant model: continuous opinions, clustered convergence

Which model would you use?

Video becoming viral
Marketing
Opinions on stock markets

Robustness and control

Learning outcomes

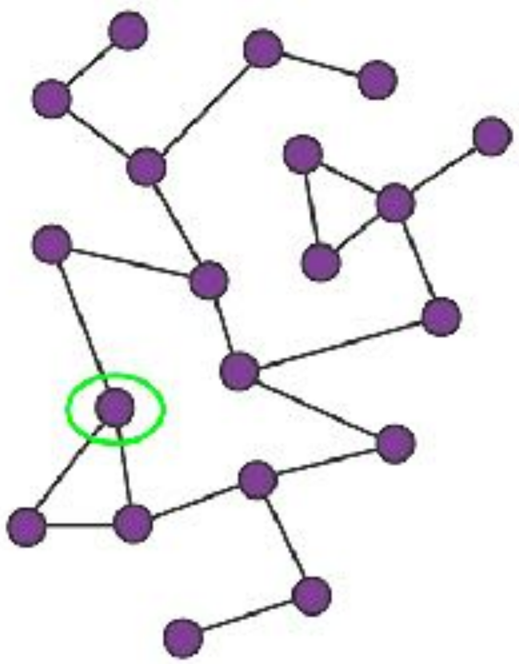
Robustness of different networks

Financial networks and systemic risk

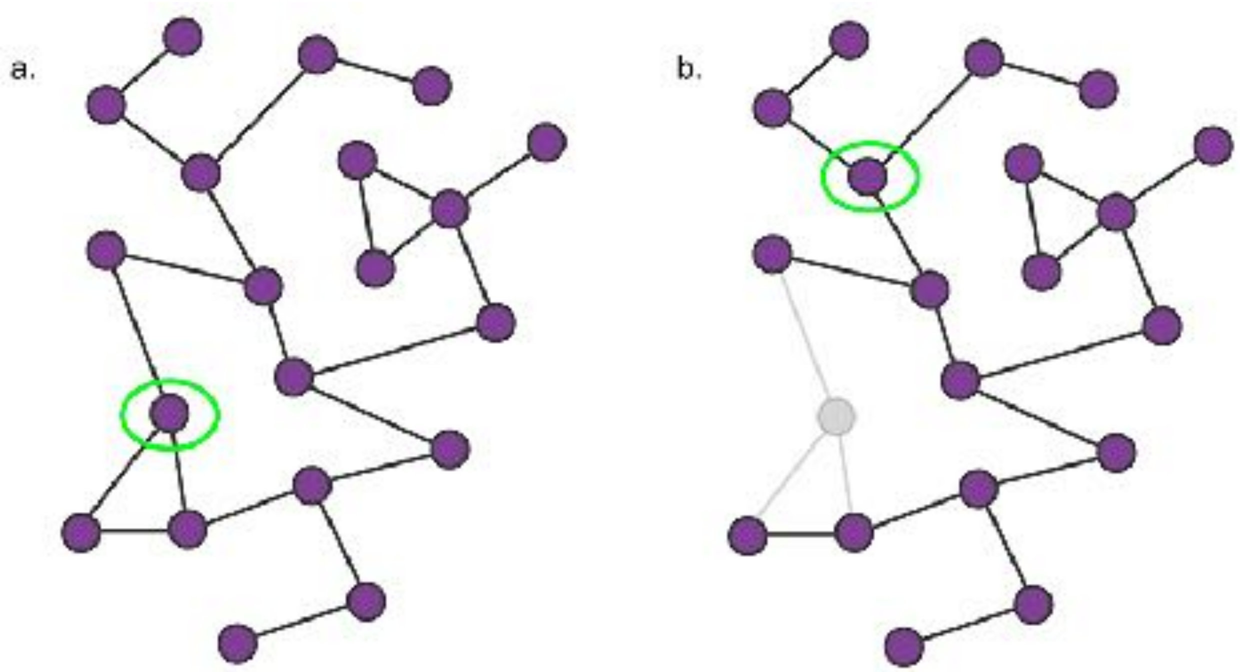
Overview of **targeting** strategies

Percolation

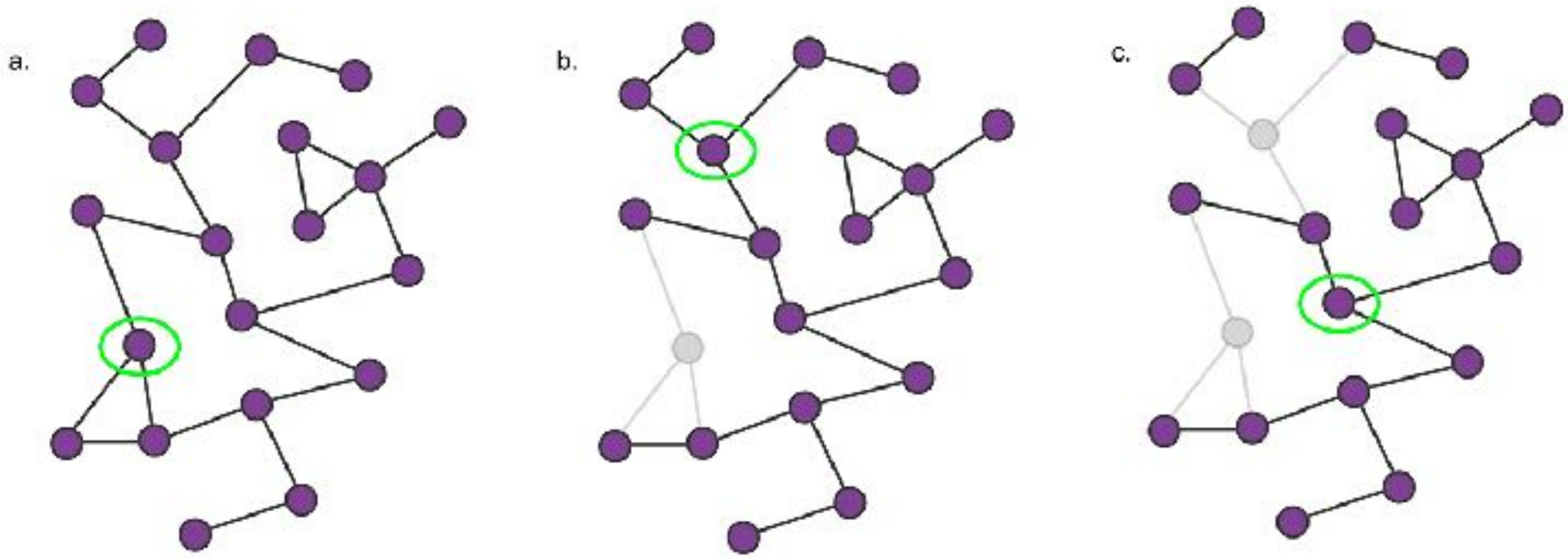
a.



