### Foundations of Natural Language Processing

Lecture 9: Distributional Semantics

#### Mirella Lapata

School of Informatics University of Edinburgh

mlap@inf.ed.ac.uk



Slides based on content from: Philipp Koehn, Alex Lascarides, Sharon Goldwater, Shay Cohen, Khalil Sima'an, Ivan Titov, Hinrich Schuetze

Two words are semantically similar if they have **similar meanings**.

 $astronaut \iff cosmonaut$ 



gobble ←⇒ devour



huge ← large



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- How about "banana" and "apple"?
- Are "car" and "flower" similar?

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- How about "banana" and "apple"?
- Are "car" and "flower" similar?
- And what do you think about "car" and "pope"?

### Why is semantic similarity interesting?

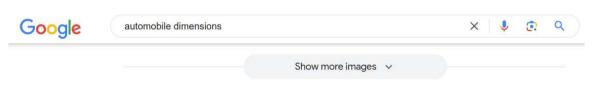
It's a **solvable problem** (see below). Many other things we want to do with language are more interesting, but much harder to solve.

- We do not need annotated data.
- There are many applications for semantic similarity.
- Two examples of applications:
  - 1. Direct use of measures of semantic similarity
  - 2. Plagiarism detection

### Application 1: Direct use of semantic similarity

- Query expansion in information retrieval
- User types in query [automobile]
- Search engine expands with semantically similar word [car]
- The search engine then uses the query [car OR automobile]
- Better results for the user

# Google: Internal model of semantic similarity



#### Car Dimensions: What Size is Your Vehicle?

https://www.carsquide.com.au > car-dimensions :

Use our free **car dimensions** tool to find the exact measurements for your vehicle. From length, width, height, ground clearance and more we have all the main ...



CarsGuide

### Compare car design and dimensions in a Virtual Showroom

This website provides simple means to compare **car dimensions** and design from a street perspective side by side for many current and old models. Limitations ...

### Application 2: Plagiarism Detection

36% SIMILARITY INDEX	11% INTERNET SOURCE	17% s publications	33% STUDENT PAPERS
PRIMARY SOURCES			
Student Pa	ted to Massey	University	18%
2 Submit	ted to GradeGu	uru	6%
3 Submit	ted to Foothill	College	4%
4 www.ge	eography.ccsu.e	edu	4%
5 Submit	ted to CSU, Ch	ico	2%
6 Submit Student Pag		rmingham Colle	ge 1%
7 Submit		ty of College Co	rk 1%
8 Submit	ted to CSU, Fu	llerton	1%
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#### Body

#### Humidity

Humidity refers to water vapour in the air. The capacity of air to hold water vapour is primarily a function of temperature. Warmer air has a greater capacity for holding water vapour than cooler air. The temperature at which a body of air becomes saturated is its dew-point temperature. Relative humidity is a ratio of the amount of water vapour that is actually in the air, compared with the maximum water vapour the air could hold at a given temperature. If

compared with the maximum water vapour the air could hold at a given temperature. If the air is saturated with all the moisture it can hold for its temperature, the relative humidity is 100%. A further increase of water vapour or a decurase in temperature results in active condensation. Relative humidity varies due to evaporation, condensation or temperature changes. All these affect both the moisture content and the capacity of the air to hold water vapour. It is highest at dawn, when air temperature is lowest and the capacity of air is less, and also lowest in late afternoon, where higher air temperatures increase the capacity of air to hold water vapour.

#### Adiabatic Processes

Adiabatic Warming and Cooling

In order for precipitation to occur, processes need to take place. Adiabatic processes are the changes in temperature that occur due to variations in the air pressure. When water

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- Perhaps we can infer meaning just by looking at the contexts a word occurs in
- Perhaps meaning IS the contexts a word occurs in (Wittgenstein!)
- Either way, similar contexts imply similar meanings
- This idea is known as the **distributional hypothesis** (Harris, 1954; Firth, 1857).

### Distributional Semantics and Word Embeddings

- Distributional semantics is an approach to semantics that is based on the contexts of words in large corpora.
- The basic notion formalized in distributional semantics is **semantic similarity**.
- Word embeddings are the modern incarnation of distributional semantics— adapted to work well with deep learning.

In this lecture, **semantic similarity** also includes **semantic relatedness** (e.g., "car" and "motorway" are related but not similar).

### Key concept: Cooccurrence count

#### **Cooccurrence Count**

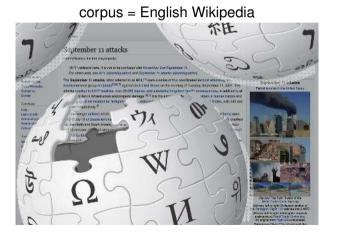
Basis for precise definition of "semantic similarity". The cooccurrence count of words  $w_1$  and  $w_2$  in a corpus is the number of times that  $w_1$  and  $w_2$  cooccur.

