

Introduction to Databases

(INFR10080)

(Equivalence of RA and RC)

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Changelog

v25.0 Initial version

Algebra $\equiv$ Safe calculus

Fundamental theorem of database theory:

Relational algebra and Safe relational calculus **equally expressive**

- For every query in safe relational calculus
there exists an equivalent query in relational algebra
- For every query in relational algebra
there exists an equivalent query in safe relational calculus

We need a **correspondence** between column **names** and **positions**

When we say that a **base relation** R is over attributes $A, B, C, \dots$
this means the 1st column is A , the 2nd is B , the 3rd is C , etc.

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From algebra to calculus

Translate each RA expression E into a FOL formula φ

Assumptions (without loss of generality)

- Each renaming operation renames **only one** attribute

Environment η

Injective map from attributes to variables

Unless stated otherwise, for an attribute A we assume $\eta(A) = x_A$

But in general the chosen **variable names can be arbitrary**

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From algebra to calculus

Base relation

R over $A_1, \dots, A_n$ is translated to $R(\eta(A_1), \dots, \eta(A_n))$

Example

If R is a base relation over A, B

$$\eta = \{ A \mapsto x_A, B \mapsto x_B, \dots \}$$

then R is translated to $R(x_A, x_B)$

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From algebra to calculus

Renaming $\rho_{\text{old} \rightarrow \text{new}}(E)$

1. Translate E to φ
2. If there is no mapping for **new** in η , add $\{\text{new} \mapsto x_{\text{new}}\}$
3. Replace every occurrence of $\eta(\text{new})$ in φ with a **fresh** variable
4. Replace every (free) occurrence of $\eta(\text{old})$ in φ by $\eta(\text{new})$

Example

If R is a base relation over A, B

then $\rho_{A \rightarrow B}(\rho_{B \rightarrow C}(R))$ is translated to $R(x_B, x_C)$

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From algebra to calculus

Projection

$\pi_\alpha(E)$ is translated to $\exists X \varphi$

where

- φ is the translation of E
- $X = \mathbf{free}(\varphi) - \eta(\alpha)$
(attributes that are **not** projected become quantified)

Example

If R is a base relation over A, B

then $\pi_A(R)$ is translated into $\exists x_B R(x_A, x_B)$

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From algebra to calculus

Selection

$\sigma_\theta(E)$ is translated to $\varphi \wedge \eta(\theta)$

where

- φ is the translation of E
- $\eta(\theta)$ is obtained from θ by replacing each attribute A by $\eta(A)$

Example

If R is a base relation over A, B

then $\sigma_{A=B}(R)$ is translated into $R(x_A, x_B) \wedge x_A = x_B$

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From algebra to calculus

Product	$E_1 \times E_2$	is translated to	$\varphi_1 \wedge \varphi_2$
Union	$E_1 \cup E_2$	is translated to	$\varphi_1 \vee \varphi_2$
Difference	$E_1 - E_2$	is translated to	$\varphi_1 \wedge \neg \varphi_2$

where

- φ_1 is the translation of E_1
- φ_2 is the translation of E_2

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Example

Customer : CustID, Name

Account : Number, CustID

Environment $\eta = \{ \text{CustID} \mapsto x_1, \text{Name} \mapsto x_2, \text{Number} \mapsto x_3 \}$

How do we translate **Customer** $\bowtie$ **Account** ? Blackboard time!

$$\exists x_4 \text{ Customer}(x_1, x_2) \wedge \text{Account}(x_3, x_4) \wedge x_1 = x_4$$

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Active domain in relational algebra

For R over attributes $A_1, \dots, A_n$

$\mathbf{adom}(R)$ is given by $\rho_{A_1 \rightarrow A}(\pi_{A_1}(R)) \cup \dots \cup \rho_{A_n \rightarrow A}(\pi_{A_n}(R))$

$$\mathbf{adom}(D) = \bigcup_{R \in D} \mathbf{adom}(R)$$

We denote by $\mathbf{Adom}_N$ the RA expression that, on a database D , returns a table

- with a single column, named N
- consisting of all elements of $\mathbf{adom}(D)$

