Informatics 1 Cognitive Science

Lecture 5: The Perceptron

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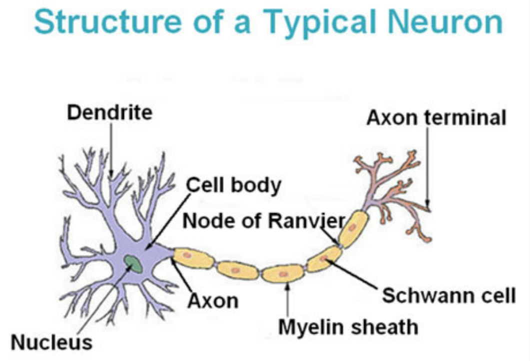
Slide credits: Frank Mollica, Chris Lucas, Mirella Lapata

Overview

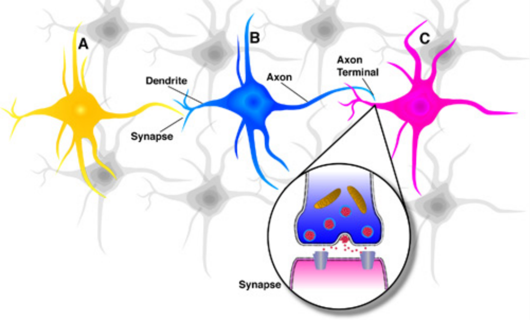
Neural Networks

A Single Neuron



- A neuron receives inputs and combines these in the cell body.
- If the input reaches a threshold, then the neuron may fire (produce an output).
- Some inputs are excitatory, while others are inhibitory.

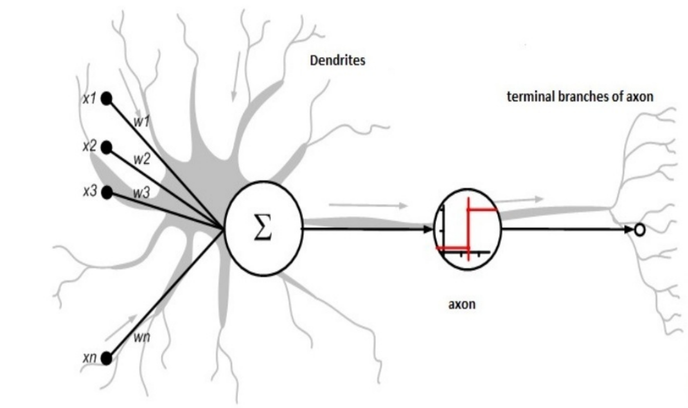
Biological Neural Networks



- In biological neural networks, neurons are connected by synapses.
- An input connection is a conduit through which a neuron receives information.
- An output connection is a conduit through which a neural sends information.

Neural Networks

Neural networks (aka deep learning) is a computer modeling approach inspired by information processing in networks of biological neurons.



Anatomy of a Neural Network

Units are to a neural net model what neurons are to a biological neural network — basic information processing structures.

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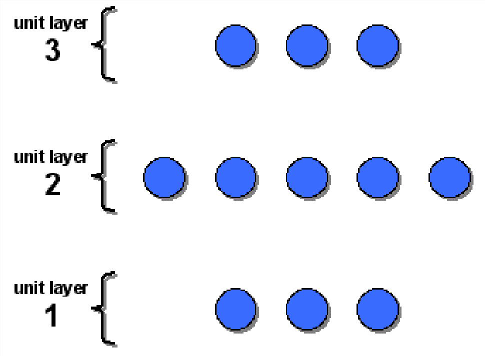
Biological neural networks are organized in layers of neurons. Neural net models are organized in layers of units, not random clusters.



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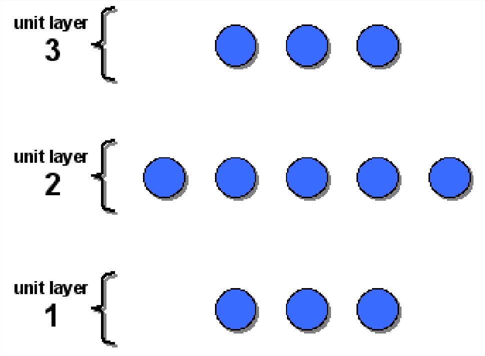
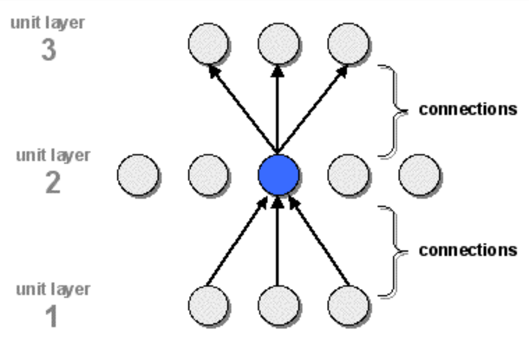


But what you see here still isn't a network. Something is missing.

