Informatics 1 – Introduction to Computation

Computation and Logic

Julian Bradfield

based on materials by

Michael P. Fourman

From Aristotle to Venn:

Aristotelian Syllogisms

and

Venn Diagrams
A universe of coloured shapes
Some statements about the universe

Every red triangle is small
Every small triangle is red
Some big triangle is green
Some small disc is red
No red thing is blue
Some statements about the universe

Every red triangle is small
Every small triangle is red
Some big triangle is green
Some small disc is red
No red thing is blue
Some statements about the universe

Every red triangle is small ✓
Every small triangle is red X
Some big triangle is green
Some small disc is red
No red thing is blue
Some statements about the universe

Every red triangle is small ✓
Every small triangle is red ×
Some big triangle is green ？
Some small disc is red ？
No red thing is blue ？

Categorical propositions say: (Every/some/no) A is (not) B.

Aristotle 384–322 B.C.
Some statements about the universe

Every red triangle is small
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Categorical propositions say:
(Every/some/no) A is (not) B.

Aristotle
384–322 B.C.
Checking categorical propositions

Every red triangle is small ✓
Every small triangle is red ✗
Some big triangle is green ?
Some small disc is red ?
No red thing is blue ?
Checking categorical propositions

- Every red triangle is small ✔
- Every small triangle is red ✗
- Some big triangle is green ?
- Some small disc is red ?
- No red thing is blue ?
Checking categorical propositions

- Every red triangle is small ✓
- Every small triangle is red X
- Some big triangle is green ?
- Some small disc is red ?
- No red thing is blue ?
Checking categorical propositions

- Every red triangle is small  
- Every small triangle is red  
- Some big triangle is green  
- Some small disc is red  
- No red thing is blue  

- Every red triangle is small: ✓
- Every small triangle is red: ×
- Some big triangle is green: ?
- Some small disc is red: ?
- No red thing is blue: ?
Checking categorical propositions

Every red triangle is small ✓
Every small triangle is red ✗
Some big triangle is green ?
Some small disc is red ?
No red thing is blue ?

triangle small triangle green disc blue
Checking categorical propositions

- Every red triangle is small: ✅
- Every small triangle is red: ✗
- Some big triangle is green: ✅
- Some small disc is red: ?
- No red thing is blue: ?

Diagram:
- Red triangle
- Small
- Green
- Triangle
- Small
- Green
- Disc
- Blue
Every red triangle is small
Every small triangle is red
Some big triangle is green
Some small disc is red
No red thing is blue
Checking categorical propositions

Every red triangle is small ✓
Every small triangle is red ✗
Some big triangle is green ✓
Some small disc is red ✗
No red thing is blue ?
Checking categorical propositions

Every red triangle is small ✔
Every small triangle is red ❌
Some big triangle is green ✔
Some small disc is red ❌
No red thing is blue ❓
Checking categorical propositions

Every red triangle is small ✓
Every small triangle is red ✗
Some big triangle is green ✓
Some small disc is red ✗
No red thing is blue ✓
Categorical propositions as predicate logic

Categorical propositions are a very restricted form of predicate logic:

- Every red thing is small
  \( \forall x. \text{isRed}(x) \rightarrow \text{isSmall}(x) \)

- Every small triangle is red
  \( \forall x. (\text{isSmall}(x) \land \text{isTriangle}(x)) \rightarrow \text{isRed}(x) \)

- Some small disc is red
  \( \exists x. (\text{isSmall}(x) \land \text{isDisc}(x)) \land \text{isRed}(x) \)
Categorical propositions as predicate logic

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  \( \exists x. (\text{isSmall}(x) \land \text{isDisc}(x)) \land \text{isRed}(x) \)

Can you write the general form of a categorical proposition?
We need names for the things in the universe:

```haskell
data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq,Show)
things = [ R, S, T, U, V, W, X, Y, Z ]
```

It's tempting to define types for the features that things have:

```haskell
data Colour = Red | Blue | Green
data Shape = Disc | Triangle
data Size = Big | Small
```

and then define functions for the features:

```haskell
colour :: Thing -> Colour
shape :: Thing -> Shape
size :: Thing -> Size

colour R = Green
```

etc. etc.

