Good afternoon!
Old school notes today- using my beautiful fountain pen!

Perceptrons

- what is a decision boundary
- represent a decision boundary
- perceptrons
- perceptron learning rule

If I am hungry enough, then I will go get food.

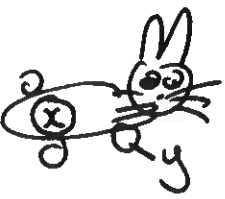
continuous variable — discrete variable

[classification problem: finding a mapping between inputs (continuous or discrete) to a discrete output.

what level of analysis (a la Marr) is this problem stated?

computational problem spec. — algorithmic — implementational

how can the bunny do this? algorithmic

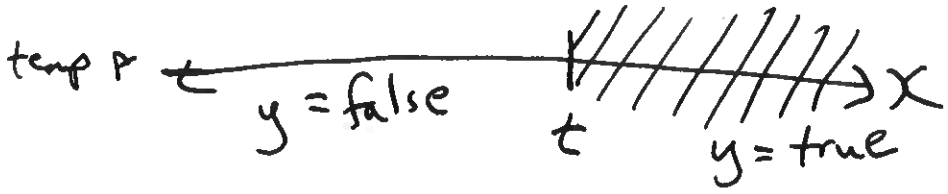


x = hunger sensor $\in \mathbb{R}$

y = decision to leave nest to get food $\in \{0, 1\}$

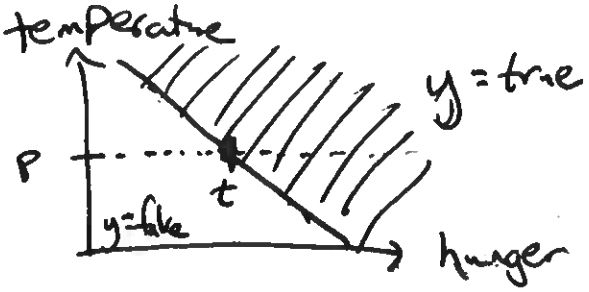
if $x >$ threshold t, then $y = 1$

else $y = 0$



point t is a **decision boundary** in a 1D space

also do this in 2D:

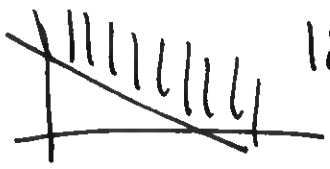


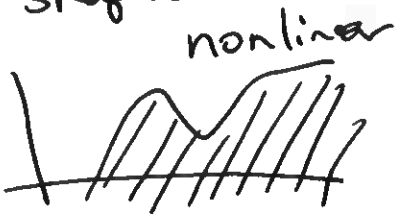
in 2D, decision boundary is a line (not just a point)

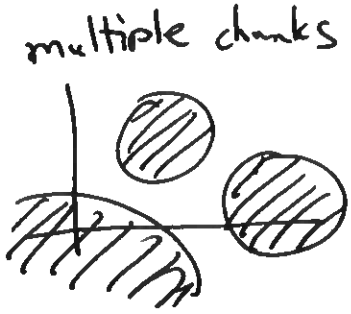
in 3D: d.b. is a plane!

decision boundary is a very powerful concept in AI & cogsci

- can be in 1D, 2D, any # of dimensions
- can have different shapes



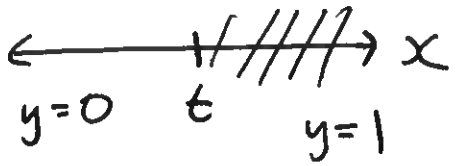




any way to carve up an n-dimensional space of inputs into 2 regions (binary decision) or more regions (k-way decision)

- binary is usually enough

II. how do we represent a decision boundary in a flexible & useful way? maths!



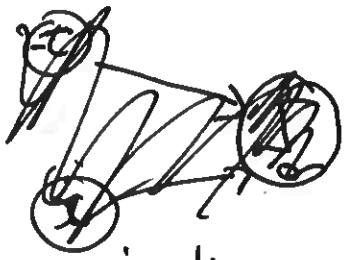
$$y = \begin{cases} 1 & \text{if } x > t \\ 0 & \text{if } x \leq t \end{cases}$$

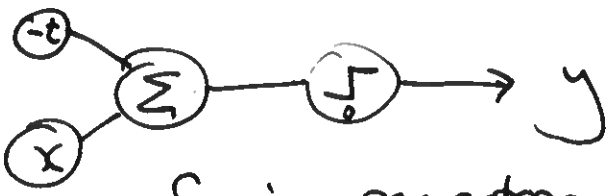
redraw

perceptron

trick: get rid of t inside the step function

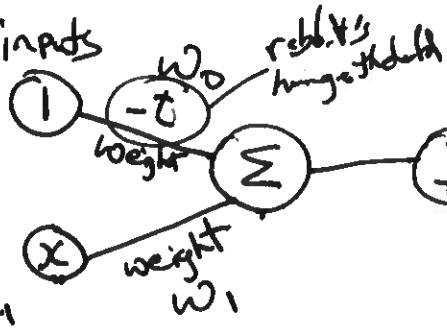
$$y = \begin{cases} 1 & \text{if } x > t \quad x - t > 0 \\ 0 & \text{if } x \leq t \quad x - t \leq 0 \end{cases}$$





