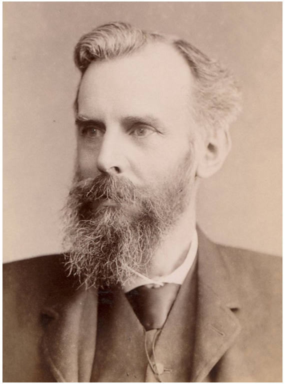
Informatics 1 – Introduction to Computation
Computation and Logic

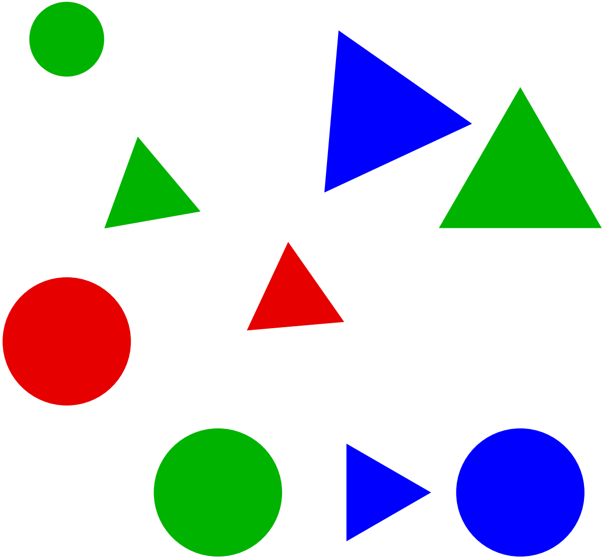
Julian Bradfield
based on materials by
Michael P. Fourman

From Aristotle to Venn: Aristotelian Syllogisms and Venn Diagrams

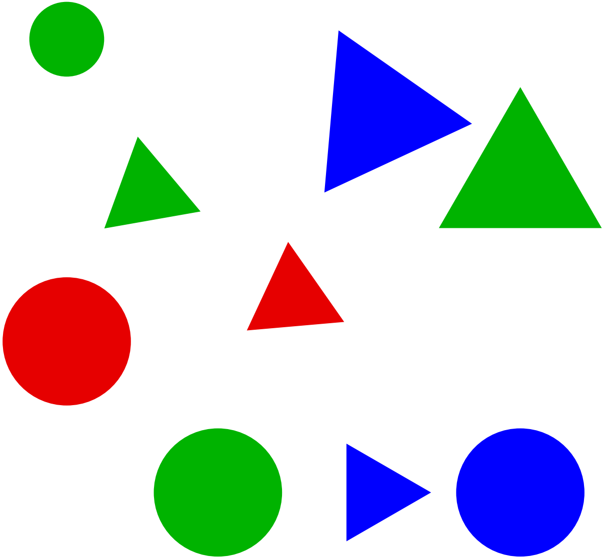


John Venn
1834–1923

A universe of coloured shapes



Some statements about the universe



Every red triangle is small

Every small triangle is red

Some big triangle is green

Some small disc is red

No red thing is blue

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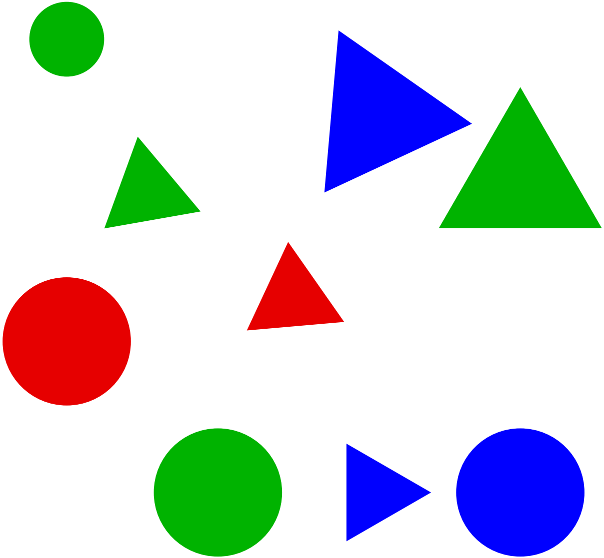