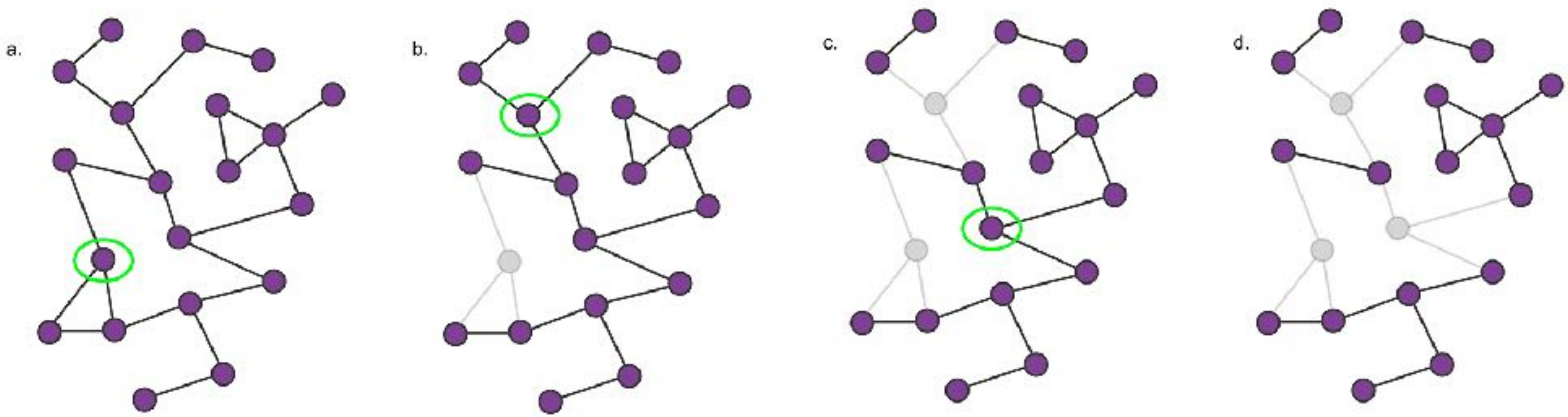
Percolation



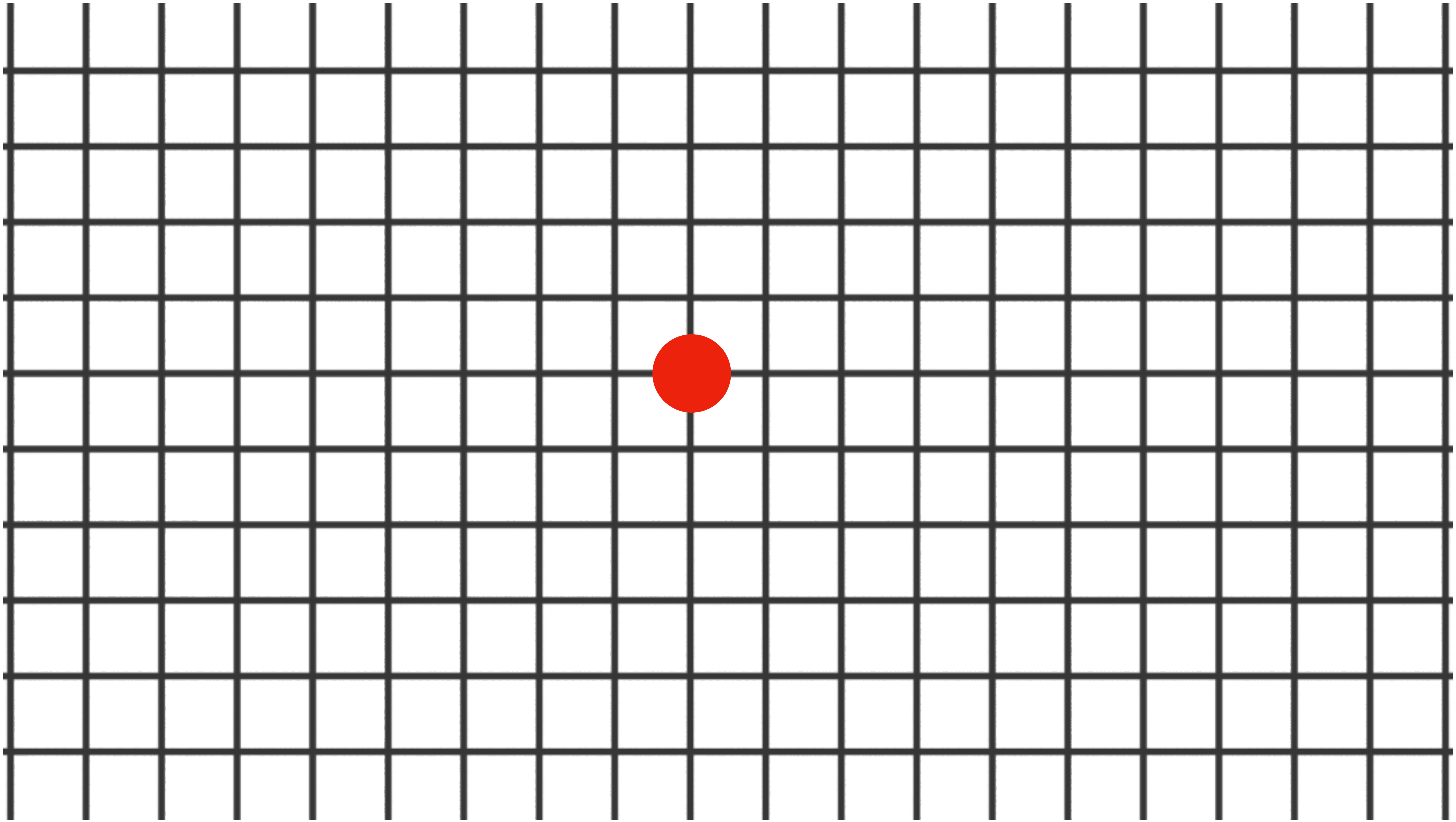
Percolation



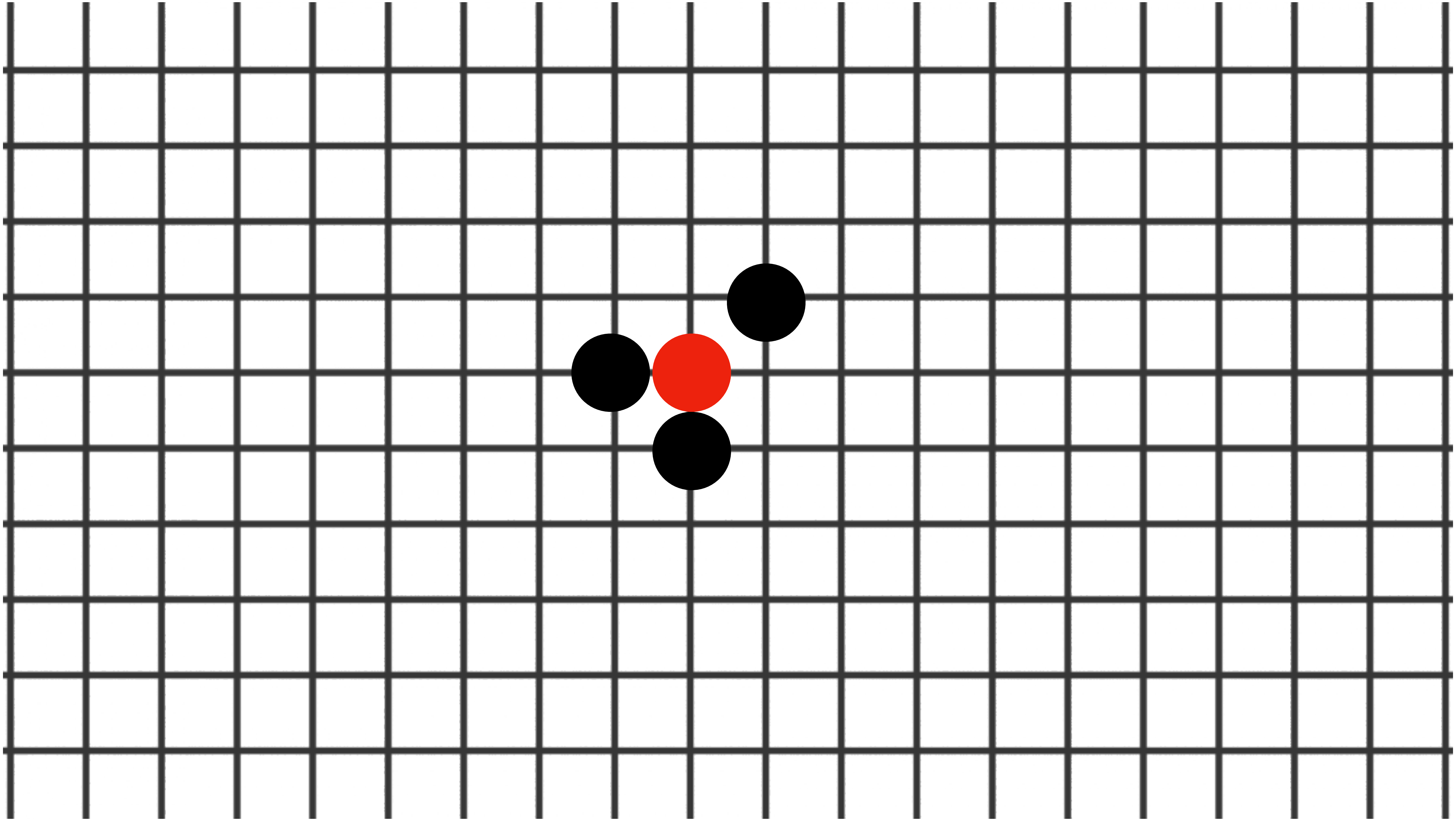
Percolation



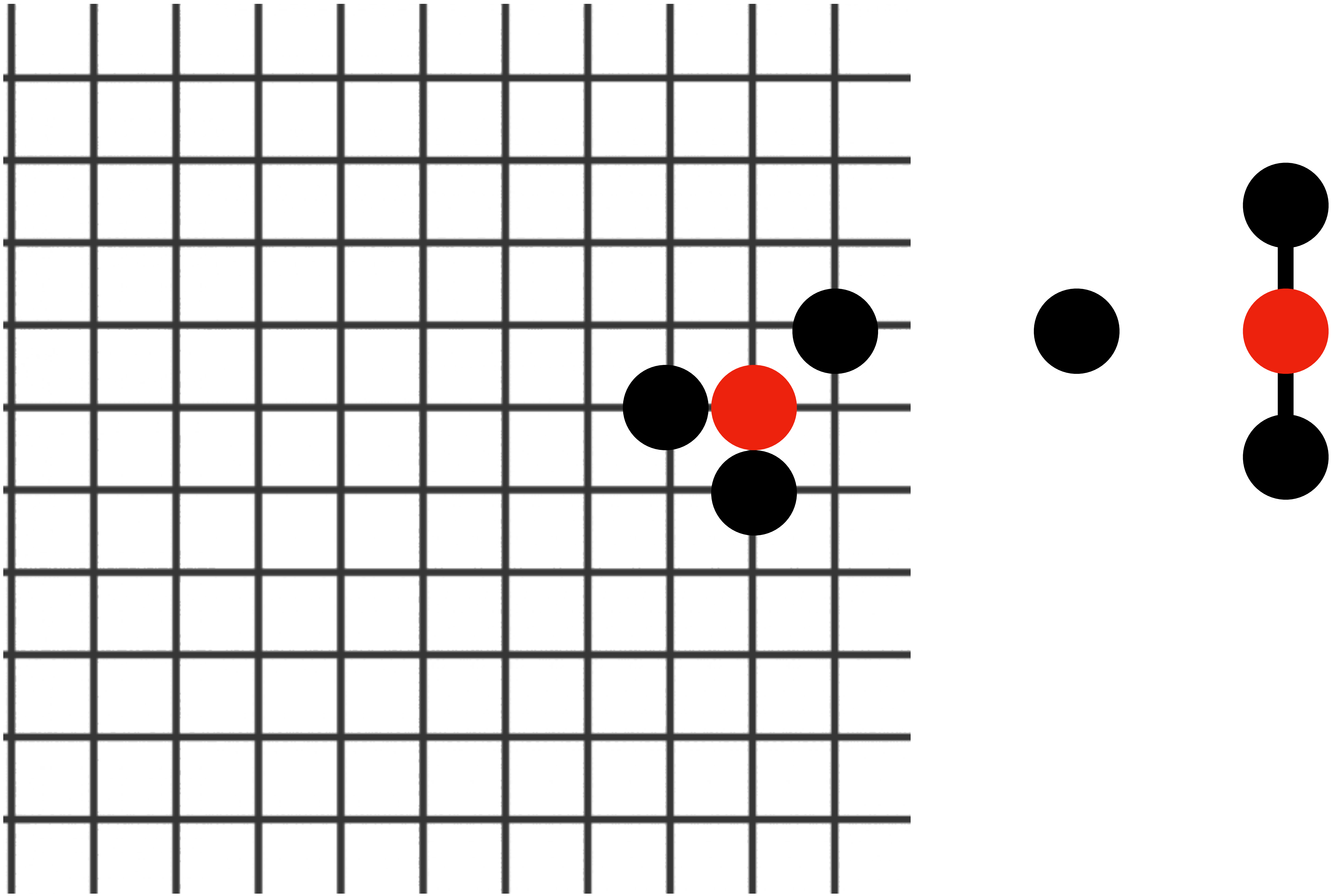
2d lattice



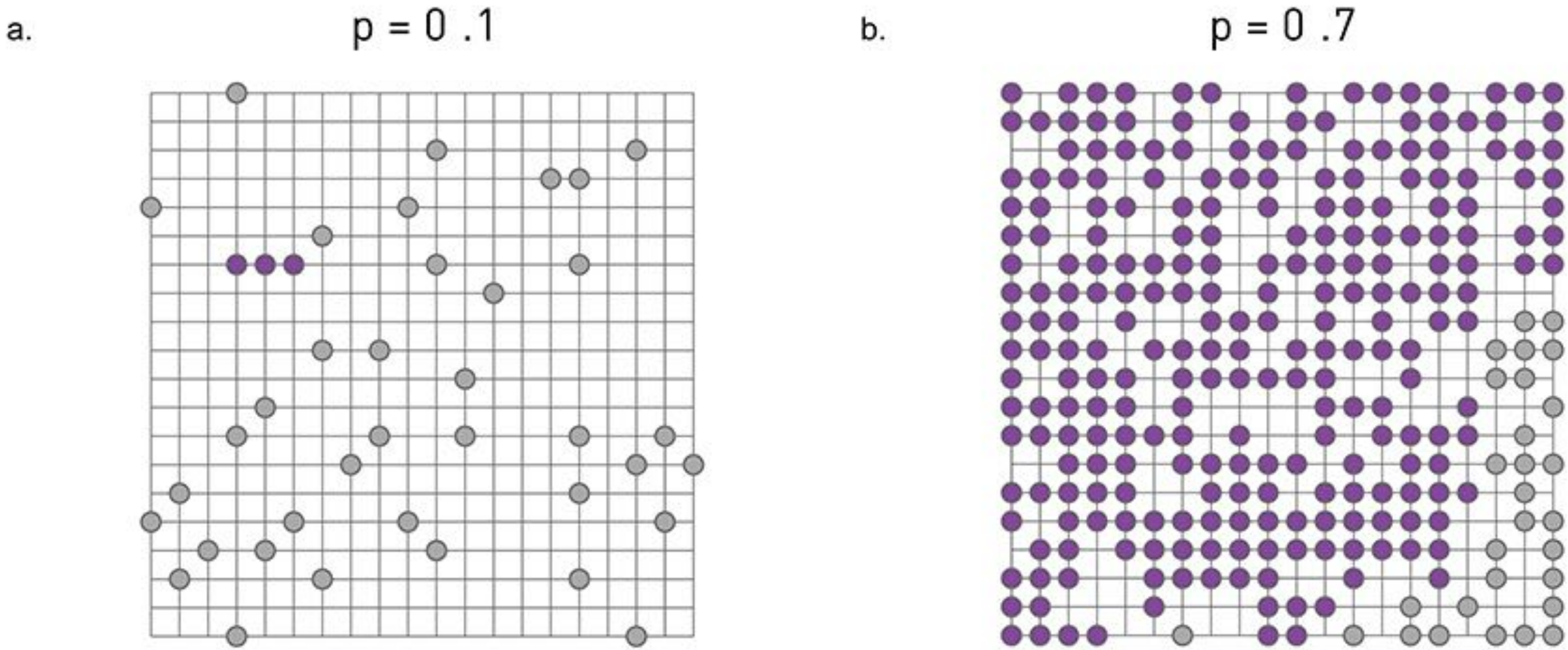
2d lattice



2d lattice



2d lattice



2d lattice

Cluster size does not grow linearly with p

2d lattice

Average cluster size

$$\langle s \rangle \sim |p - p_c|^{-\gamma_p}$$

Probability that a randomly chosen pebble belongs to the largest cluster

Order parameter

$$p_\infty \sim (p - p_c)^{\beta_p}$$

Correlation length