Anatomy of a Neural Network

Network connections are conduits through which information flows between the units in a network:

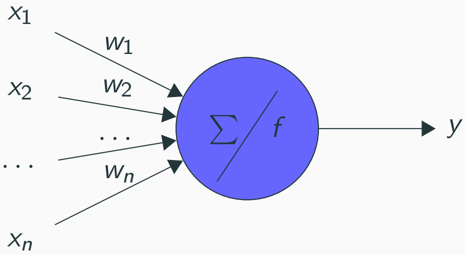


- Connections are represented with lines
- Arrows in a neural net indicate the flow of information from one unit to the next.

The Perceptron

Perceptron: An Artificial Neuron

The perceptron was developed by Frank Rosenblatt in 1957. It's the simplest kind of artificial neural network.



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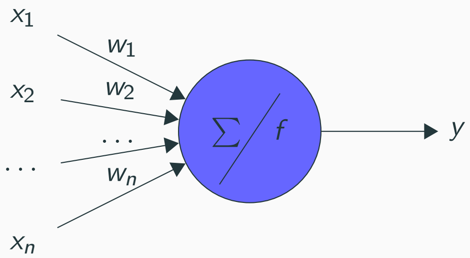

Input function:

$$u(x) = \sum_{i=1}^{n} w_i x_i$$

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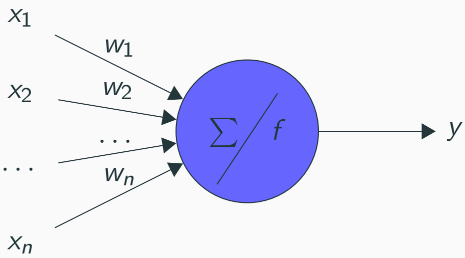

Input function:

$$u(\mathrm{x}) = \sum_{i=1}^{n} w_i x_i$$

Activation function: threshold

$$y = f(u(\mathrm{x})) = \begin{cases} 1, & \text{if } u(\mathrm{x}) > \theta \\ 0, & \text{otherwise} \end{cases}$$

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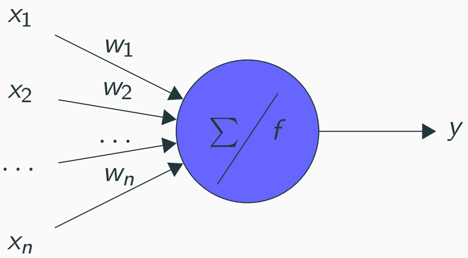

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Activation function: threshold

$$y = f(u(\mathrm{x})) = \begin{cases} 1, & \text{if } u(\mathrm{x}) > \theta \\ 0, & \text{otherwise} \end{cases}$$

Activation state:
0 or 1 (-1 or 1)

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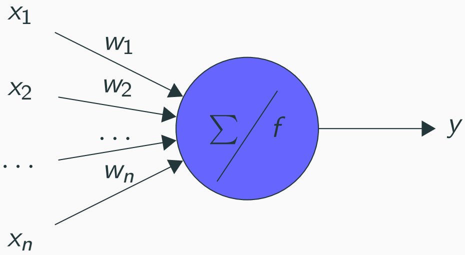
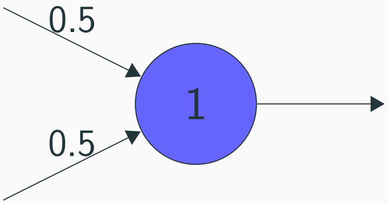


- Inputs are in the range [0, 1], where 0 is "off" and 1 is "on".
- Weights can be any real number (positive or negative).