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From calculus to algebra

Translate each FOL formula φ into an RA expression E

Assumptions (without loss of generality)

- No universal quantifiers, implications, double negations
- No distinct pair of quantifiers binds the same variable name
- No variable name occurs both free and bound
- No variable name is repeated within a predicate
- No constants in predicates
- No atoms of the form $x \mathbf{op} x$ or $c_1 \mathbf{op} c_2$

Environment η

Injective map from variables to attributes

Unless stated otherwise, for a variable x we assume $\eta(x) = A_x$

But in general the chosen attribute names can be arbitrary

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From calculus to algebra

Let R be over attributes $A_1, \dots, A_n$

Predicate

$R(x_1, \dots, x_n)$ is translated to $\rho_{A_1 \rightarrow \eta(x_1), \dots, A_n \rightarrow \eta(x_n)}(R)$

Example

For R over attributes A, B, C ,

$R(x, y, z)$ is translated into $\rho_{A \rightarrow A_x, B \rightarrow A_y, C \rightarrow A_z}(R)$

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From calculus to algebra

Existential quantification

$\exists x \varphi$ is translated to $\pi_{\eta(X - \{x\})}(E)$

where

- E is the translation of φ
- $X = \mathbf{free}(\varphi)$

Example

For φ with free variables x, y, z and translation E ,

$\exists y \varphi$ is translated to $\pi_{A_x, A_z}(E)$

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From calculus to algebra

Comparisons

$x \text{ op } y$ is translated to $\sigma_{\eta(x) \text{ op } \eta(y)}(\mathbf{Adom}_{\eta(x)} \times \mathbf{Adom}_{\eta(y)})$

$x \text{ op } c$ is translated to $\sigma_{\eta(x) \text{ op } c}(\mathbf{Adom}_{\eta(x)})$

Example

$x = y$ is translated to $\sigma_{A_x = A_y}(\mathbf{Adom}_{A_x} \times \mathbf{Adom}_{A_y})$

$x > 1$ is translated to $\sigma_{A_x > 1}(\mathbf{Adom}_{A_x})$

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From calculus to algebra

Negation

$\neg\varphi$ is translated into $\left(\bigtimes_{x \in \text{free}(\varphi)} \mathbf{Adom}_{\eta(x)} \right) - E$

where E is the translation of φ

Example

For φ with free variables x, y and translation E

$\neg\varphi$ is translated to $\mathbf{Adom}_{A_x} \times \mathbf{Adom}_{A_y} - E$

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From calculus to algebra

Disjunction: $\varphi_1 \vee \varphi_2$ is translated to

$$E_1 \times \left(\bigtimes_{x \in X_2 - X_1} \mathbf{Adom}_{\eta(x)} \right) \cup E_2 \times \left(\bigtimes_{x \in X_1 - X_2} \mathbf{Adom}_{\eta(x)} \right)$$

where, for $i \in \{1, 2\}$,

- E_i is the translation of φ_i
- $X_i = \mathbf{free}(\varphi_i)$

Conjunction: same as disjunction, but use $\cap$ instead of $\cup$

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Example

Customer : CustID, Name

Account : Number, CustID

Translate $\exists x_4 \text{ Customer}(x_1, x_2) \wedge \text{Account}(x_3, x_4) \wedge x_1 = x_4$

Environment $\eta = \{ x_1 \mapsto A, x_2 \mapsto B, x_3 \mapsto C, x_4 \mapsto D \}$

$$\pi_{A,B,C} \left(\left(E_1 \times \mathbf{Adom}_C \times \mathbf{Adom}_D \right) \cap \left(\mathbf{Adom}_A \times \mathbf{Adom}_B \times E_2 \right) \cap \left(\sigma_{A=D}(\mathbf{Adom}_A \times \mathbf{Adom}_D) \times \mathbf{Adom}_B \times \mathbf{Adom}_C \right) \right)$$

where

- $E_1 = \rho_{\text{CustID} \rightarrow A, \text{Name} \rightarrow B}(\text{Customer})$
- $E_2 = \rho_{\text{Number} \rightarrow C, \text{CustID} \rightarrow D}(\text{Account})$

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