fancier perceptron

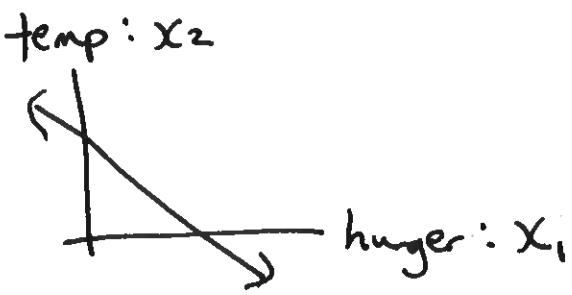
another trick: x_0

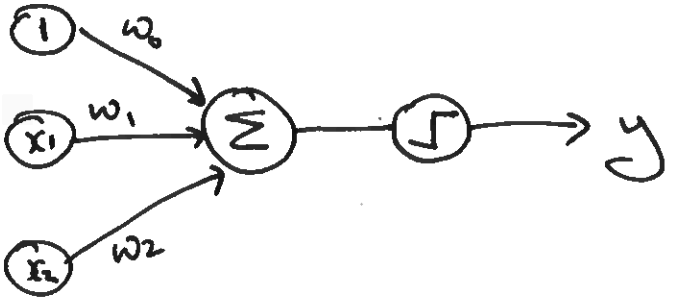


hunger ← x_1

classical definition of perceptron

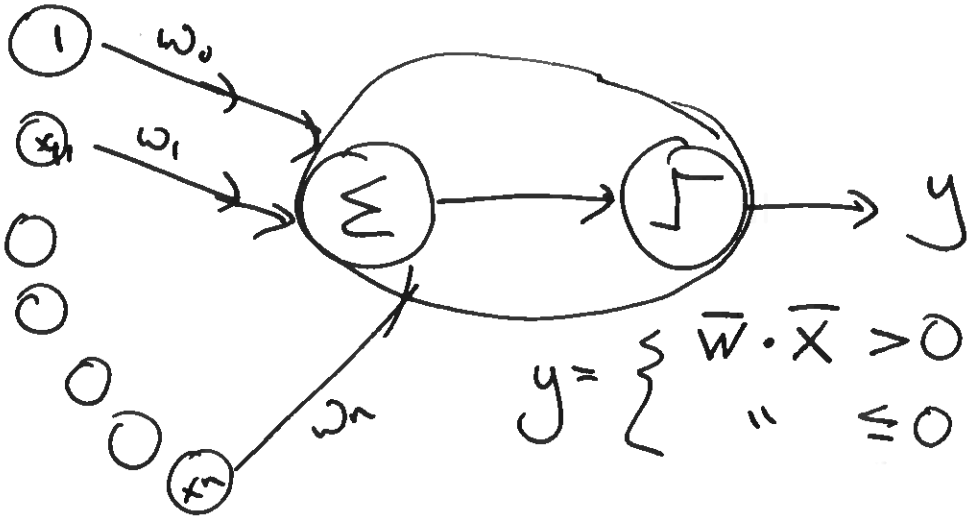
$$y = \begin{cases} 1 & \text{if } w_0 x_0 + w_1 x_1 > 0 \\ 0 & \text{if } w_0 x_0 + w_1 x_1 \leq 0 \end{cases}$$



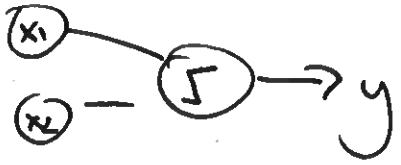


$$y = \begin{cases} 1 & \text{if } w_0 + w_1 x_1 + w_2 x_2 > 0 \\ 0 & \text{"} \quad \leq 0 \end{cases}$$

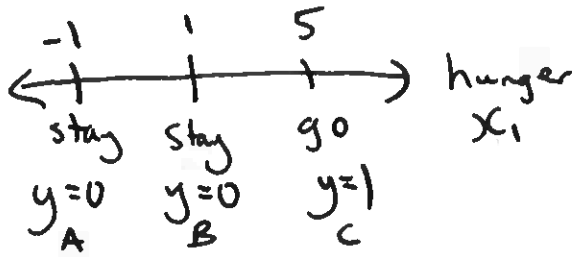
can have perceptron with n dimensions



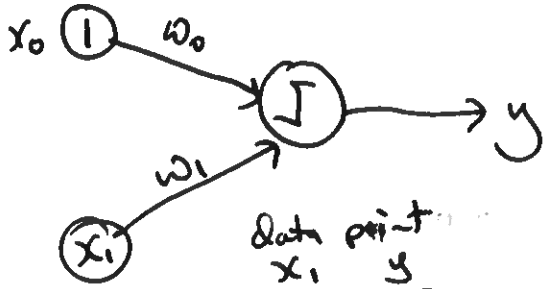
$$y = \begin{cases} \bar{w} \cdot \bar{x} > 0 : y = 1 \\ " \quad \leq 0 : y = 0 \end{cases}$$



suppose want to infer rabbit threshold t (unknown)
but we can observe rabbit's behavior



data points!
observe (x, y) for a few values of x



$w_0 \cdot 1 + w_1 \cdot x_1$

data point x_1	y
B: 1	0

w_0	w_1	$\Sigma w_i x_i$	compare to 0	y
3	2	5	$5 > 0$	1