#### Different definitions of cooccurrence:

- in a linguistic relationship with each other (e.g.,  $w_1$  is a modifier of  $w_2$ ) or
- in the same sentence or
- in the same document or
- within a distance of at most k words (where k is a parameter)

### Word cooccurrence in Wikipedia: Examples

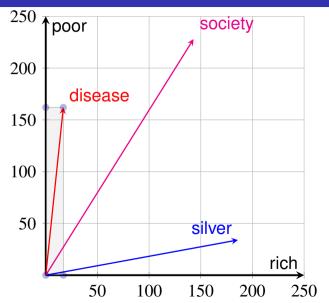
We define cooccurrence in this example as occurrence within k = 10 words of each other.



```
cooc.(rich,silver) = 186
cooc.(rich,society) = 143
cooc.(rich,disease) = 17
```

```
cooc.(poor,silver) = 34
cooc.(poor,society) = 228
cooc.(poor,disease) = 162
```

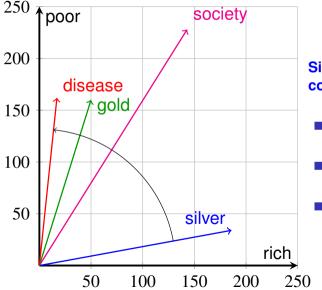
### Cooccurrence counts → Count vectors



```
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```

```
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```

## Cooccurrence counts $\rightarrow$ Vectors $\rightarrow$ Similarity



**Similarity** between two words is the **cosine** of the angle between them.

- Small angle: silver and gold are similar.
- Medium-size angle: silver and society are not very similar.
- Large angle: silver and disease are even less similar.

### Dimensionality of vectors

- Up to now we've only used two dimension words: rich and poor
- Now do this for a very large number of dimension words: hundreds, thousands, or even millions of dimension words.
- This is now a very high-dimensional space with a large number of vectors represented in it.
- But formally, there is no difference to a two-dimensional space with four vectors.

**Note:** a word has **dual role** in the vector space

- (1) each word is a **dimension word**, an axis of the space.
- (2) but each word is also a **vector** in that space.

### Measures of Similarity

The cosine of the angle between two vectors  $\mathbf{x}$  and  $\mathbf{y}$  is:

$$\cos(\mathbf{x}, \mathbf{y}) = \frac{\mathbf{x} \cdot \mathbf{y}}{||\mathbf{x}|| \cdot ||\mathbf{y}||} = \frac{\sum_{i=1}^{n} x_{i} y_{i}}{\sqrt{\sum_{i=1}^{n} x_{i}^{2}} \sqrt{\sum_{i=1}^{n} y_{i}^{2}}}$$

The Euclidean distance of two vectors x and y is:

$$||\mathbf{x} - \mathbf{y}|| = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

Many more similarity measures exist.

# Cooccurrence count (CC) matrix

		$w_2$										
		rich	poor	silver	society	disease						
	rich											
142.	poor silver											
$w_1$	society											
	disease											

# Cooccurrence count (CC) matrix

				$w_2$		
		rich	poor	silver	society	disease
	rich	$CC(w_1, w_2)$				
	poor	$CC(w_1, w_2)$				
$w_1$	silver	$CC(w_1, w_2)$				
	society	$CC(w_1, w_2)$				
	disease	$CC(w_1, w_2)$				

### Cases where distributional semantics fails

- Antonyms are judged to be similar: "disease" and "cure".
- Ambiguity: "Cambridge"
- Non-specificity (occurs in a large variety of different contexts and has few/no specific semantic associations): "person"
- The corpus meaning is different from the meaning that comes to mind when the word is encountered without context: "umbrella".
- Tokenization issues: "metal"

### Pointwise Mutual Information

Pointwise Mutual Information (PMI): weighting of cooccurrence counts. We are replacing the raw cooccurrence count with PMI, a measure of surprise.

$$PMI(w_1, w_2) = \log \frac{P(w_1, w_2)}{P(w_1)P(w_2)}$$

- If  $w_1, w_2$  independent:  $PMI(w_1, w_2) = 0$
- If  $w_1, w_2$  perfectly correlated:

$$P(w_1, w_2) = P(w_1) = P(w_2), \text{ PMI}(w_1, w_2) = \log \frac{P(w_2)}{P(w_2)P(w_2)} = \log \frac{1}{P(w_2)}$$

- If  $w_1, w_2$  positively correlated: PMI( $w_1, w_2$ ) is large and positive.
- If  $w_1, w_2$  negatively correlated: PMI( $w_1, w_2$ ) is large and negative.