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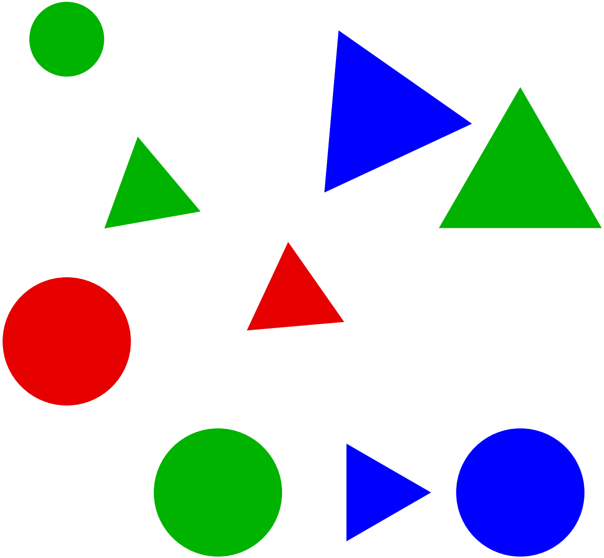

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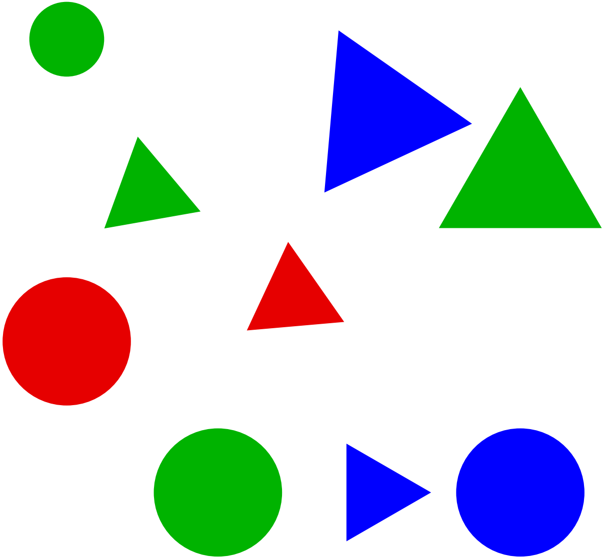

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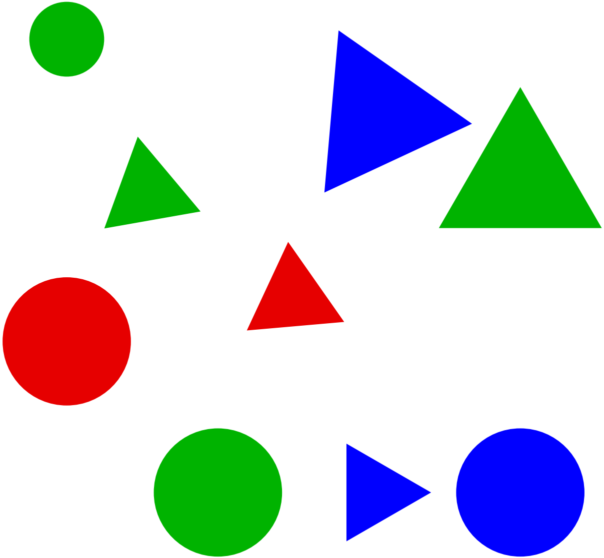

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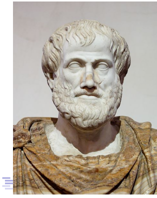
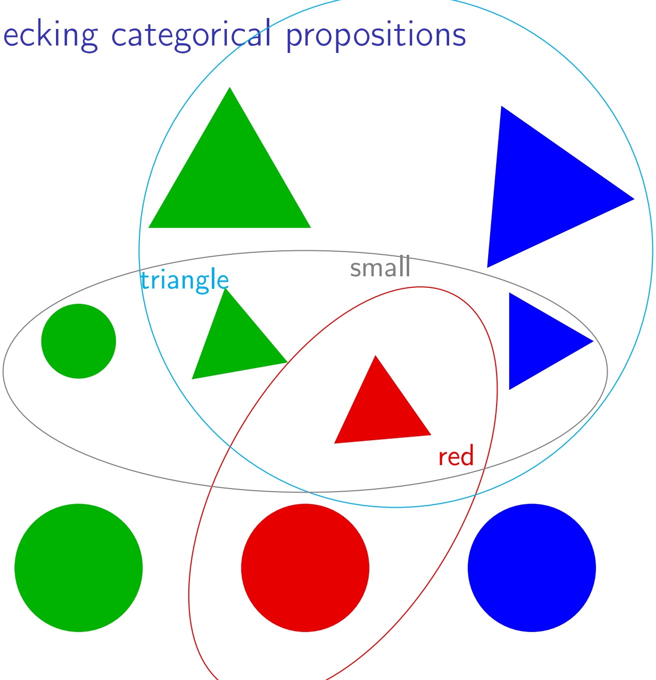
Categorical propositions say:
(Every/some/no) A is (not) B.

