Foundations of Natural Language Processing
Lecture 7
More smoothing and the Noisy Channel Model

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Recap: Smoothing for language models

- N-gram LMs reduce sparsity by assuming each word only depends on a fixed-length history.
- But even this assumption isn't enough: we still encounter lots of unseen N-grams in a test set or new corpus.
- If we use MLE, we'll assign 0 probability to unseen items: useless as an LM.
- **Smoothing** solves this problem: move probability mass from seen items to unseen items.

Smoothing methods so far

- Add-α smoothing: ($\alpha = 1$ or < 1) very simple, but no good when vocabulary size is large.

- Good-Turing smoothing:
  - estimate the probability of seeing (any) item with N_c counts (e.g., 0 count) as the proportion of items already seen with N_{c+1} counts (e.g., 1 count).
  - Divide that probability evenly between all possible items with N_c counts.

Good-Turing smoothing

- If n is count of history, then $P_{GT} = \frac{c^*}{n}$ where

$$c^* = (c+1)\frac{N_{c+1}}{N_c}$$

  - N_c number of N-grams that occur exactly c times in corpus
  - N_0 total number of unseen N-grams

- Ex. for trigram probability $P_{GT}(\text{three}|\text{I spent})$, then n is count of I spent and c is count of I spent three.

Problems with Good-Turing

- Assumes we know the vocabulary size (no unseen words)
  [but again, use UNK: see J&M 4.3.2]

- Doesn't allow "holes" in the counts (if $N_i > 0$, $N_{i-1} > 0$)
  [can estimate using linear regression: see J&M 4.5.3]

- Applies discounts even to high-frequency items
  [but see J&M 4.5.3]

- But there's a more fundamental problem...

Remaining problem

- In training corpus, suppose we see Scottish beer but neither of
  - Scottish beer drinkers
  - Scottish beer eaters

- If we build a trigram model smoothed with Add-α or G-T, which example has higher probability?

Remaining problem

- Previous smoothing methods assign equal probability to all unseen events.
- Better: use information from lower order N-grams (shorter histories).
  - beer drinkers
  - beer eaters
- Two ways: **interpolation** and **backoff**.

Interpolation

- **Combine** higher and lower order N-gram models, since they have different strengths and weaknesses:
  - high-order N-grams are sensitive to more context, but have sparse counts
  - low-order N-grams have limited context, but robust counts

- If P_N is N-gram estimate (from MLE, GT, etc; $N = 1 - 3$), use:

$$P_{\text{INT}}(w_3|w_1, w_2) = \lambda_1 \; P_1(w_3) + \lambda_2 \; P_2(w_3|w_2) + \lambda_3 \; P_3(w_3|w_1, w_2)$$

$$P_{\text{INT}}(\text{three}|\text{I, spent}) = \lambda_1 \; P_1(\text{three}) + \lambda_2 \; P_2(\text{three}|\text{spent}) + \lambda_3 \; P_3(\text{three}|\text{I, spent})$$

Interpolation

- Note that λ_is must sum to 1:

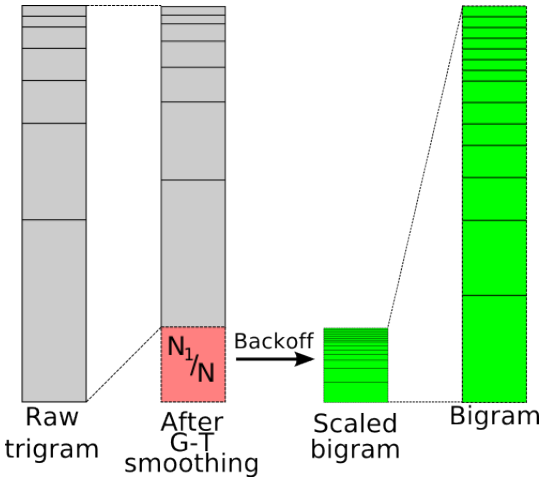
$$
\begin{aligned}
1 &= \sum_{w_3} P_{\text{INT}}(w_3|w_1,w_2) \\
&= \sum_{w_3} [\lambda_1\ P_1(w_3) + \lambda_2\ P_2(w_3|w_2) + \lambda_3\ P_3(w_3|w_1,w_2)] \\
&= \lambda_1 \sum_{w_3} P_1(w_3) + \lambda_2 \sum_{w_3} P_2(w_3|w_2) + \lambda_3 \sum_{w_3} P_3(w_3|w_1,w_2) \\
&= \lambda_1 + \lambda_2 + \lambda_3
\end{aligned}
$$

Fitting the interpolation parameters

- In general, any weighted combination of distributions is called a **mixture model**.
- So λ_is are **interpolation parameters** or **mixture weights**.
- The values of the λ_is are chosen to optimize perplexity on a held-out data set.

