Learning and Memory - Associative Memory

Informatics 1 Cognitive Science

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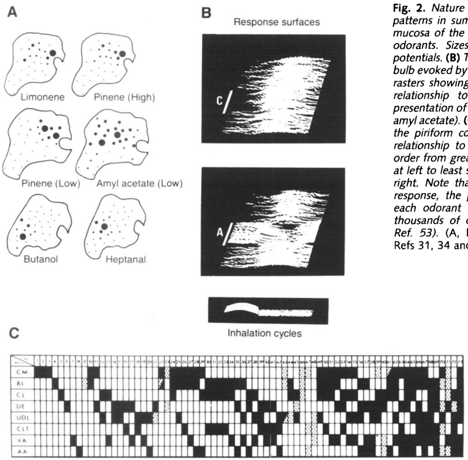
Topics

- Associative memory
- Auto-associative memory
- Hopfield networks
- Auto-associative ensembles in the brain

Associative Memory

- Retrieval of computer memory is address-based
  - localised: one address
  - error-prone: gone if one bit flipped in address
  - reliability through check-sums etc.
- In the brain memory retrieval appears content-addressable
  - associative: partial cues sufficient for recall
  - distributed: neurons may participate in multiple memories
  - error correcting: '*An American politician who was very intelligent and whose politician father did not like broccoli*.' (MacKay, 2003)
  - robust: tolerates loss of neurons

Associative encoding of odorands



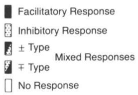


Fig. 2. *Nature of the olfactory code.* **(A)** *Broad spatial patterns in summed receptor potentials in the olfactory mucosa of the tiger salamander in response to different odorants. Sizes of dots denote relative amplitudes of potentials.* **(B)** *Temporal response patterns in the olfactory bulb evoked by two different odorants. Plots are response rasters showing changes in single unit firing patterns in relationship to the inhalation cycle (bottom) during presentation of odors (white bars at left; C = cineole, A = amyl acetate).* **(C)** *Specificity in responses of single units in the piriform cortex and adjacent amygdaloid cortex in relationship to eight odorants. Results are arranged in order from greatest specificity (response to one odorant) at left to least specificity (response to seven odorants) at right. Note that while there is a tendency for specific response, the pattern is essentially an ensemble code: each odorant excites or inhibits the firing of tens of thousands of cells in all parts of the cortex (also see Ref. 53).* (A, B, C reproduced, with permission, from Refs 31, 34 and 54, respectively.)

The models we will discuss were developed before experimental data became available.

Haberly, L. B., & Bower, J. M. (1989). Olfactory cortex: model circuit for study of associative memory?. Trends in Neurosciences, 12(7), 258-264.

The Willshaw Network for Associative Memories (1969)

Non-Holographic Associative Memory

by
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The features of a hologram that commend it as a model of associative memory can be improved on by other devices.

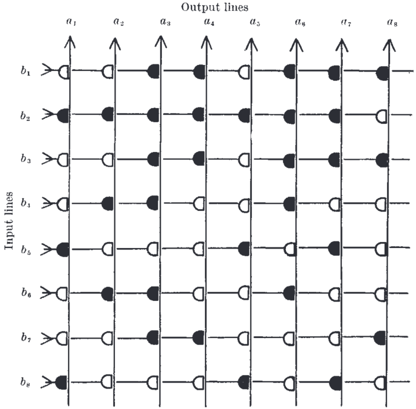

Fig. 4. An associative net.

A Model for Auto-Associative Memory

Aim: To store "patterns" in a network of neurons. Each pattern will be associated with itself, hence *auto-associative*. This model should be able to retrieve a memory also from partial cues.

Let's first create a simple network

Network of M binary (McCulloch-Pitts) neurons s_i connected by weights w_{ij}:

$$
\begin{aligned}
s_i(t+1) &= \Theta\left(\sum_{j=1}^{M} w_{ij} s_j(t) - \theta_i\right) \\
\Theta(a) &= \begin{cases} 1 & a \geq 0 \\ 0 & a < 0 \end{cases}
\end{aligned}
$$

- Symmetric weights: $w_{ij} = w_{ji}$
- Updates can be synchronous or asynchronous.
- The bias value θ_i determines the average activity.
- Converges to stable fixed point under fairly general conditions.
- Aim: activities s_i should reflect a stored pattern when presented with similar inputs.

M Neurons $S = \{s_i\}$
N Pattern $P = \{p_i\}$
Weights $W = \{w_{ij}\}$

$$w_{ij} = \frac{1}{N}\sum_{n=1}^{N} p_i^n p_j^n$$

This is a simple Hebbian plasticity rule!

Recall in the Hopfield Network

Recall in the Hopfield Network

Recall in the Hopfield Network

Recall in the Hopfield Network

Recall in the Hopfield Network

Recall in the Hopfield Network

Recall in the Hopfield Network

Recall in the Hopfield Network

(a) 