Perceptrons for Logic

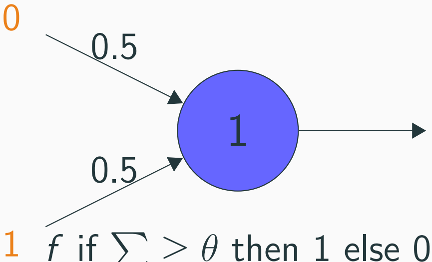
Perceptron for AND



f if $\sum \geq \theta$ then 1 else 0

input		output $y =$
x_1	x_2	x_1 AND x_2
0	0	0
0	1	0
1	0	0
1	1	1

Perceptron for AND



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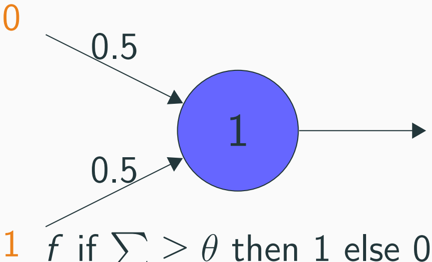
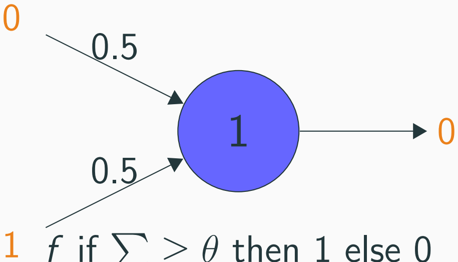


f if $\sum \geq \theta$ then 1 else 0

$0 \cdot 0.5 + 1 \cdot 0.5 = 0.5$

input x_1	input x_2	output $y = x_1$ AND x_2
0	0	0
0	1	0
1	0	0
1	1	1

Perceptron for AND



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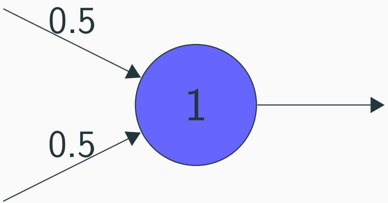

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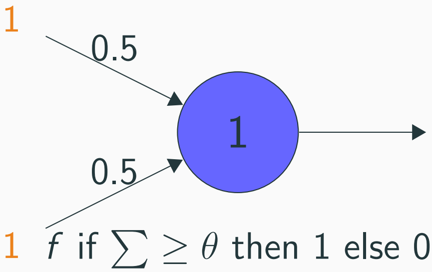
Perceptrons for Logic

Perceptron for AND



input		output $y =$
x_1	x_2	x_1 AND x_2
0	0	0
0	1	0
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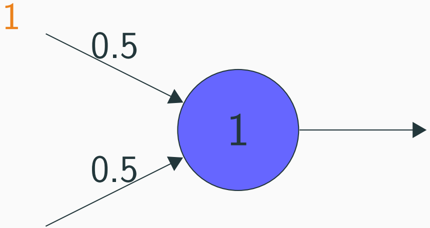
Perceptrons for Logic

Perceptron for AND



f if $\sum \geq \theta$ then 1 else 0

$$1 \cdot 0.5 + 1 \cdot 0.5 = 1$$

input		output $y =$
x_1	x_2	x_1 AND x_2
0	0	0
0	1	0
1	0	0
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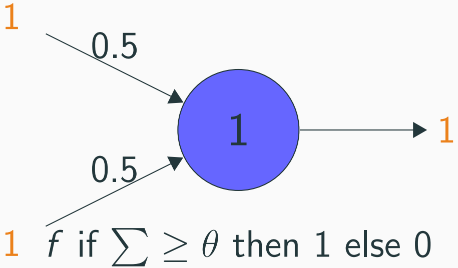
Perceptrons for Logic

Perceptron for AND



f if $\sum \geq \theta$ then 1 else 0

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input x_1	input x_2	output $y = x_1$ AND x_2
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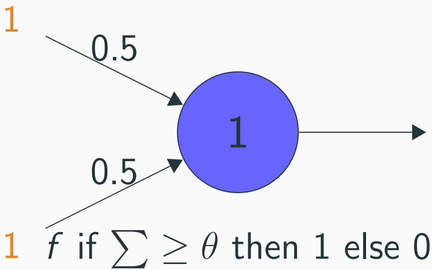
Perceptron for AND



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input		output $y =$
x_1	x_2	x_1 AND x_2
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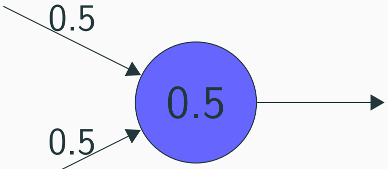
1	0	0
1	1	1

We can easily devise perceptrons that compute the logical functions AND and OR.

Can we compute all logical functions? What about XOR?