However, because of all the types, this ends up being hard to work with.
A universe in Haskell (1)

We need names for the *things* in the universe:

```haskell
data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq,Show)

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data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq, Show)

things = [ R, S, T, U, V, W, X, Y, Z ]

Instead of features, we define predicates, the basic propositions of logic. Every feature has a predicate, e.g. isGreen.
A universe in Haskell (2)

data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq,Show)
things = [ R, S, T, U, V, W, X, Y, Z ]

Instead of features, we define predicates, the basic propositions of logic. Every feature has a predicate, e.g. `isGreen`.
We could define the type of predicates on things:

```
type ThingPredicate = Thing -> Bool
isGreen :: ThingPredicate
```
data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq, Show)

things = [ R, S, T, U, V, W, X, Y, Z ]

Instead of features, we define predicates, the basic propositions of logic. Every feature has a predicate, e.g. `isGreen`.

We could define the type of predicates on things:

```haskell
type ThingPredicate = Thing -> Bool
isGreen :: ThingPredicate
```

but it’s more general and convenient to do:

```haskell
type Predicate u = u -> Bool
isGreen :: Predicate Thing
```
Defining the predicates

This is the simplest way to establish the predicates:

- `isGreen R = True`
- `isGreen S = True`
- `isGreen T = False`

A lazier way is:

```hs
isGreen x = x `elem` [R, S, W, Y]
isRed x = x `elem` [U, V]
```

Is this too lazy? (What happens when we extend the universe?)

- `isBlue x = not (isGreen x || isRed x)`

The three chief virtues of a programmer are laziness, impatience, and hubris – Larry Wall
Defining the predicates

This is the simplest way to establish the predicates:

\[
\begin{align*}
\text{isGreen } R &= \text{ True} \\
\text{isGreen } S &= \text{ True} \\
\text{isGreen } T &= \text{ False}
\end{align*}
\]

A lazier\(^1\) way is:

\[
\begin{align*}
\text{isGreen } x &= x \ `\text{elem}` \ [ R, S, W, Y ] \\
\text{isRed } x &= x \ `\text{elem}` \ [ U, V ]
\end{align*}
\]

\(^1\) The three chief virtues of a programmer are laziness, impatience, and hubris
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A lazier\(^1\) way is:

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\begin{align*}
\text{isGreen } x &= x \ `\text{elem}` [ R, S, W, Y ] \\
\text{isRed } x &= x \ `\text{elem}` [ U, V ]
\end{align*}
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Is this too lazy? (What happens when we extend the universe?)

\[
\text{isBlue } x = \text{not} (\text{isGreen } x \text{ || isRed } x)
\]

\(^1\) *The three chief virtues of a programmer are laziness, impatience, and hubris*  
– Larry Wall
Representing statements with list comprehension

Haskell’s *list comprehension* gives a powerful way of representing statements:

\[
[ x \mid x \leftarrow \text{things}, \text{isBlue} \ x \ || \ (\text{isBig} \ x \ \&\& \ \text{isDisc} \ x) ]
\]

‘the set (list) of things that are either blue or are big discs’
Haskell’s *list comprehension* gives a powerful way of representing statements:

```plaintext
[ x | x <- things, isBlue x || (isBig x && isDisc x) ]
```

‘the set (list) of things that are either blue or are big discs’
Combining list comprehension with boolean operators on lists lets us express categorical statements.

Every small triangle is red. \( \times \)
Combining list comprehension with boolean operators on lists lets us express categorical statements.

Every small triangle is red. \( \times \)

\[ [ x \mid x \gets \text{things}, \text{isTriangle}(x) \land \text{isSmall}(x) ] \]
\[ [S,V,X] \]

‘The set of things that are small triangles.’
Combining list comprehension with boolean operators on lists lets us express categorical statements.

Every small triangle is red.  \( \times \)

\[
[ x \mid x \leftarrow \text{things}, \text{isTriangle}(x) \land \text{isSmall}(x) ]
\]
\[S,V,X] \]

'The set of things that are small triangles.'

\[
[ \text{isRed}(x) \mid x \leftarrow \text{things}, \text{isTriangle}(x) \land \text{isSmall}(x) ]
\]
\[\text{False, True, False}\]

'Whether each small triangle is red.'
Combining list comprehension with boolean operators on lists lets us express categorical statements.

Every small triangle is red. \( \times \)

\[
[ x | x \leftarrow \text{things}, \text{isTriangle}(x) \&\& \text{isSmall}(x) ]
\]
\([S,V,X]\)

‘The set of things that are small triangles.’

\[
[ \text{isRed}(x) | x \leftarrow \text{things}, \text{isTriangle}(x) \&\& \text{isSmall}(x) ]
\]
\([\text{False, True, False}]\)

‘Whether each small triangle is red.’

\text{and} \ [ \text{isRed}(x) | x \leftarrow \text{things}, \text{isTriangle}(x) \&\& \text{isSmall}(x) ]
\text{False}

‘Every small triangle is red.’
A syllogism is discourse (logos) in which, certain things being stated, something other than what is stated follows of necessity from those things.

- All Greeks are human
- All humans are mortal
- ∴ All Greeks are mortal
Syllogisms

Can be expressed in many forms:

- \( \{ x \mid \text{isGreek}(x) \} \subseteq \{ x \mid \text{isHuman}(x) \} \)
- \( \{ x \mid \text{isHuman}(x) \} \subseteq \{ x \mid \text{isMortal}(x) \} \)
- \( \therefore \{ x \mid \text{isGreek}(x) \} \subseteq \{ x \mid \text{isMortal}(x) \} \)

All Greeks are human
All humans are mortal
\( \therefore \) all humans are mortal
Syllogisms

Can be expressed in many forms:

1. \{ x \mid \text{isGreek}(x) \} \subseteq \{ x \mid \text{isHuman}(x) \}
2. \{ x \mid \text{isHuman}(x) \} \subseteq \{ x \mid \text{isMortal}(x) \}
3. \therefore \{ x \mid \text{isGreek}(x) \} \subseteq \{ x \mid \text{isMortal}(x) \}

In modern logic, we write it as:

\text{isGreek} \vdash \text{isHuman} \quad \text{isHuman} \vdash \text{isMortal}

\text{isGreek} \vdash \text{isMortal}

All Greeks are human
All humans are mortal
\therefore all humans are mortal
Syllogisms

Can be expressed in many forms:

- $\{ x \mid \text{isGreek}(x) \} \subseteq \{ x \mid \text{isHuman}(x) \}$
- $\{ x \mid \text{isHuman}(x) \} \subseteq \{ x \mid \text{isMortal}(x) \}$

$\therefore \{ x \mid \text{isGreek}(x) \} \subseteq \{ x \mid \text{isMortal}(x) \}$

In modern logic, we write it as:

\[
\begin{align*}
\text{isGreek} \models \text{isHuman} & \quad \text{isHuman} \models \text{isMortal} \\
\text{isGreek} \models \text{isMortal}
\end{align*}
\]

All Greeks are human
All humans are mortal
$\therefore$ all humans are mortal

