Informatics 1 Cognitive Science – Tutorial 5 Solutions

Frank Keller, Chris Lucas, Frank Mollica

Week 6

In class we discussed categorization and how conceptual knowledge is organized. The aim of this tutorial is to understand theories of categorization and relations between categories. The second part of the tutorial deals with judgment and decision making.

1 Classical Concepts

This exercise is designed to shed light on some challenges facing classical (or definitional) accounts of concepts and some phenomena that are relevant to theories of concepts and categories.

Under classical accounts, concepts are represented using sets of necessary and jointly sufficient properties. For instance, we might say that a triangle is a polygon with three vertices. If something isn't a polygon, or if it doesn't have three vertices, it isn't a triangle (necessity)[1]. If it has both of these properties, it must be a triangle (joint sufficiency). In the case of mathematical concepts, this can work fairly well, but we'll see it's a bit more difficult for everyday objects and events. If you want a trickier task, replace the word "rabbit" below with "art", or "game". Here's the exercise:

1. Get a sheet of paper (or text editor) and write down (as best you can) the necessary and jointly sufficient properties something must have to be a **rabbit**. When you're done, pass your list to the person on your left (or below you in the tutorial chat).
2. When you receive the list from someone else, come up with a counterexample. This is either (1) something you would call a rabbit, which does not have one of the listed properties, or (2) something that isn't a rabbit but has all of the properties on the list. Update the list of properties to account for your counterexample.
3. Pass the updated list to the next person and repeat the process from step 2. When your original list comes back to you, you're done, at which point it's time to discuss the lists.

Discussion

Compare your lists and discuss the following questions.

- Are the lists effectively the same – do they pick out the same sets of things as being rabbits?
- For each list, what's the last counterexample that someone came up with? Does everyone agree that this counterexample is, or isn't, a rabbit?
- Are there counterexamples where people agree that something is a rabbit, but disagree about the extent to which it's a good, or typical example of a rabbit?
- Does everyone agree that the lists (or some of them) are complete, and admit no new counterexamples?

[1] Naturally, we'll need to define vertices and polgyons as well

Solution There aren't right and wrong answers in this exercise, but hopefully it'll be a useful medium for discussing the following points (and others):

- It's difficult to define concepts in terms of necessary and sufficient properties. This doesn't logically imply that our mental representations aren't definitional, but it nonetheless bodes ill for the theory.
- Concept membership tends to be graded. A penguin and a robin are both birds in almost everyone's assessment, but people show other behaviors suggesting that a robin is a better, more prototypical example of a bird than a penguin is. The classical theory of concepts needs to introduce additional machinery to explain this.
- Not everyone agrees about what goes into a particular category. This isn't a problem for definitional theories in particular – people might vary in their mental representations whatever they are – but it's good to keep in mind when assessing theories of concepts and explanations of how we learn or construct concepts, and to think about how variability in people's concepts might have implications for communication.

2 Similarity-based Concept Models

General guidance: The discussion of this question could start with a recap of the problems with the classical view of concepts, and an explanation of how the similarity-based view addresses these problems. Then remind students that similarity-based theories crucially rely on how features of exemplars and the similarity between exemplars is defined. The point of the exercise is for students to work through a concrete example that illustrates this, and to compare the prototype-based view and the exemplar-based view. In this example, we only have one feature, weight, and similarity can simply be defined in terms of difference in weight.

Imagine you've just been introduced to two new categories for the first time: "daxes" and "grobs". As far as you can tell, daxes and grobs are all identical except for their weights. A person gives you 100 daxes that weigh around 10 kg, and 100 daxes that weigh around 100 kg. The person also gives you 200 grobs that have widely varying weights between 1 kg and 200 kg, with an average of weight of 105 kg.

You now find two new objects (object P and object Q). P weighs 101 kg, and Q weighs 57 kg.

1. According to prototype theory as discussed in lecture, is P more likely to be a dax or a grob? Why?

   Solution: The average dax weight is around 55kg, while the average grob is 105kg. At 101kg, P is closer to the average grob, so it's more likely to be a grob.

2. According to prototype theory as discussed in lecture, is Q more likely to be a dax or a grob? Why?

   Solution: The average dax weight is around 55kg, while the average grob is 105kg. At 57 kg, Q is closer to the average dax, so it's more likely to be a dax.

3. According to exemplar theory as discussed in lecture, is P more likely to be a dax or a grob? Why?

   Solution: P is close in weight to a large number of dax exemplars so we might expect that it's a dax.

4. According to exemplar theory as discussed in lecture, is Q more likely to be a dax or a grob? Why?

   Solution: Q is far from most dax exemplars and likely to be comparatively close to several grob exemplars, so we might expect it's a grob. That said, it depends on one's weighting function, and students haven't been exposed to classical weighting functions (which tend to involve exponential decay).

3 Lotteries

General guidance: The aim of this question is for students to apply the concept of expected utility to a concrete example. The examples assume that expected utility is just the expected value of the monetary gain (a concept that many students be familiar with). However, the question is designed to encourage discussion of cases where a more complicated utility function needs to be assumed (as in prospect theory).

We discussed lotteries in the context of framing effects and prospect theory. When reasoning about the subjective value of decisions in an uncertain environment, it is common to focus on **expected utility,** or the sum of the utilities of different possible outcomes weighted by their probabilities. For example, if we assign a utility of 10 to a coin coming up heads, and a utility of 5 to the coin coming up tails, and believe a heads will come up with probability 0.6, then the expected utility is $0.6 \times 10 + 0.4 \times 5 = 8$. In the following example, we will focus first on monetary gains and losses, rather than utilities.

Suppose we have a lottery in which there are 1000 tickets printed with unique numbers. After all of the tickets have been purchased, one of the ticket numbers will be called out randomly (all are equally likely), and the person with that ticket will win £500.

Exercises

1. What is the expected monetary gain or loss for someone who purchases one ticket for £1?
   Solution: If all tickets are equally likely to win, then the probability of winning is 1/1000. The expected reward is

   $$\sum_{i=1}^{N} R(o_i)P(o_i)$$

   where $N = 2$, o_1 is the outcome of winning, $R(o_1) = 500 - 1$, $P(o_1) = 0.001$, $R(o_2) = -1$, and $P(o_2) = 0.999$. That comes out to $0.499 - 0.999 = -0.5$.

2. One way to assess whether a person takes a rational approach to decision-making is to see if there are situations where their beliefs lead to guaranteed, avoidable losses. Consider a person who thinks £1 is a fair ticket price for buying or selling a ticket and is willing and able to buy any number of tickets. Is there a way for someone to take this person's money without incurring any risk?
   Solution: Yes. In this case, the lottery operator can make guaranteed money by selling at least 501 tickets to the person. If the person is sold **all** of the tickets, the operator makes a profit of £500: £1000 guaranteed ticket revenue minus £500 from paying out on the lottery.

3. What if this person is only willing to buy a single ticket?
   Solution: Here the bookie cannot guarantee the person will lose money, so it isn't risk free. This suggests there are utility functions where this value assignment could make sense.

4. Under what circumstances, if any, might it be rational for a person to purchase a single ticket for £1, considering only selfish economic concerns, e.g., setting aside any pleasure a person might

take in gambling and the possibility that ticket proceeds go to charity? Feel free to use contrived scenarios if necessary.

Solution: Sometimes a person's utility can be a non-linear function of monetary gains or losses. For example, if a person is starving and lives in a society where food is unavailable for £1, then that person might reasonably assign a utility of zero to any amount of money that doesn't prevent starvation. In that case, a small chance of winning enough money to buy food is better than the alternative.