Perceptrons for Logic

Perceptron for XOR

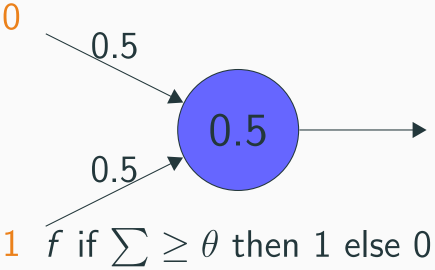


f if $\sum \geq \theta$ then 1 else 0

input x_1	x_2	output $y =$ x_1 XOR x_2
0	0	0
0	1	1
1	0	1
1	1	0

XOR is an exclusive OR because it only returns a true value of 1 if the two values are exclusive, i.e., they are both different.

Perceptron for XOR



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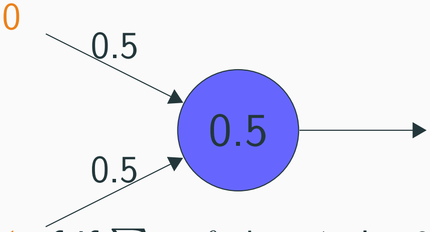

f if $\sum \geq \theta$ then 1 else 0

$$0 \cdot 0.5 + 1 \cdot 0.5 = 0.5$$

input x_1	input x_2	output $y = x_1$ XOR x_2
0	0	0
0	1	1
1	0	1
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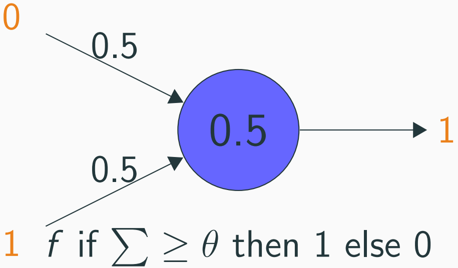

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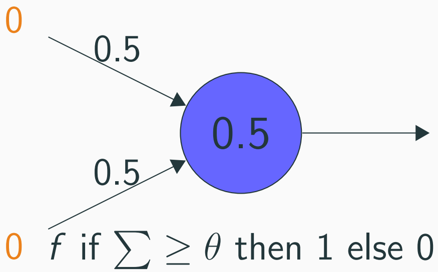

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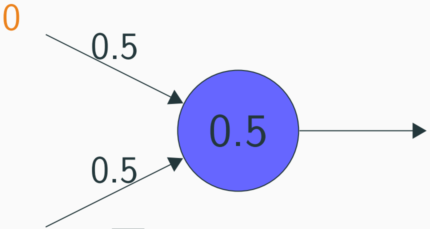
Perceptron for XOR



0 f if $\sum \geq \theta$ then 1 else 0

$$0 \cdot 0.5 + 0 \cdot 0.5 = 0$$

input		output $y =$
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Perceptrons for Logic

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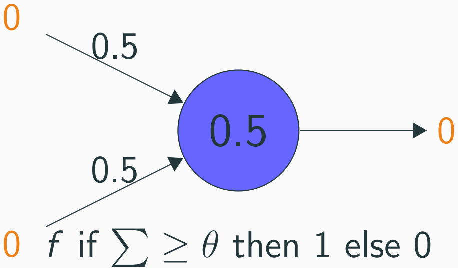

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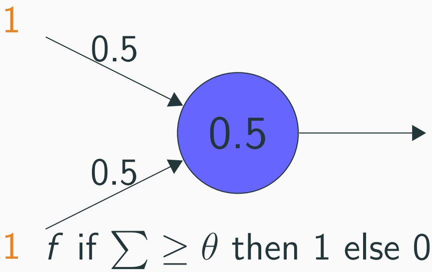
Perceptron for XOR



input x_1	input x_2	output $y = x_1$ XOR x_2
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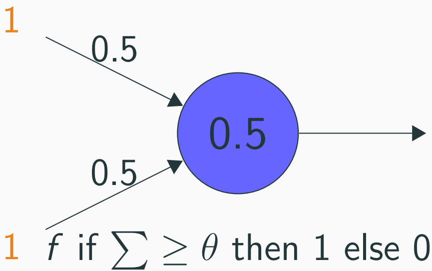
Perceptrons for Logic

Perceptron for XOR



f if $\sum \geq \theta$ then 1 else 0

$$1 \cdot 0.5 + 1 \cdot 0.5 = 1$$

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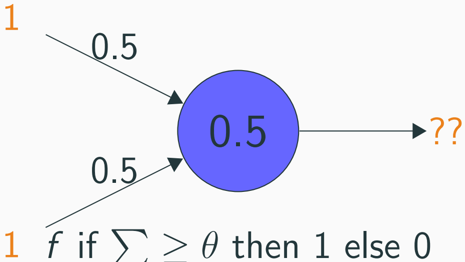