Aristotle
384–322 B.C.



Checking categorical propositions



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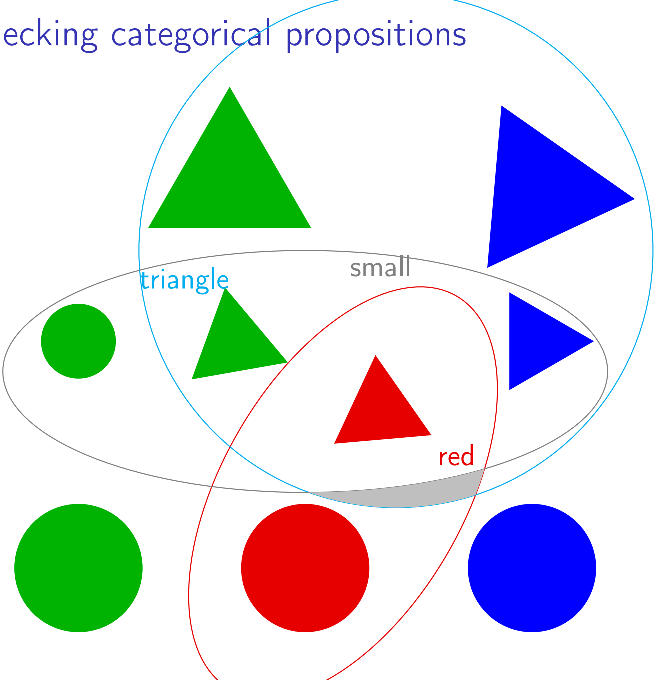

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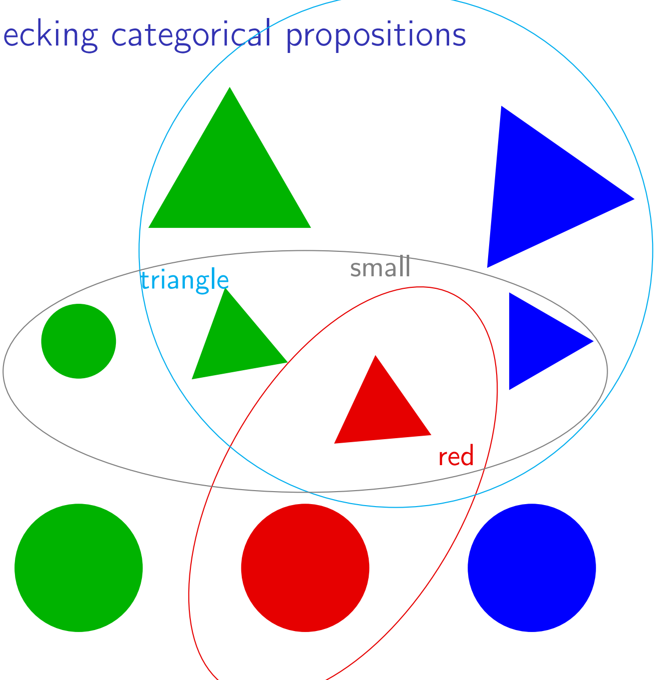