$$\xi \sim |p - p_c|^{-\nu}$$

2d lattice

Average cluster size

$$\langle s \rangle \sim |p - p_c|^{-\gamma_p}$$

Order parameter

$$p_\infty \sim (p - p_c)^{\beta_p}$$

Correlation length

$$\xi \sim |p - p_c|^{-\nu}$$

Average distance between two pebbles in the same cluster

2d lattice

Average cluster size

$$\langle s \rangle \sim |p - p_c|^{-\gamma_p}$$

Order parameter

$$p_\infty \sim (p - p_c)^{\beta_p}$$

Correlation length

$$\xi \sim |p - p_c|^{-\nu}$$

p_c **Critical probability**

γ_p, β_p, ν **Critical exponents**

2d lattice

Average cluster size

$$\langle s \rangle \sim |p - p_c|^{-\gamma_p}$$

Order parameter

$$p_\infty \sim (p - p_c)^{\beta_p}$$

Correlation length

$$\xi \sim |p - p_c|^{-\nu}$$

Depends on lattice geometry

p_c **Critical probability**

γ_p, β_p, ν **Critical exponents**

Depend on lattice dimension (eg 2d, 3d) up to 6d

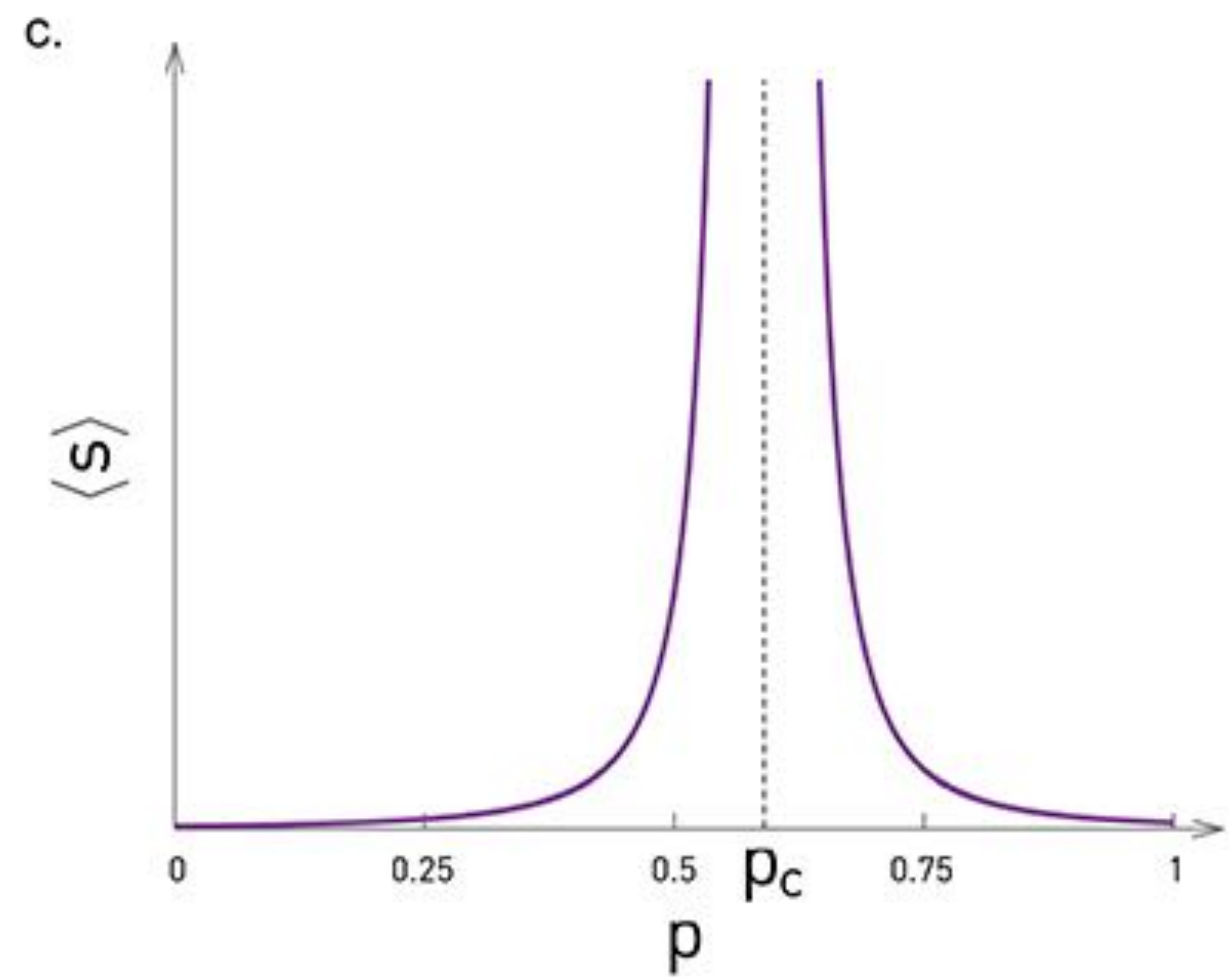
2d lattice

Average cluster size

$$\langle s \rangle \sim |p - p_c|^{-\gamma_p}$$

Correlation length

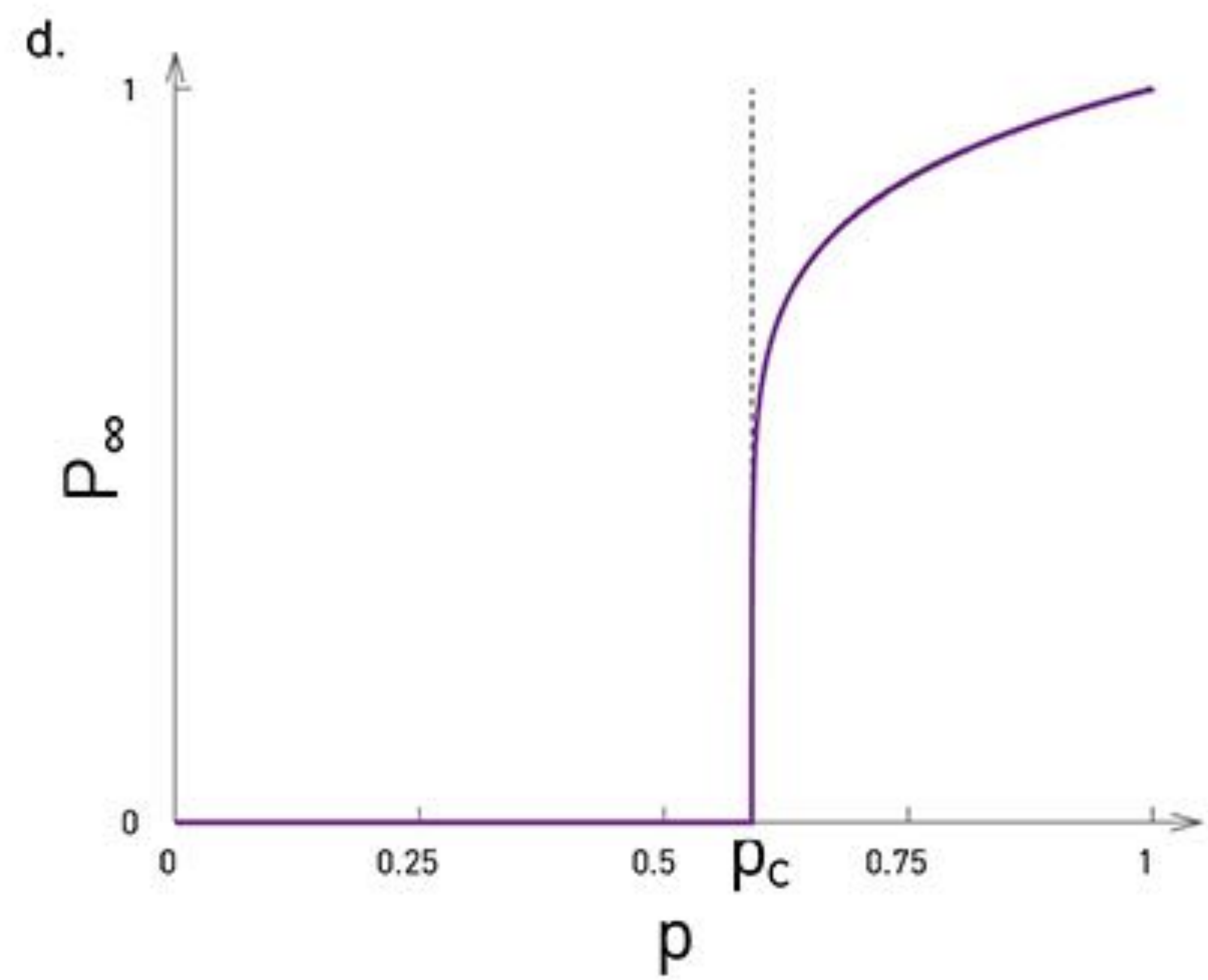
$$\xi \sim |p - p_c|^{-\nu}$$



2d lattice

Order parameter

$$p_\infty \sim (p - p_c)^{\beta_p}$$



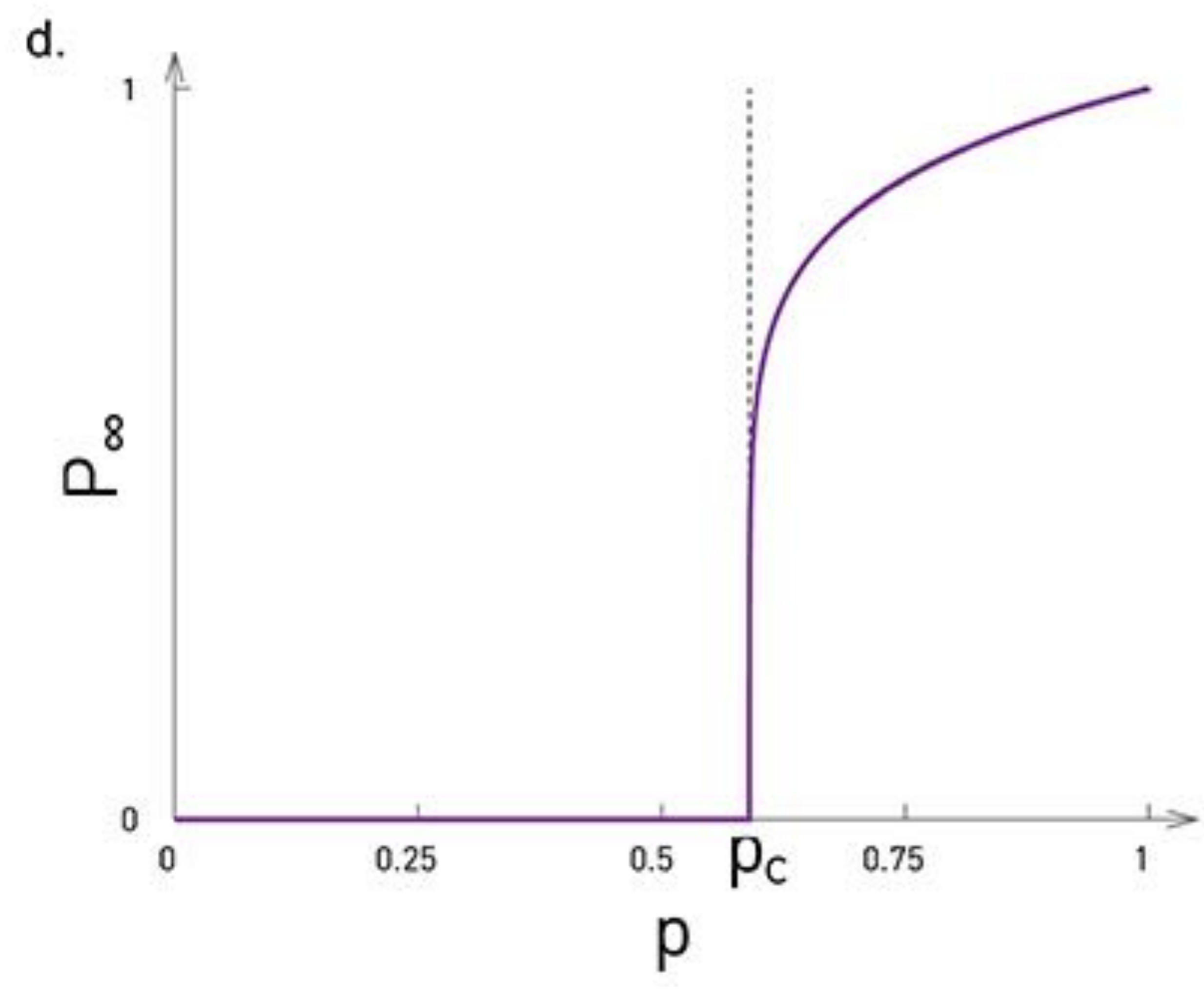

OMG Who The Hell Cares!

