

Introduction to Databases

Predicate Logic

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Problem 1. Let R and S be ternary and binary predicate names, respectively, and consider the following first-order logic formulas:

$$\exists x R(x, y, z) \wedge \forall y \exists x S(y, x) \rightarrow S(x, y) \quad (\varphi_1)$$

$$\exists x \forall y (S(y, x) \rightarrow S(x, y)) \rightarrow \exists y R(x, y, z) \quad (\varphi_2)$$

$$(\exists x S(y, x) \vee S(x, y)) \wedge \neg \exists y R(x, y, z) \quad (\varphi_3)$$

Rewrite each of the above formulae into equivalent ones that use only existential quantification, conjunction, and negation. Use the equivalences presented in class, and indicate which equivalence is applied at each rewriting step.

Problem 2. Consider the unary predicates P for PERSON and C for CAKE, and the binary predicates H for HAVE and E for EAT, with the following understanding:

- $P(x)$ means that x is a person;
- $C(x)$ means that x is a cake;
- $H(x, y)$ means that x possesses y ; and
- $E(x, y)$ means that x eats y .

- Write a first-order logic formula that expresses the saying “No person can have a cake and eat it too” without using universal quantifiers.
- Rewrite the above formula into an equivalent one of the form $\forall x \forall y \varphi \rightarrow \neg \psi$, where φ and ψ are conjunctions of predicates.

Problem 3. Let $\{\text{John, Mary, Jane, Scooby}\} \cup \mathbb{N}$ be the underlying set of constants. Consider the binary predicate Person and the unary predicate Happy, and take the semantic function $\mathcal{I}$ such that

$$\text{Person}^{\mathcal{I}} = \{(\text{John}, 24), (\text{Jane}, 20), (\text{Mary}, 26)\}$$

$$\text{Happy}^{\mathcal{I}} = \{\text{Scooby, Jane, Mary}\}$$

For each of the formulas below, indicate whether there is an assignment ν such that the interpretation $(\mathcal{I}, \nu)$ satisfies the formula. If yes, provide a suitable assignment; otherwise, explain why no such assignment exists.

- $\text{Happy}(x) \wedge \neg \exists y \text{ Person}(x, y)$
- $\exists x, y \text{ Person}(x, z) \wedge \text{Person}(y, z)$
- $\exists x, y \text{ Person}(x, z) \wedge \text{Person}(y, z) \wedge \neg(x = y)$
- $\forall x \text{ Happy}(x) \rightarrow \exists y \text{ Person}(x, y)$