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input		output $y =$
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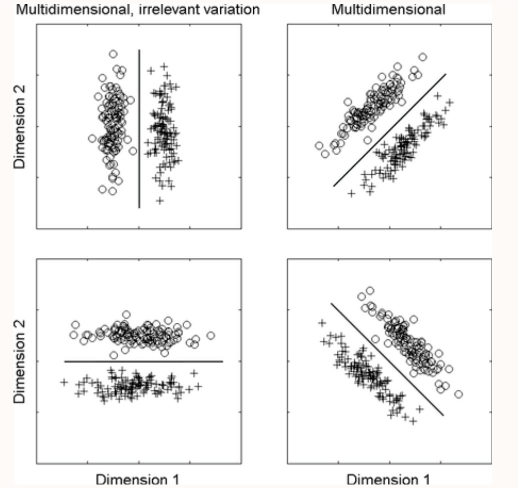
Time for a short quiz on Wooclap!

https://app.wooclap.com/GEKKBD

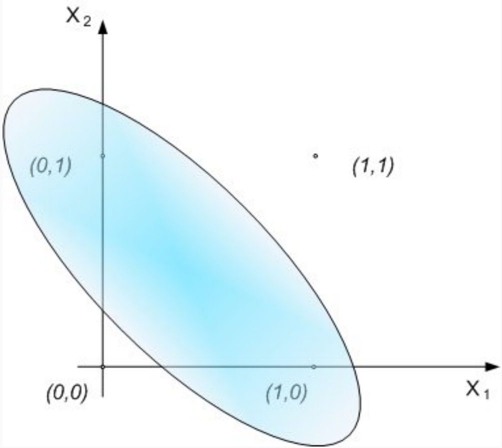
Perceptrons as Classifiers

Perceptrons are linear classifiers, i.e., they can only separate points with a hyperplane (a straight line in two dimensions).



The XOR problem again

The XOR function is not linearly separable, as more than one line is required to separate the two classes {(0,0), (1,1)} and {(0,1), (1,0)}. A single-layer perceptron cannot compute XOR.



Learning in Perceptrons

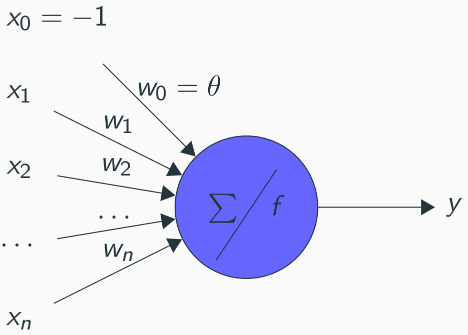
Learning

Q_1: But choosing weights and threshold θ for the perceptron is not easy! How to learn the weights and threshold from examples?

A_1: We can use a learning algorithm that adjusts the weights and threshold based on examples.

http://www.youtube.com/watch?v=vGwemZhPlsA&feature=youtu.be

Learning: A trick to learn θ



- We can consider θ as a weight to be learned!
- The input is fixed as -1. The activation function is then:

$$y = f(u(\mathrm{x})) = \begin{cases} 1, & \text{if } u(\mathrm{x}) > 0 \\ 0, & \text{otherwise} \end{cases}$$

What is the Perceptron Really Seeing?

Sequence of exemplars presented to the perceptron during training:

N	**input** x	**target** t
1	(0,1,0,0)	1
2	(1,0,0,0)	0
3	(0,1,1,1)	0
4	(1,0,1,0)	0
5	(1,1,1,1)	1
6	(0,1,0,0)	1

- This perceptron has 4 inputs (binary) $\approx$ feature vector representing exemplars
- The perceptron sees 6 exemplars or training items

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Sequence of exemplars presented to the perceptron during training:

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- This perceptron has 4 inputs (binary) $\approx$ feature vector representing exemplars
- The perceptron sees 6 exemplars or training items
- We don't know the weights/threshold!

What is the Perceptron Really Seeing?