The general form of this syllogism is

\[
\begin{align*}
a \models b & \quad b \models c \\
\therefore a \models c
\end{align*}
\]
Syllogisms

Can be expressed in many forms:

- \( \{ x \mid \text{isGreek}(x) \}\subseteq \{ x \mid \text{isHuman}(x) \}\)
- \( \{ x \mid \text{isHuman}(x) \}\subseteq \{ x \mid \text{isMortal}(x) \}\)
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In modern logic, we write it as:

\[
\text{isGreek} \models \text{isHuman} \quad \text{isHuman} \models \text{isMortal} \quad \therefore \text{isGreek} \models \text{isMortal}
\]

The general form of this syllogism is

\[
\frac{a \models b \quad b \models c}{a \models c}
\]

Is this syllogism sound? I.e. valid in every universe?

All Greeks are human
All humans are mortal
\( \therefore \) all humans are mortal
Every red triangle is small  
Every small triangle is red  
Some big triangle is green  
Some small disc is red  
No red thing is blue
Venn diagrams

Venn diagrams show every possible combination

A rotationally symmetric Venn diagram for $n > 1$ sets exists iff $n$ is prime
Venn diagrams

We use light shading to show emptiness of a region

\[ \overline{A} \cap B \cap C = \emptyset \]

A rotationally symmetric Venn diagram for \( n > 1 \) sets exists iff \( n \) is prime
A rotationally symmetric Venn diagram for \( n > 1 \) sets exists iff \( n \) is prime.
Venn interpretation of \( a \models b \)

\[
\begin{align*}
\neg a \land \neg b & \quad \text{\( a \models b \) \ 'every a is b' } \\
\neg a \land b & \quad \text{\( a \models b \) \ 'no a is not b' } \\
a \land \neg b & \quad \text{\( a \models b \) \ 'nothing is a and not b' } \\
a \land b & \quad \text{\( a \models b \) \ \( a \cap \overline{b} = \emptyset \) } \\
\neg (a \land \neg b) & \quad \text{\( a \models b \) \ \( \neg(a \land \neg b) \) } \\
b \lor \neg a & \quad \text{\( a \models b \) \ \( b \lor \neg a \) }
\end{align*}
\]
Venn interpretation of $a \models b$

- $a \models b$ ‘every $a$ is $b$’
- $a \models b$ ‘no $a$ is not $b$’
- $a \models b$ ‘nothing is $a$ and not $b$’
- $a \models b$ $a \cap \bar{b} = \emptyset$
- $a \models b$ $\neg(a \wedge \neg b)$
- $a \models b$ $b \vee \neg a$

$\neg a \wedge \neg b$

$a \wedge \neg b$

$a \wedge b$

$\neg a \wedge b$

a

b
Venn syllogism

every $a$ is $b$    every $b$ is $c$

every $a$ is $c$
Combining diagrams

\[ a \models b \quad b \models c \quad \frac{}{a \models c} \]
Combining diagrams

\[
\frac{a \models b \quad b \models c}{a \models c}
\]
Combining diagrams

\[ a \models b \quad b \models c \]

\[ \therefore a \models c \]
Combining diagrams

\[ a \models b \quad b \models c \quad a \models c \]
Combining diagrams

\[
\begin{align*}
& a \models b \\
& b \models c \\
& \therefore a \models c
\end{align*}
\]
Barbara is sound!

\[
\text{barbara} \quad \frac{a \models b \quad b \models c}{a \models c}
\]

This rule, as we’ve seen, is sound:
for any predicates \( a, b, c \) in any universe, we have:
if the premises (above the line) are valid
then the conclusion (below the line) is valid.

Mediaeval logicians gave mnemonic names to syllogisms. This one is \textit{barbara}. Consult Wikipedia to find out what that means – but only if you don’t value your sanity!
Aristotle’s *universal* propositions