Katz Back-Off

- Solve the problem in a similar way to Good-Turing smoothing.
- Discount the trigram-based probability estimates.
- This leaves some probability mass to share among the estimates from the lower-order model(s).
- Katz backoff: Good-Turing discount the observed counts, but
- instead of distributing that mass uniformly over unseen items, use it for backoff estimates.



Back-Off Formulae

- Trust the highest order language model that contains N-gram

$$P_{BO}(w_i|w_{i-N+1},...,w_{i-1}) =$$

$$= \begin{cases} P^*(w_i|w_{i-N+1},...,w_{i-1}) \\ \qquad \text{if count}(w_{i-N+1},...,w_i) > 0 \\ \alpha(w_{i-N+1},...,w_{i-1})\ P_{BO}(w_i|w_{i-N+2},...,w_{i-1}) \\ \qquad \text{else} \end{cases}$$

Back-Off

- Requires
  - adjusted prediction model $P^*(w_i|w_{i-N+1}, ..., w_{i-1})$
  - backoff weights $\alpha(w_1, ..., w_{N-1})$
- Exact equations get complicated to make probabilities sum to 1.
- See textbook for details if interested.

Do our smoothing methods work here?

Example from MacKay and Peto (1995):

> Imagine, you see, that the language, you see, has, you see, a frequently occurring couplet, 'you see', you see, in which the second word of the couplet, see, follows the first word, you, with very high probability, you see. Then the marginal statistics, you see, are going to become hugely dominated, you see, by the words you and see, with equal frequency, you see.

- $P(\text{see})$ and $P(\text{you})$ both high, but see nearly always follows you.
- So $P(\text{see}|novel)$ should be much lower than $P(\text{you}|novel)$.

Diversity of histories matters!

- A real example: the word York
  - fairly frequent word in Europarl corpus, occurs 477 times
  - as frequent as foods, indicates and providers

  → in unigram language model: a respectable probability

- However, it almost always directly follows New (473 times)

- So, in unseen bigram contexts, York should have low probability
  - lower than predicted by unigram model as used in interpolation/backoff.

Kneser-Ney Smoothing

- Kneser-Ney smoothing takes diversity of histories into account
- Count of distinct histories for a word:

$$N_{1+}(\bullet w_i) = |\{w_{i-1} : c(w_{i-1}, w_i) > 0\}|$$

- Recall: maximum likelihood est. of unigram language model:

$$P_{ML}(w_i) = \frac{C(w_i)}{\sum_w C(w)}$$

- In KN smoothing, replace raw counts with count of histories:

$$P_{KN}(w_i) = \frac{N_{1+}(\bullet w_i)}{\sum_w N_{1+}(\bullet w)}$$

Kneser-Ney in practice

- Original version used backoff, later "modified Kneser-Ney" introduced using interpolation.
- Fairly complex equations, but until recently the best smoothing method for word n-grams.
- See Chen and Goodman (1999) for extensive comparisons of KN and other smoothing methods.
- KN (and other methods) implemented in language modelling toolkits like SRILM (classic), KenLM (good for really big models), OpenGrm Ngram library (uses finite state transducers), etc.

Are we done with smoothing yet?

We've considered methods that predict rare/unseen words using

- Uniform probabilities (add-α, Good-Turing)
- Probabilities from lower-order n-grams (interpolation, backoff)
- Probability of appearing in new contexts (Kneser-Ney)

What's left?

Word similarity

- Suppose we have two words with $C(w_1) \gg C(w_2)$
  - salmon
  - swordfish
- Can $P(\text{salmon}|\text{caught two})$ tell us anything about $P(\text{swordfish}|\text{caught two})$?
- N-gram models: no.