Systemic risk

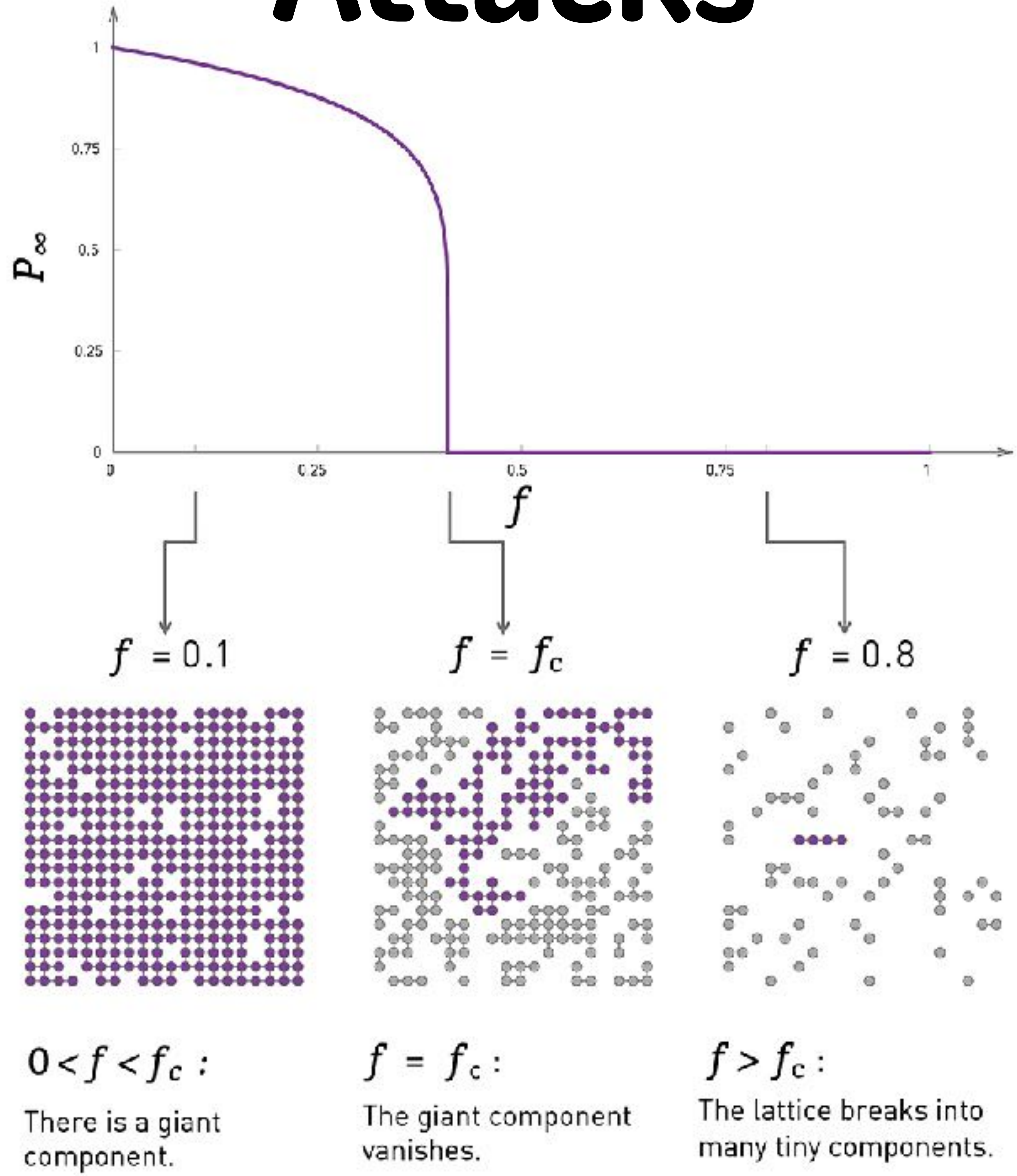
2d lattice

Order parameter

$$p_\infty \sim (p - p_c)^{\beta_p}$$



Attacks



P_∞
1
0.75
0.5
0.25
0
0
0.25
0.5
0.75
1
f
$f = 0.1$
$f = f_c$
$f = 0.8$
$0 < f < f_c$:
There is a giant component.
$f = f_c$:
The giant component vanishes.
$f > f_c$:
The lattice breaks into many tiny components.

Network structure comparison

What network do you think is more robust?

Network structure comparison

Scale-free networks are **more robust***

Most nodes have **low degrees**

Hubs are highly connected and central

Targeted removal

Robustness of different networks

Targeting strategies

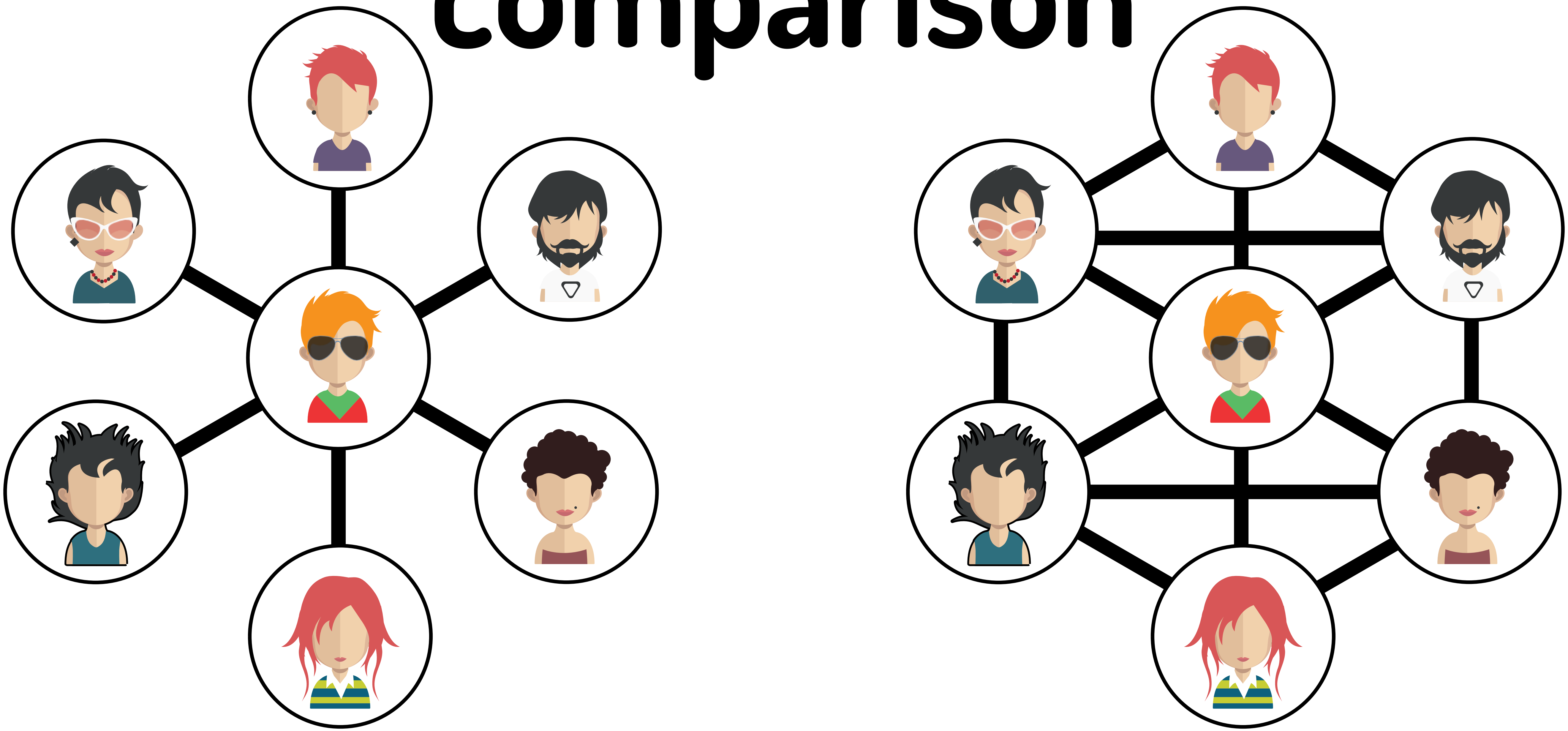
Financial networks and systemic risk

Targeted removal

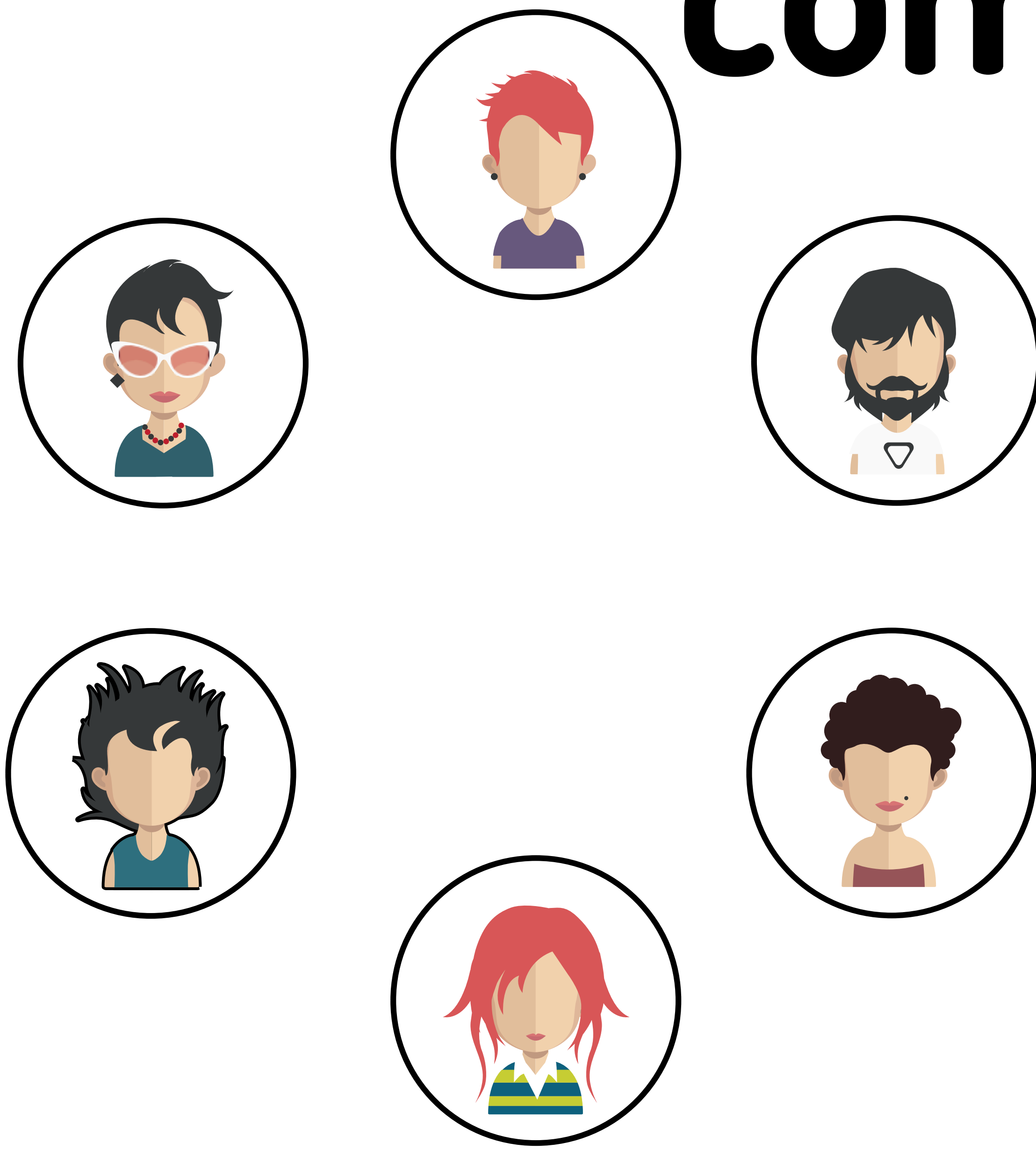
If we consider **targeted attacks** everything changes!