Sequence of exemplars presented to the perceptron during training:

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1	(0,1,0,0)	1	0
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3	(0,1,1,1)	0	1
4	(1,0,1,0)	0	1
5	(1,1,1,1)	1	0
6	(0,1,0,0)	1	1

- This perceptron has 4 inputs (binary) $\approx$ feature vector representing exemplars
- The perceptron sees 6 exemplars or training items
- We don't know the weights/threshold!
- But we know the perceptron's output o and can compare it to the correct answer, the target t

The Perceptron Learning Rule

Learning Rule

Key idea: Adjust the weights so that o (the output of the perceptron) moves closer to t (the target, i.e., the desired correct output):

Perceptron Learning Rule

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta(t - o)x_i$$

- η, $0 < \eta \leq 1$ is a constant called learning rate.
- t is the target for the current example.
- o is the perceptron output for the current example.

Learning Rule

Perceptron Learning Rule

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta(t - o)x_i$$

$$o = 1 \text{ and } t = 1$$
$$o = 0 \text{ and } t = 1$$

- Learning rate η is positive; controls how big changes Δw_i are.
- If $x_i > 0$, $\Delta w_i > 0$ then w_i increases in an attempt to make $w_i x_i$ become larger than θ.
- If $x_i < 0$, $\Delta w_i < 0$ then w_i reduces.

Perceptron Learning Rule

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i \ = \eta(t - o)x_i$$

$$\begin{aligned} o = 1 \text{ and } t = 1 \quad \Delta w_i \ &= \eta(t - o)x_i = \eta(1 - 1)x_i = 0 \\ o = 0 \text{ and } t = 1 \quad \Delta w_i \ &= \eta(t - o)x_i = \eta(1 - 0)x_i = \eta x_i \end{aligned}$$

- Learning rate η is positive; controls how big changes Δw_i are.
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Perceptrons for Logic

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Learning Rule: Exercise

Perceptron Learning Rule

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i \; = \eta(t - o)x_i$$

Consider a perceptron with only one input x_1, weight $w_1 = 0.5$, threshold $\theta = 0$ and learning rate $\eta = 0.6$. Consider also the training example $\{x_1 = -1, t = 1\}$. For now, let's temporarily ignore the learning of the threshold and consider it fixed.

Learning Rule: Exercise

Perceptron Learning Rule

$$w_i \leftarrow w_i + \Delta w_i$$

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Consider a perceptron with only one input x_1, weight $w_1 = 0.5$, threshold $\theta = 0$ and learning rate $\eta = 0.6$. Consider also the training example $\{x_1 = -1, t = 1\}$. For now, let's temporarily ignore the learning of the threshold and consider it fixed.

- Determine the output of the perceptron for the input -1:

Learning Rule: Exercise

Perceptron Learning Rule

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta(t - o)x_i$$

Consider a perceptron with only one input x_1, weight $w_1 = 0.5$, threshold $\theta = 0$ and learning rate $\eta = 0.6$. Consider also the training example $\{x_1 = -1, t = 1\}$. For now, let's temporarily ignore the learning of the threshold and consider it fixed.

- Determine the output of the perceptron for the input -1:
  $w_1 x_1 = 0.5(-1) = -0.5 \leq \theta \rightarrow o = 0$
- The new weight w_1 after applying the learning rule:

Learning Rule: Exercise

Perceptron Learning Rule

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i \ = \eta(t - o)x_i$$

Consider a perceptron with only one input x_1, weight $w_1 = 0.5$, threshold $\theta = 0$ and learning rate $\eta = 0.6$. Consider also the training example $\{x_1 = -1, t = 1\}$. For now, let's temporarily ignore the learning of the threshold and consider it fixed.

- Determine the output of the perceptron for the input -1:
  $w_1 x_1 = 0.5(-1) = -0.5 \leq \theta \rightarrow o = 0$
- The new weight w_1 after applying the learning rule:
  $\Delta w_1 = 0.6(1 - 0)(-1) = -0.6 \rightarrow w_1 = 0.5 - 0.6 = -0.1$

Learning Rule: Exercise

Perceptron Learning Rule

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- The new output of the perceptron for the input -1:
  $w_1 x_1 = -0.1(-1) = 0.1 \geq \theta \rightarrow o = 1$

Learning Algorithm

```
Initialize all weights randomly.
repeat
    for each training example do
        Apply the learning rule.
    end for
until the error is acceptable or a certain number
of iterations is reached
```

This algorithm is guaranteed to find a solution with zero error in a limited number of iterations as long as the examples are linearly separable.

Summary

- Neural networks (aka deep learning) is a computer modeling approach inspired by networks of biological neurons.
- A neural net consists of units and connections.
- The perceptron is the simplest neural network model; it is a linear classifier.
- A learning algorithm for perceptrons exists.
- **Key limitation:** only works for linearly separable data.