Word similarity in language modeling

- Early version: class-based language models (J&M 4.9.2)
  - Define classes c of words, by hand or automatically
  - $P_{CL}(w_i|w_{i-1}) = P(c_i|c_{i-1})P(w_i|c_i)$ (an HMM)
- Recent version: **distributed** language models
  - Use neural networks to project words into a continuous space, so words that appear in similar contexts have similar representations (e.g., Mikolov 2012).

Distributed word representations

- Each word represented as high-dimensional vector (50-500 dims)

E.g., salmon is $[0.1, 2.3, 0.6, -4.7, \ldots]$

- Similar words represented by similar vectors

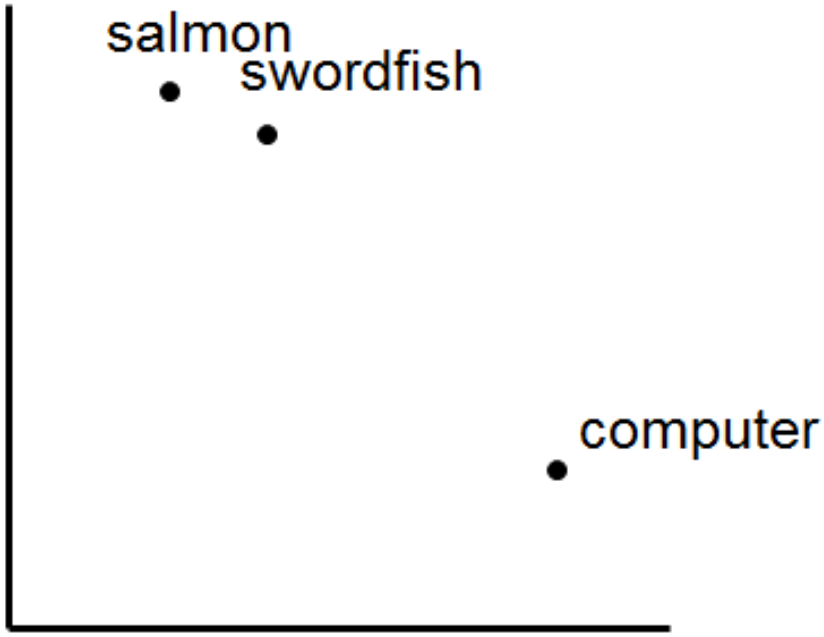
E.g., swordfish is $[0.3, 2.2, 1.2, -3.6, \ldots]$

- More about this later in the course.

Training the model

- Goal: learn word representations (**embeddings**) such that words that behave similarly are close together in high-dimensional space.

- 2-dimensional example:



Training the model

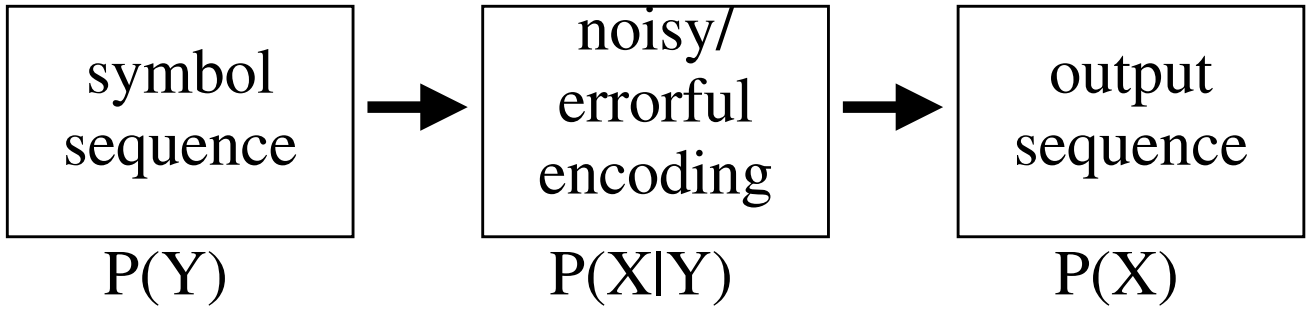
- N-gram LM: collect counts, maybe optimize some parameters
  - (Relatively) quick, especially these days (minutes-hours)
- distributed LM: learn the representation for each word
  - Use ML methods like neural networks that iteratively improve embeddings
  - Can be extremely time-consuming (hours-days)
  - Learned embeddings capture both semantic and syntactic similarity.
  - Embeddings building blocks for LLMs:
    More later in the course.