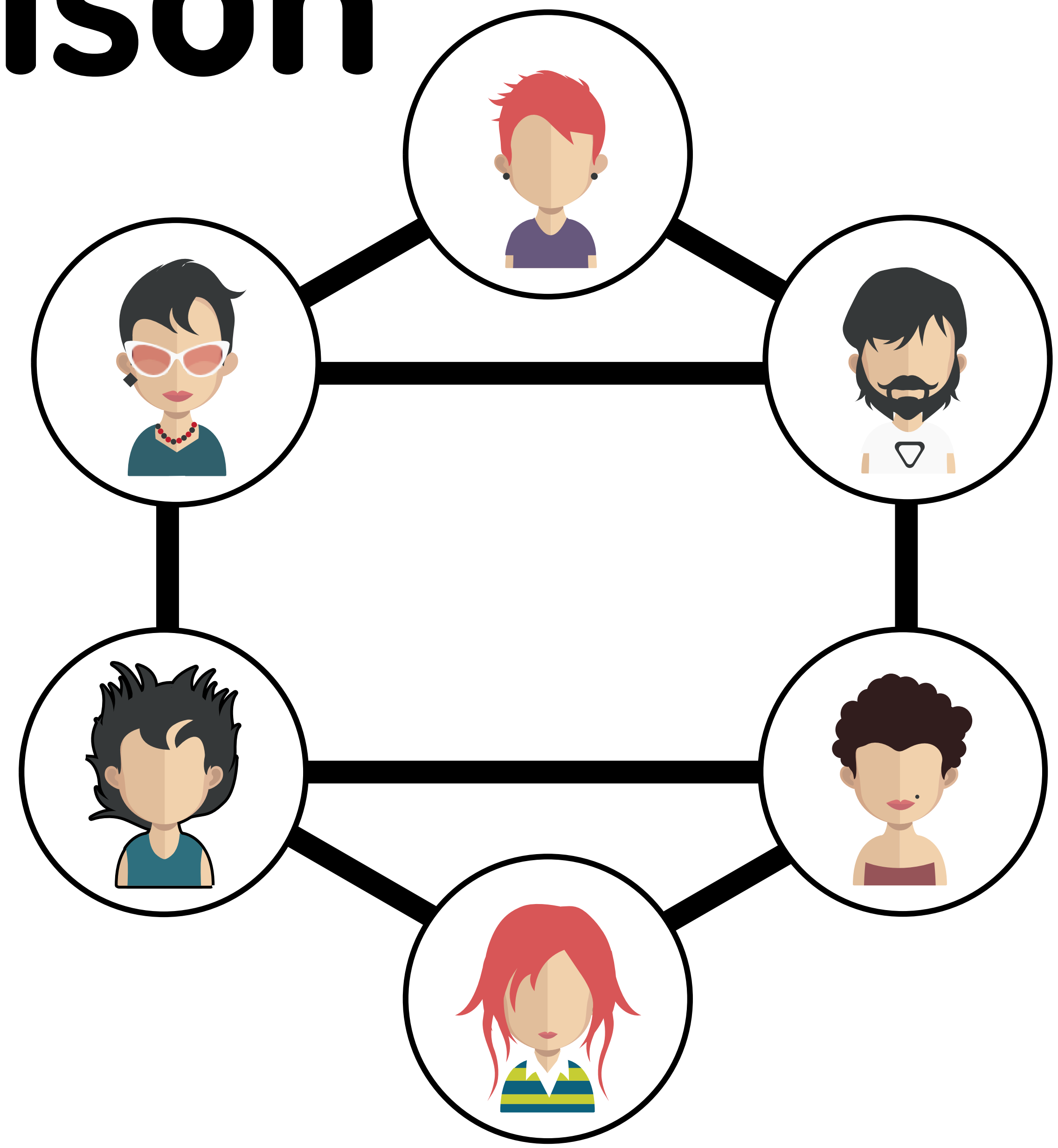
Hubs are highly connected and central

Network structure comparison



Network structure comparison

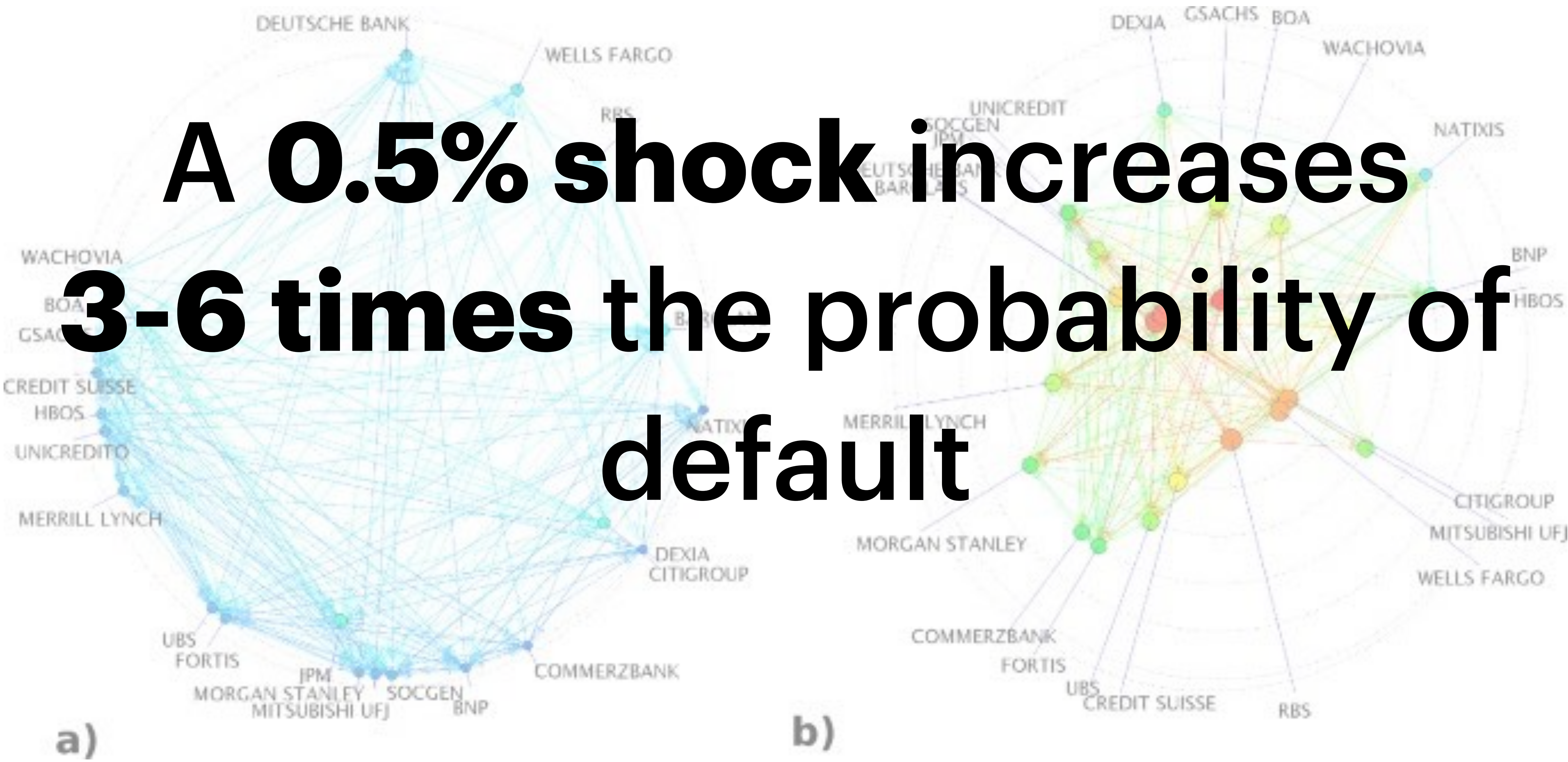




Example: Systemic risk

risk that default or stress **of one** or more financial institutions ("banks") **will trigger default** or stress **of further** banks.

Systemic risk



A **0.5% shock** increases **3-6 times** the probability of default

Battiston, S., Puliga, M., Kaushik, R. *et al*. DebtRank: Too Central to Fail? Financial Networks, the FED and Systemic Risk. *Sci Rep* 2, 541 (2012). https://doi.org/10.1038/srep00541

Think of a topic you like

Think of a topic you like

Think of an example of maximising/ minimising propagation

Influence maximisation

Selection of k nodes that best trigger a cascade

Heuristic strategies

Rule of thumb strategies that make sense

Heuristic strategies

What are some easy strategies that you think would be effective?