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- What does it mean to have a negative PMI?

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  $PPMI(w_1, w_2) = \max \left(\log \frac{P(w_1, w_2)}{P(w_1)P(w_2)}, 0\right)$ 

- If  $w_1, w_2$  independent:  $PMI(w_1, w_2) = 0$
- If  $w_1, w_2$  perfectly correlated:  $P(w_1, w_2) = P(w_1) = P(w_2)$ ,  $PMI(w_1, w_2) = \log \frac{P(w_2)}{P(w_2)P(w_2)} = \log \frac{1}{P(w_2)}$
- If  $w_1, w_2$  positively correlated: PMI( $w_1, w_2$ ) is large and positive.
- If  $w_1, w_2$  negatively correlated: PMI( $w_1, w_2$ ) is large and negative.
- What does it mean to have a negative PMI? Replace negative PMI values with zero.

### Summary: Constructing Vector Spaces

Informal algorithm for constructing vector spaces:

- Select a corpus
- Select n target words which will be represented as vectors in the space;
- Select *k* dimension words (they are found around target word in the context window)
- **compute**  $k \times n$  **cooccurrence** matrix
- Compute (PPMI): weighted cooccurrence matrix
- Compute similarity of any two focus words as the cosine of their vectors

### Bag of words model

- We do not consider the **order** of words in a context.
- John is quicker than Mary and Mary is quicker than John give rise to same cooccurrence counts.
- This is called a bag of words model.
- More sophisticated models: compute dimension features based on the parse of a sentence – the feature "is object of the verb cook" would be recovered from both "John cooked the ham" and "the ham was cooked".

# **Embeddings**

#### Definition

The embedding of a word w is a dense vector  $\vec{v}(w) \in \mathcal{R}^k$  that represents semantic and other properties of w. Typical values are  $50 \le k \le 1,000$ .

- It appears there is little difference to count vectors: Both embeddings and count vectors are representations of words, primarily semantic, but also capturing other properties.
- Embeddings have much lower dimensionality than count vectors.
- Count vectors are sparse (most entries are 0), embeddings dense (almost never happens that an entry is 0).
- Embeddings are **lower-dimensional** (e.g., 100–300 dimensions).

### **Embedding Learning Algorithms**

#### **Singular Value Decomposition (SVD)**

- Also called Latent Semantic Indexing (LSI)
- Factorization of cooccurrence matrix
- Least squares objective optimized by power method

#### Word2Vec

- A group of related models used to generate word embeddings
- Word2Vect models are optimized by gradient descent
- Skip-gram model predicts surrounding words (context) given a target word

## Embedding Learning Algorithms

#### Singular Value Decomposition (SVD)

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# Linear Algebra: Recap

### **Dot product**

$$\mathbf{x} \cdot \mathbf{y} = \sum_{i=1}^{n} x_i y_i$$

#### **Example**

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = x_1 y_1 + x_2 y_2 + x_3 y_3$$

### **Matrix Multiplication**

$$\begin{array}{ccc}
A & \cdot & B & = & AB \\
m \times n & n \times p & m \times p
\end{array}$$

#### Example

$$\begin{pmatrix} a_1 & b_1 \\ c_2 & d_1 \end{pmatrix} \cdot \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} a_1 a_2 + b_1 c_2 & a_1 b_2 + b_1 d_2 \\ c_1 a_2 + d_1 c_2 & c_1 b_2 + d_1 d_2 \end{pmatrix}$$

### Length of vector

$$\mathbf{d}| = \sqrt{\sum_{i=1}^n d_i^2}$$

### **Orthogonal Vectors**

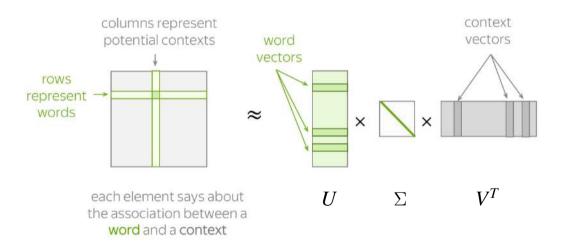
 ${f c}$  and  ${f d}$  are orthogonal iff

$$\sum_{i=1}^{n} c_i \cdot d_i = 0$$

## Matrix Factorization: Embeddings

- We will decompose the cooccurrence matrix into a product of matrices.
- The particular decomposition we'll use: singular value decomposition (SVD).
- SVD:  $C = U\Sigma V^T$  (where C = cooccurrence matrix, with PPMI weighting)
- We will then use the SVD to compute a new, improved cooccurrence matrix C'.
- We'll get better and more compact word representations out of C' (compared to C).