make statements about *all* of something: ‘all $a$ are $b$’. We can make universal *negative* statements: ‘no $a$ is $b$’. 

‘no $a$ is $b$’ iff ‘every $a$ is $\neg b$’ iff $a \models \neg b$
Aristotle’s *universal* propositions

make statements about *all* of something: ‘all $a$ are $b$’. We can make universal *negative* statements: ‘no $a$ is $b$’.

‘no $a$ is $b$’ iff ‘every $a$ is $\neg b$’ iff $a \models \neg b$

Here is a syllogism involving universal negatives:

$\frac{s \models r \quad r \models \neg f}{s \models \neg f}$

*All snakes are reptiles*

*No reptile has fur*

$\therefore$ *No snake has fur*

Is this an instance of *barbara* (and so valid)?
Aristotle’s *universal* propositions

make statements about *all* of something: ‘all *a* are *b*’. We can make universal *negative* statements: ‘no *a* is *b*’.

‘no *a* is *b*’ iff ‘every *a* is ¬*b*’ iff \( a \models \neg b \)

Here is a syllogism involving universal negatives:

\[
\begin{align*}
  s & \models r & r & \models \neg f & \quad & \text{All snakes are reptiles} \\
  s & \models \neg f & & & \quad & \text{No reptile has fur} \\
  \therefore & & & & \quad & \text{No snake has fur}
\end{align*}
\]

Is this an instance of *barbara* (and so valid)?

For us modern logicians, it is: \( a \equiv s, b \equiv r, c \equiv \neg f \).

A negated predicate is also a predicate.

Aristotle differed from us moderns on the relation between ‘all’ and ‘no’. For him, this syllogism contained a universal affirmative and two universal negatives. The mediaeval logicians called it *celarent*.

The key difference was the ‘existential assumption’ – see later.
Universal statements

whether affirmative or negative, say that some region is *empty*:

- all *a* are *b*  
  \( a \models b \)

- no *a* is *b*  
  \( a \models \neg b \)

We can observe:

- \( a \models b \) and \( \neg a \models \neg b \) are reflections of each other: so \( \neg a \models \neg b \) is the same as \( b \models a \).

- \( \neg a \models \neg b \) is the contrapositive of \( b \models a \).

- \( a \models \neg b \) is symmetrical, so is the same as \( b \models \neg a \) – they are contrapositives. Likewise \( \neg a \models b \) and \( \neg b \models a \).
Universal statements

whether affirmative or negative, say that some region is *empty*:
- all $a$ are $b$
  - $a \models b$
- no $a$ is $b$
  - $a \models \neg b$

What about $\neg a \models b$ and $\neg a \models \neg b$?
Universal statements

whether affirmative or negative, say that some region is *empty*:

all $a$ are $b$

$\forall a \, a \models b$  

no $a$ is $b$

$\forall a \, a \not\models b$

We can observe:

$\neg a \models \neg b$ is the contrapositive of $b \models a$.

$\neg a \models \neg b$ is symmetrical, so is the same as $b \models \neg a$ – they are contrapositives. Likewise $\neg a \models b$ and $\neg b \models a$.
Universal statements

whether affirmative or negative, say that some region is *empty*:

- all $a$ are $b$
  \[ a \models b \]
- no $a$ is $b$
  \[ a \models \neg b \]

We can observe:

- $a \models b$ and $\neg a \models \neg b$ are reflections of each other: so $\neg a \models \neg b$ is the same as $b \models a$.
- $\neg a \models \neg b$ is the contrapositive of $b \models a$. 

\[ \neg a \models b \]
\[ \neg a \models \neg b \]
Universal statements

whether affirmative or negative, say that some region is *empty*:

```
all a are b
  a ⊨ b

no a is b
  a ⊨ ¬b
```

We can observe:

- **a ⊨ b** and **¬a ⊨ ¬b** are reflections of each other: so
  - ¬a ⊨ ¬b is the same as **b ⊨ a**.
  - ¬a ⊨ ¬b is the **contrapositive** of **b ⊨ a**.

- **a ⊨ ¬b** is symmetrical, so is the same as **b ⊨ ¬a** – they are **contrapositives**. Likewise ¬a ⊨ b and ¬b ⊨ a.
Negation and contraposition

Negation can be tricky – modern classical logic makes it simple.

Natural languages differ, within and between themselves, on how they treat multiple negatives: ‘I didn’t never do nothing to nobody!’.

How does your native language/dialect treat multiple negatives?
Negation and contraposition

Negation can be tricky – modern classical logic makes it simple.

The law of double negation: \( \neg \neg a = a \) (two negatives make a positive).

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Negation and contraposition

Negation can be tricky – modern classical logic makes it simple.

The law of double negation: $\neg\neg a = a$ (two negatives make a positive).

The law of contraposition: $a \vdash b$ iff $\neg b \vdash \neg a$.

Natural languages differ, within and between themselves, on how they treat multiple negatives: ‘I didn’t never do nothing to nobody!’.
How does your native language/ dialect treat multiple negatives?
Negation and contraposition

Negation can be tricky – modern classical logic makes it simple.

The law of double negation: \( \neg \neg a = a \) (two negatives make a positive).

The law of contraposition: \( a \models b \iff \neg b \models \neg a \).

Thus we get \( a \models b \iff \neg b \models \neg a \iff \neg \neg a \models \neg \neg b \iff a \models b \).

The double line means the rule works both ways.

Natural languages differ, within and between themselves, on how they treat multiple negatives: ‘I didn’t never do nothing to nobody!’.

How does your native language/dialect treat multiple negatives?
So far, we have seen (and hopefully agreed on) these sound rules about predicates and $\models$:

1. $\neg
\neg a = a \text{ or } \frac{a}{\neg \neg a}$ (double negation)

2. $a \models b \quad b \models c \quad \frac{a \models c}{\text{(barbara)}}$

3. $\frac{a \models b}{\neg b \models \neg a}$ (contraposition)
So far, we have seen (and hopefully agreed on) these sound rules about predicates and $\models$:

- $\neg \neg a = a$ or $\frac{a}{\neg \neg a}$ (double negation)
- $a \models b \quad b \models c \quad (barbara)$
- $a \models b \quad \frac{\neg b}{\neg a}$ (contraposition)

We also saw a ‘different’ (for Aristotle) syllogism with negatives got from $barbara$ by putting $\neg c$ for $c$:

\[
\frac{a \models b \quad b \models \neg c}{a \models \neg c} \quad All\ snakes\ are\ reptiles
\]
\[
\frac{No\ reptile\ has\ fur}{\therefore\ No\ snake\ has\ fur}
\]
More syllogisms

By using (un)negated predicates in *barbara*, we get 8 syllogisms:

\[
\begin{align*}
\frac{a \vdash b \quad b \vdash c}{a \vdash c} & \quad \frac{\neg a \vdash b \quad b \vdash c}{\neg a \vdash c} \\
\frac{a \vdash b \quad b \vdash \neg c}{a \vdash \neg c} & \quad \frac{\neg a \vdash b \quad b \vdash \neg c}{\neg a \vdash \neg c} \\
\frac{a \vdash \neg b \quad \neg b \vdash c}{a \vdash c} & \quad \frac{\neg a \vdash \neg b \quad \neg b \vdash c}{\neg a \vdash c} \\
\frac{a \vdash \neg b \quad \neg b \vdash \neg c}{a \vdash \neg c} & \quad \frac{\neg a \vdash \neg b \quad \neg b \vdash \neg c}{\neg a \vdash \neg c}
\end{align*}
\]

Aristotle only considered negative predicates on the right of $\vdash$ (negating a means 'no a is b', so he viewed it as a negative statement about positive predicates). This leaves ...
By using (un)negated predicates in *barbara*, we get 8 syllogisms:

\[
\begin{align*}
&\text{If } a \vdash b, \quad b \vdash c, \quad \text{then } a \vdash c \\
&\text{If } \neg a \vdash b, \quad b \vdash c, \quad \text{then } \neg a \vdash c \\
&\text{If } a \vdash b, \quad b \vdash \neg c, \quad \text{then } a \vdash \neg c \\
&\text{If } \neg a \vdash b, \quad b \vdash \neg c, \quad \text{then } \neg a \vdash \neg c \\
&\text{If } a \vdash \neg b, \quad \neg b \vdash c, \quad \text{then } a \vdash c \\
&\text{If } \neg a \vdash \neg b, \quad \neg b \vdash c, \quad \text{then } \neg a \vdash c \\
&\text{If } a \vdash \neg b, \quad \neg b \vdash \neg c, \quad \text{then } a \vdash \neg c \\
&\text{If } \neg a \vdash \neg b, \quad \neg b \vdash \neg c, \quad \text{then } \neg a \vdash \neg c
\end{align*}