Kempe et al

First “influence maximisation” algorithm

Greedy algorithm - Theoretical **guarantee**

Works well with **unrealistic** assumptions

Kempe et al

Algorithm 1 Greedy Approximation Algorithm

1: Start with $A = \emptyset$.
2: **while** $|A| \leq k$ **do**
3: For each node x, use repeated sampling to approximate $\sigma(A \cup \{x\})$ to within $(1 \pm \varepsilon)$ with probability $1 - \delta$.
4: Add the node with largest estimate for $\sigma(A \cup \{x\})$ to A.
5: **end while**
6: Output the set A of nodes.

Kempe et al

Set of nodes

Algorithm 1 Greedy Approximation Algorithm

Start with $A = \emptyset$.
while $|A| \leq k$ **do**
    For each node x, use repeated sampling to approximate $\sigma(A \cup \{x\})$ to within $(1 \pm \varepsilon)$ with probability $1 - \delta$.
    Add the node with largest estimate for $\sigma(A \cup \{x\})$ to A.
end while
Output the set A of nodes.

Kempe et al

Set of nodes

Maximum n. of nodes in seed

Algorithm 1 Greedy Approximation Algorithm

Start with $A = \emptyset$

while $|A| \leq k$ **do**

    For each node x, use repeated sampling to approximate $\sigma(A \cup \{x\})$ to within $(1 \pm \varepsilon)$ with probability $1 - \delta$.

    Add the node with largest estimate for $\sigma(A \cup \{x\})$ to A.

end while

Output the set A of nodes.

Kempe et al

Set of nodes

Maximum n. of nodes in seed

Algorithm 1 Greedy Approximation Algorithm

Start with $A = \emptyset$

while $|A| \leq k$ **do**

  For each node x, use repeated sampling to approximate $\sigma(A \cup \{x\})$ to within $(1 \pm \varepsilon)$ with probability $1 - \delta$.

  Add the node with largest estimate for $\sigma(A \cup \{x\})$ to A.

end while

Output the set A of nodes.

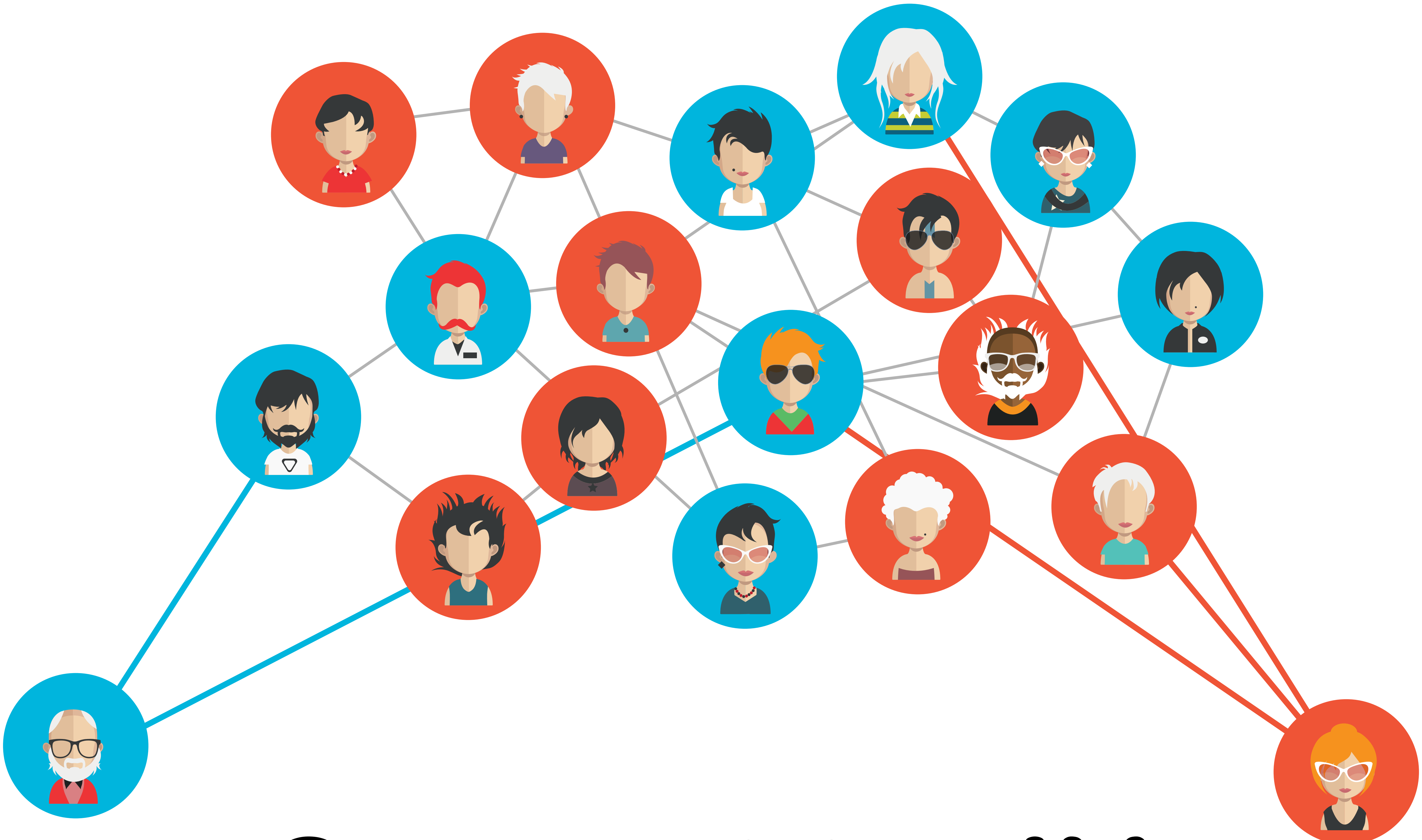
Influence of set of nodes a+x

Competitive IM

Two or more parties **compete** for influence

Classical setting: **2 parties**, opposite sides

Easy to study on **voter model**



Competitive IM

Competitive IM on voter model

$$\Delta_i \frac{dx_i}{dt} = (1 - x_i)(\sum_j a_{ji} x_j + p_{A,i}) - x_i(\sum_j a_{ji}(1 - x_j) + p_{B,i})$$

Competitive IM on voter model

Probability being in state A

$$\Delta_i \frac{dx_i}{dt} = (1 - x_i)(\sum_j a_{ji}x_j + p_{A,i}) - x_i(\sum_j a_{ji}(1 - x_j) + p_{B,i})$$

Competitive IM on voter model

Probability being in state A

$$\Delta_i \frac{dx_i}{dt} = (1 - x_i)(\sum_j a_{ji}x_j + p_{A,i}) - x_i(\sum_j a_{ji}(1 - x_j) + p_{B,i})$$

Influence of zealot A

Competitive IM on voter model

Probability being in state A

$$\Delta_i \frac{dx_i}{dt} = (1 - x_i)(\sum_j a_{ji} x_j + p_{A,i}) - x_i(\sum_j a_{ji}(1 - x_j) + p_{B,i})$$

Influence of zealot A

Influence of neighbours

Competitive IM on voter model

Probability being in state A

$$\Delta_i \frac{dx_i}{dt} = (1 - x_i)(\sum_j a_{ji}x_j + p_{A,i}) - x_i(\sum_j a_{ji}(1 - x_j) + p_{B,i})$$