### **SVD** Visualization



### **SVD Summary**

- We decompose the cooccurrence matrix C into a product of three matrices:  $U^T$
- The input word matrix U consists of one (row) vector for each word
- The context word matrix  $V^T$  consists of one (column) vector for each context word
- The singular value matrix  $\Sigma$  is a diagonal matrix with singular values, reflecting importance of each dimension
- We only keep first k dimensions and set the others to zero.

### Property of SVD that we exploit here

- **Key property:** each singular value tells us how important its dimension is.
- By setting less important dimensions to zero, we keep the important information, but get rid of the "details".
- These details may be noise in that case, reduced SVD vectors are a better representation because they are less noisy or make things dissimilar that should be similar again, reduced SVD vectors are a better representation because they represent similarity better.
- Analogy for "fewer details is better": Image of a blue flower Image of a yellow flower, Omitting color makes is easier to see the similarity

# Example of $C = U\Sigma V^T$ : All four matrices

SVD is decomposition of C into a representation of the input words, a representation of the context and a representation of the importance of the "semantic" dimensions

C		$w_1$	$w_2$	$w_3$	$W_4$	$w_5$	$w_6$		U		1	2	3	4	5
rich	ı	1	0	1	0	0	0	-	rich	-0.4	44 –	0.30	0.57	0.58	0.25
pod	or	0	1	0	0	0	0		poor	-0.	13 –	-0.33 -	-0.59	0.00	0.73
silv	er	1	1	0	0	0	0	=	silver	-0.4	48 –	-0.51 -	-0.37	0.00 -	$-0.61 \times$
soc	ciety	1	0	0	1	1	0		society	-0.	70	0.35	0.15 -	-0.58	0.16
dis	ease	0	0	0	1	0	1		disease	-0.2	26	0.65 -	-0.41	0.58 -	-0.09
	,									'					
$\sum$	1	2		3	4	į	5		$V^T$	$w_1$	$w_2$	$w_3$	$w_4$	. W	$w_6$
$\frac{\Sigma}{1}$	2.16	0.0	0 0	3	0.00		5 00		· .				<u>'</u>	•	$\frac{w_6}{8-0.12}$
$rac{\Sigma}{1}$	2.16 0.00		_			0.			1 (		0.28	-0.20	-0.45	0.33	$\frac{3}{8} - 0.12$
1		0.0	9 (	0.00	0.00	0. 0.	00	×	1 0	).75 –(	0.28 0.53	-0.20 $-0.19$	-0.45	0.22 0.22	$\frac{3}{8} - 0.12$
1 2	0.00	0.0 1.5	9 ( 0 1	0.00	0.00	0. 0. 0.	00	×	1 0 2 -0 3 0	).75 –( ).29 –( ).28 –(	0.28 0.53	-0.20 $-0.19$	-0.45 $0.63$ $-0.20$	0.22 0.22	3 - 0.12 2 0.41 2 -0.33
1 2 3	0.00	0.0 1.5 0.0	9 0 0 1 0 0	0.00 0.00 1.28	0.00 0.00 0.00	0. 0. 0.	00 00 00	×	1 0 2 -0 3 0 4 0	).75 – ( ).29 – ( ).28 – ( ).00 (	0.28 0.53 0.75	-0.20 $-0.19$ $0.45$	-0.45 $0.63$ $-0.20$ $0.00$	0.033 $0.22$ $0.12$ $0.058$	3 - 0.12 2 0.41 2 -0.33

### Embeddings = Left Singular Vectors

U	1	2	3	4	5
rich	-0.44				
poor	-0.13	-0.33	0.00	0.00	0.00
silver	-0.48	-0.51	0.00	0.00	0.00
society	-0.70	0.35	0.00	0.00	0.00
disease	-0.26	0.65	0.00	0.00	0.00

$\sum$	1	2	3	4	5		$V^T$	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$	$w_6$
1	2.16	0.00	0.00	0.00	0.00	•	1	0.75	-0.28	-0.20	-0.45	-0.33	-0.12
2	0.00	1.59	0.00	0.00	0.00		2	-0.29	-0.53	-0.19	0.63	0.22	0.41
3	0.00	0.00	0.00	0.00	0.00	$\times$	3	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00		4	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00		5	0.00	0.00	0.00	0.00	0.00	0.00

### Summary

- The meaning of a word is learned from its contexts in a large corpus.
- The main analysis method of contexts is co-occurrence.
- Distributional semantics is a good model of semantic similarity. There is a lot more in semantics that distributional semantics is not a good model for.
- Embeddings have lower-dimensionality than count vectors
- Singular value decomposition is one method to obtain dense vector representations.

**Next time:** generating embeddings with Word2Vec.