\]

Aristotle only considered negative predicates on the right of \( \vdash \) (\( a \vdash \neg b \) means ‘no \( a \) is \( b \)’, so he viewed it as a negative statement about positive predicates). This leaves . . .
More syllogisms

By using (un)negated predicates in *barbara*, we get 8 syllogisms:

\[
\begin{array}{c}
  a \vdash b \\
  b \vdash c \\
  \hline
  a \vdash c
\end{array}
\]

\[
\begin{array}{c}
  a \vdash b \\
  b \vdash \neg c \\
  \hline
  a \vdash \neg c
\end{array}
\]

*barbara* and *celarent*

Aristotle only considered negative predicates on the right of $\vdash$
($a \vdash \neg b$ means ‘no $a$ is $b$’, so he viewed it as a negative statement about positive predicates). This leaves . . .
Even more syllogisms

*Contraposition* lets us generate three more (Aristotelian) syllogisms from *celarent*:

\[
\begin{align*}
\frac{a \models b \quad c \models \neg b}{a \models \neg c} & \quad \frac{a \models b \quad b \models \neg c}{c \models \neg a} & \quad \frac{a \models b \quad c \models \neg b}{c \models \neg a}
\end{align*}
\]

That brings us to 5 sound universal syllogisms. That’s all!
Unsound syllogisms

\[ a \vdash b \quad b \vdash \neg c \quad a \vdash c \]

*All snakes are reptiles*

*No reptile has fur*

∴ *All snakes have fur*

To disprove a syllogism, we need just one universe where it's invalid (e.g. Edinburgh zoo). Is there a universe where this syllogism is valid? (Aristotle said 'no'; we moderns differ. Hint: St Patrick.)
Unsound syllogisms

All snakes are reptiles
No reptile has fur
∴ All snakes have fur

To disprove a syllogism, we need just one universe where it's invalid (e.g. Edinburgh zoo). Is there a universe where this syllogism is valid?

(Aristotle said ‘no’; we moderns differ. Hint: St Patrick.)
Unsound syllogisms

\[
\begin{align*}
  a &\equiv b \\
  b &\equiv \neg c \\
  a &\equiv c
\end{align*}
\]

All snakes are reptiles
No reptile has fur
∴ All snakes have fur

To disprove a syllogism, we need just one universe where it's invalid (e.g. Edinburgh zoo).
Is there a universe where this syllogism is valid?
(Aristotle said 'no'; we moderns differ. Hint: St Patrick.)
Unsound syllogisms

All snakes are reptiles
No reptile has fur
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To disprove a syllogism, we need just one universe where it's invalid (e.g. Edinburgh zoo).

Is there a universe where this syllogism is valid?
(Aristotle said 'no'; we moderns differ. Hint: St Patrick.)
Unsound syllogisms

\[
\begin{align*}
a \models b & \quad b \models \neg c \\
a \models c &
\end{align*}
\]

All snakes are reptiles
No reptile has fur
∴ All snakes have fur

To disprove a syllogism, we need just one universe where it’s invalid (e.g. Edinburgh zoo).
Unsound syllogisms

\[ a \models b \quad b \models \neg c \quad a \models c \]

All snakes are reptiles
No reptile has fur
∴ All snakes have fur
To disprove a syllogism, we need just one universe where it’s invalid (e.g. Edinburgh zoo).
Is there a universe where this syllogism is valid?
(Aristotle said ‘no’; we moderns differ. Hint: St Patrick.)
Reprise: Sound universal syllogisms

From *barbara*, contraposition, and double negation, we have five sound syllogisms about universal statements:

\[
\begin{align*}
  a &\models b & b &\models c \\
  &\models c \\
  a &\models b & b &\models \neg c \\
  &\models \neg c \\
  a &\models b & c &\models \neg b \\
  &\models \neg c \\
  a &\models b & b &\models \neg c \quad \text{equivalently} & c &\models b & b &\models \neg a \\
  &\models \neg a \\
  a &\models b & c &\models \neg b \quad \text{equivalently} & c &\models b & a &\models \neg b \\
  &\models \neg a &\models \neg c
\end{align*}
\]

Note that the conclusion is negative iff exactly one of the premises is negative – compare the unsound syllogism on the previous slide.
From *barbara*, contraposition, and double negation, we have five sound syllogisms about universal statements:

- $a \models b \quad b \models c \quad \Rightarrow \quad a \models c$
- $a \models b \quad b \models \neg c \quad \Rightarrow \quad a \models \neg c$
- $a \models b \quad c \models \neg b \quad \Rightarrow \quad a \models \neg c$
- $a \models b \quad b \models \neg c \quad \Rightarrow \quad c \models \neg a$
- $a \models b \quad c \models \neg b \quad \Rightarrow \quad c \models \neg a$

Note that the conclusion is negative iff exactly one of the premises is negative – compare the unsound syllogism on the previous slide.