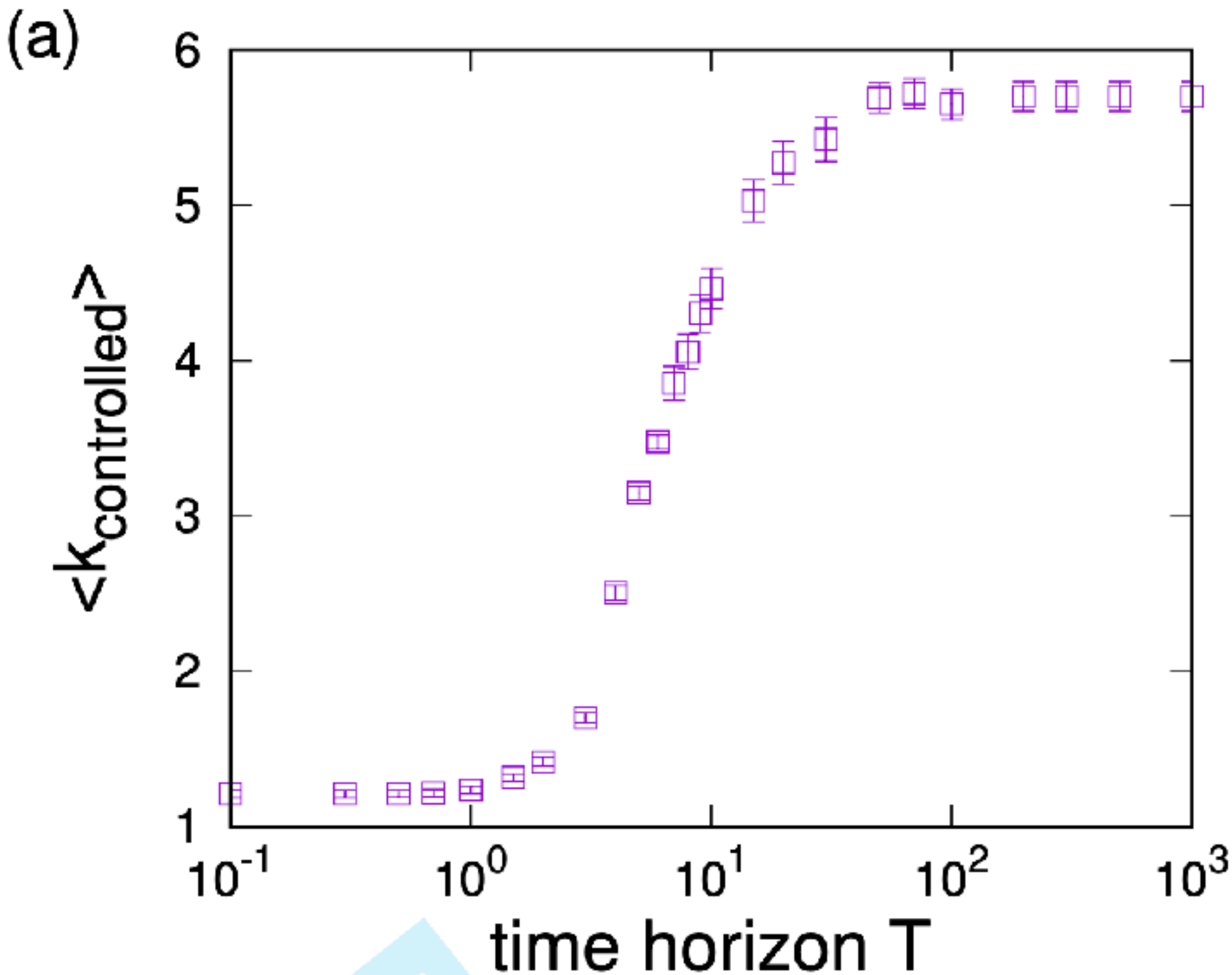
Influence of zealot A

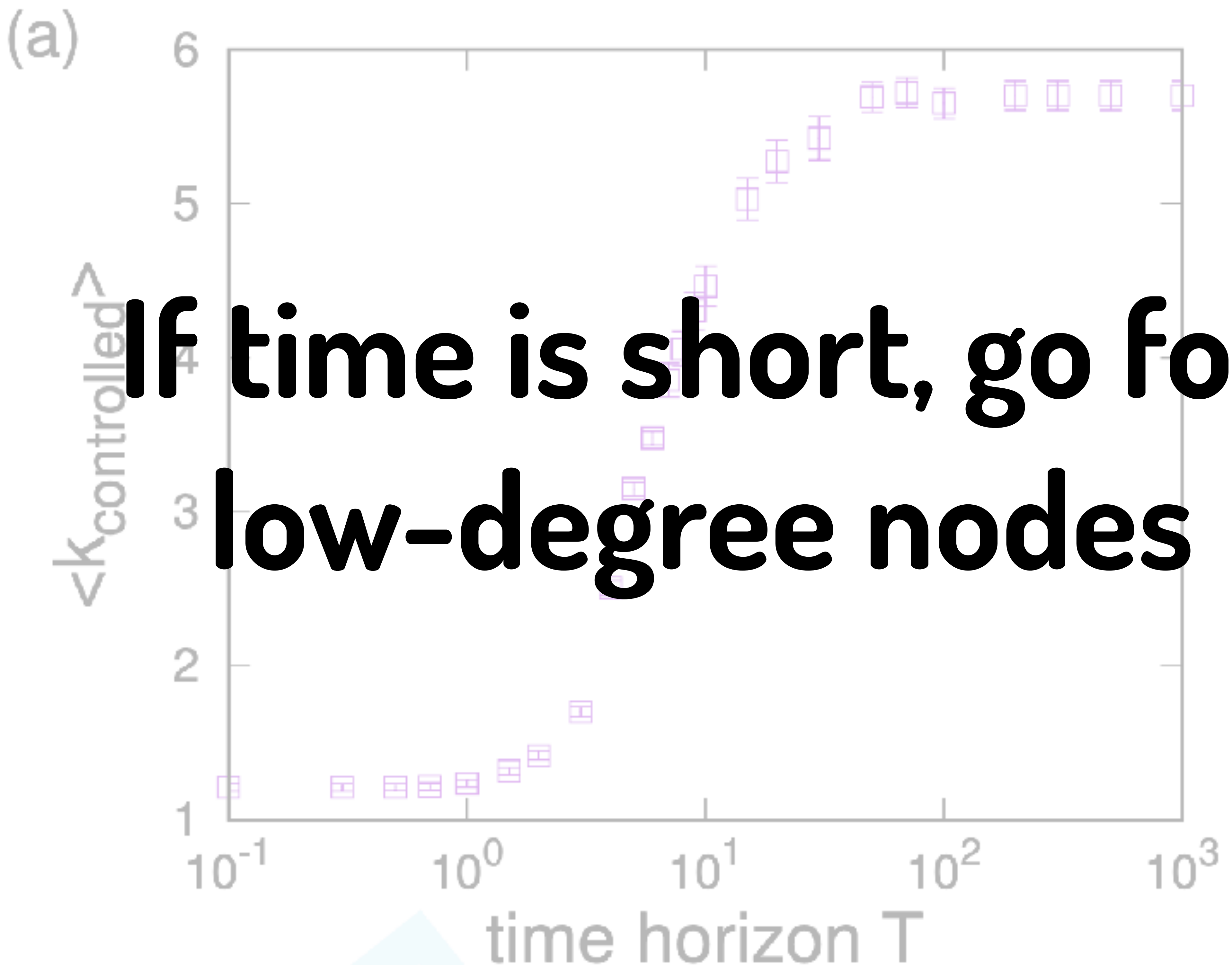
Normalisation factor

Influence of neighbours

$$\Delta_i = \sum_j a_{ji} + p_{A,i} + p_{B,i}$$

Competitive IM on voter model



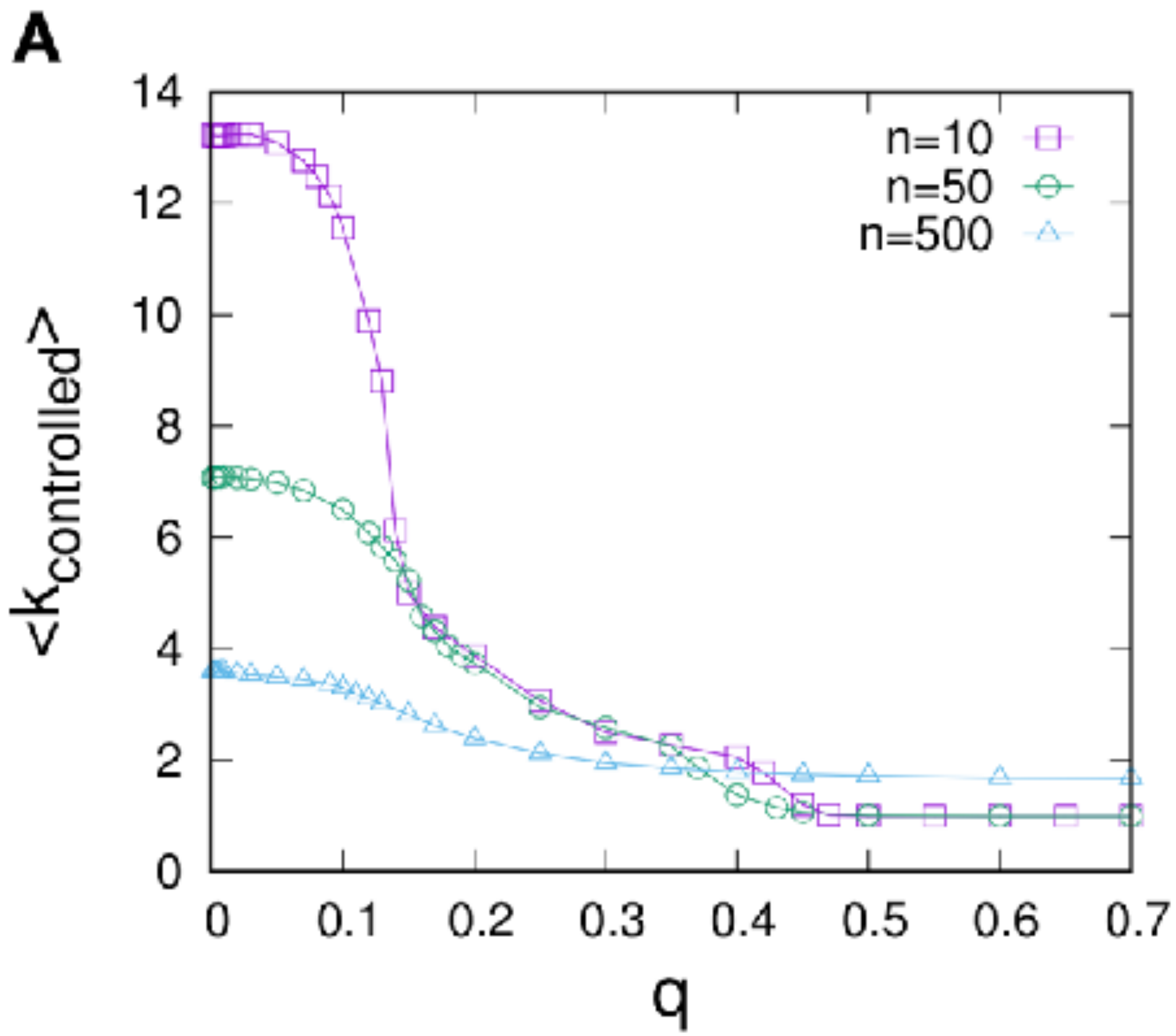


If time is short, go for low-degree nodes

Temporary influence

q = probability of flipping back to pre-influence state

Temporary influence



Scale-free network

Summary

Percolation and its implications
Systemic risk and instability of finance
Influence maximisation