Back to the big picture

- However we train our LM, we will want to use it in some application.
- Now, a bit more detail about how that can work.
- We need another concept from information theory: the **Noisy Channel Model**.

Noisy channel model

- We imagine that someone tries to communicate a sequence to us, but noise is introduced. We only see the output sequence.



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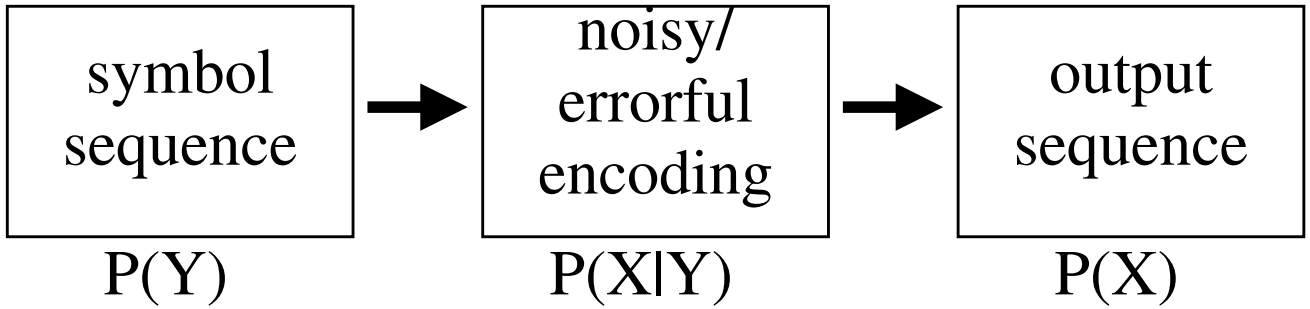

Application	Y	X
Speech recognition	spoken words	acoustic signal
Machine translation	words in L_1	words in L_2
Spelling correction	intended words	typed words

Example: spelling correction

- $P(Y)$: Distribution over the words the user intended to type. A language model!

- $P(X|Y)$: Distribution describing what user is **likely** to type, given what they **meant**. Could incorporate information about common spelling errors, key positions, etc. Call it a **noise model**.

- $P(X)$: Resulting distribution over what we actually see.

- Given some particular observation x (say, effert), we want to recover the most probable y that was intended.

Noisy channel as probabilistic inference

- Mathematically, what we want is $\text{argmax}_y P(y|x)$.
  - Read as "the y that maximizes $P(y|x)$"
- Rewrite using Bayes' Rule:

$$\begin{aligned} \underset{y}{\text{argmax}}\, P(y|x) &= \underset{y}{\text{argmax}}\, \frac{P(x|y)P(y)}{P(x)} \\ &= \underset{y}{\text{argmax}}\, P(x|y)P(y) \end{aligned}$$

Noisy channel as probabilistic inference

- So to recover the best y, we will need
  - a language model, which will be fairly similar for different applications
  - a noise model, which depends on the application: acoustic model, translation model, misspelling model, etc.

- Both are normally trained on corpus data.

You may be wondering

If we can train $P(X|Y)$, why can't we just train $P(Y|X)$? Who needs Bayes' Rule?

- Answer 1: sometimes we do train $P(Y|X)$ directly. Stay tuned...
- Answer 2: training $P(X|Y)$ or $P(Y|X)$ requires **input/output pairs**, which are often limited:
  - Misspelled words with their corrections; transcribed speech; translated text

  But LMs can be trained on huge unannotated corpora: a better model. Can help improve overall performance.

Model versus algorithm

- We defined a probabilistic model, which says **what** we should do.
  - E.g., for spelling correction: given trained LM and noise model (we haven't said yet how to acquire the noise model), find the intended word that is most probable given the observed word.
- We haven't considered **how** we would do that.
  - A **search problem**: there are (infinitely) many possible inputs that could have generated what we saw; which one is best?
  - We need to design an algorithm that can solve the problem.

Summary

- Different smoothing methods account for different aspects of sparse data and word behaviour.
  - Interpolation/backoff: leverage advantages of both higher and lower order N-grams.
  - Kneser-Ney smoothing: accounts for diversity of history.
  - Distributed representations: account for word similarity.
- Noisy channel model combines LM with noise model to define the "best" solution for many applications.
- Next time, will consider how to find this solution.