.	0	0	0	0	-2	2	-2	2	2	-2	0	0	0	2	0	0	-2	0	2	2	0	0	-2	-2
0	.	4	4	0	-2	-2	-2	-2	-2	-2	0	-4	0	-2	0	0	-2	0	-2	-2	4	4	2	-2
0	4	.	4	0	-2	-2	-2	-2	-2	-2	0	-4	0	-2	0	0	-2	0	-2	-2	4	4	2	-2
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0	0	0	0	.	2	-2	-2	2	-2	2	-4	0	0	-2	4	-4	-2	0	-2	2	0	0	-2	2
-2	-2	-2	-2	2	.	0	0	0	0	4	-2	2	-2	0	2	-2	0	-2	0	0	-2	-2	0	4
2	-2	-2	-2	-2	0	.	0	0	4	0	2	2	-2	4	-2	2	0	-2	4	0	-2	-2	0	0
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2	-2	-2	-2	2	0	0	0	.	0	0	-2	2	2	0	2	-2	0	2	0	4	-2	-2	-4	0
2	-2	-2	-2	-2	0	4	0	0	.	0	2	2	-2	4	-2	2	0	-2	4	0	-2	-2	0	0
-2	-2	-2	-2	2	4	0	0	0	0	.	-2	2	-2	0	2	-2	0	-2	0	0	-2	-2	0	4
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-2	-2	-2	-2	-2	0	0	4	0	0	0	2	2	2	0	-2	2	.	2	0	0	-2	-2	0	0
0	0	0	0	0	-2	-2	2	2	-2	-2	0	0	4	-2	0	0	2	.	-2	2	0	0	-2	-2
2	-2	-2	-2	-2	0	4	0	0	4	0	2	2	-2	4	-2	2	0	-2	.	0	-2	-2	0	0
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-2	2	2	2	-2	0	0	0	-4	0	0	2	-2	-2	0	-2	2	0	-2	0	-4	2	2	.	0
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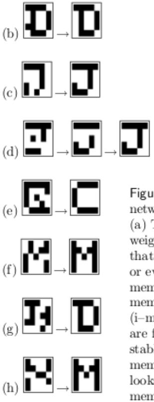


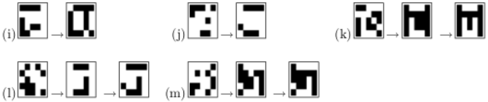
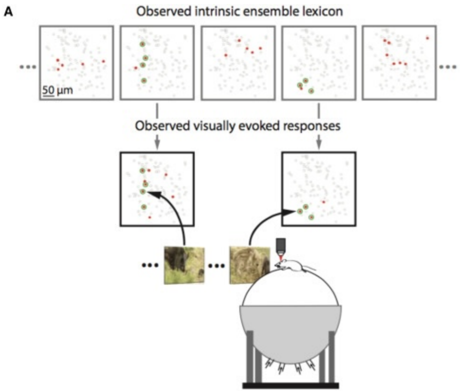


Figure 42.3. Binary Hopfield network storing four memories. (a) The four memories, and the weight matrix. (b–h) Initial states that differ by one, two, three, four, or even five bits from a desired memory are restored to that memory in one or two iterations. (i–m) Some initial conditions that are far from the memories lead to stable states other than the four memories; in (i), the stable state looks like a mixture of two memories, 'D' and 'J'; stable state (j) is like a mixture of 'J' and 'C'; in (k), we find a corrupted version of the 'M' memory (two bits distant); in (l) a corrupted version of 'J' (four bits distant) and in (m), a state which looks spurious until we recognize that it is the inverse of the stable state (l).

Properties of the Hopfield Network

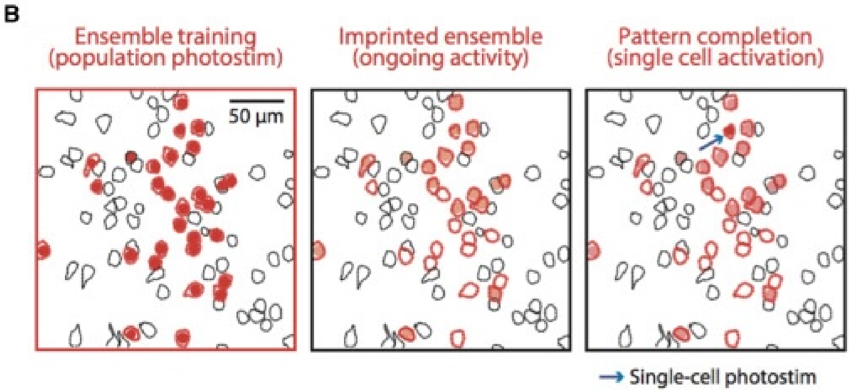
- Capacity: $0.138M$ if 0/1 have equal probability in each pattern (sparseness $s = 0.5$). Capacity increases dramatically for sparse patterns ($s < 0.5$).
- Exhibits catastrophic forgetting: adding new patterns may destroy old ones.
- Not entirely robust to noise: a single flipped bit may lead to a completely different pattern.
- Not biologically plausible: it requires symmetric weights.

Auto-associative ensembles in the brain



Cortical ensembles activated spontaneously (top) or by naturalistic visual stimuli (bottom) in mouse primary visual cortex in vivo (red: members of an ensemble, green: active in both conditions.

Yuste, R., Cossart, R., & Yaksi, E. (2024). Neuronal ensembles: Building blocks of neural circuits. Neuron.



Repeated optogenetic activation of a group of neurons (red, left) leads to spontaneous activity in this group (middle). This pattern can now be recalled through partial stimulation (arrow, right), demonstrating pattern completion.

Yuste, R., Cossart, R., & Yaksi, E. (2024). Neuronal ensembles: Building blocks of neural circuits. Neuron.

Summary

- Associative memory is content-addressable and distributed.
- The Willshaw network is a simple model for associative memory.
- The Hopfield network is a more complex model where recall is based on network dynamics.
- Auto-associative ensembles are a plausible model for associative memory in the brain.