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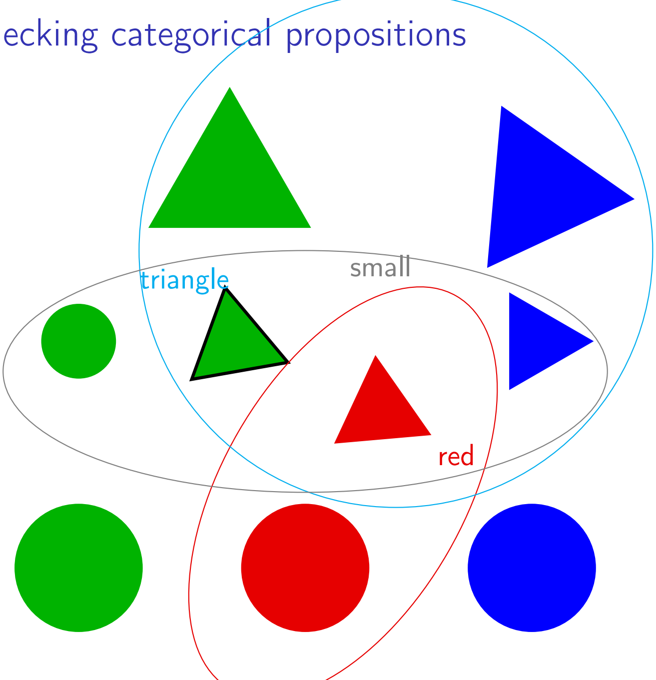

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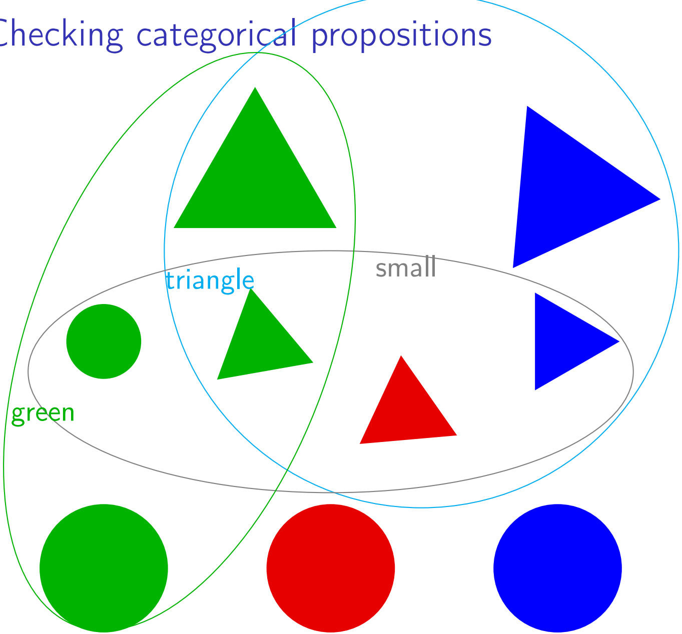

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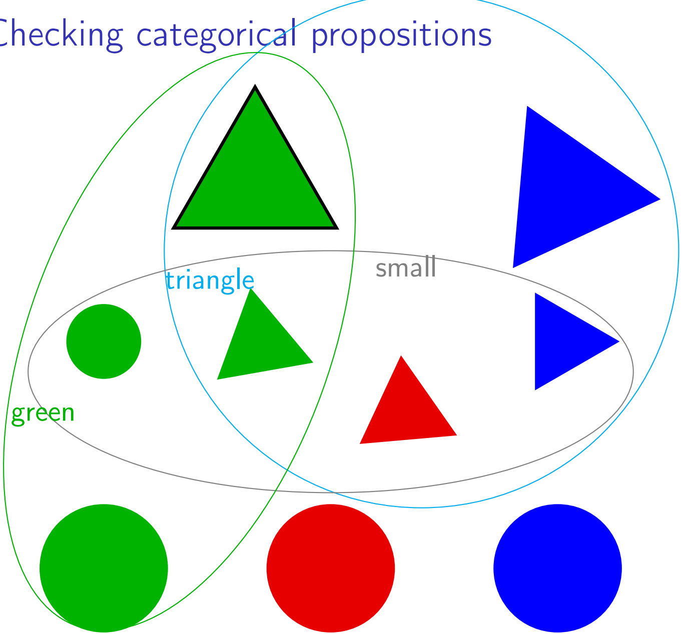

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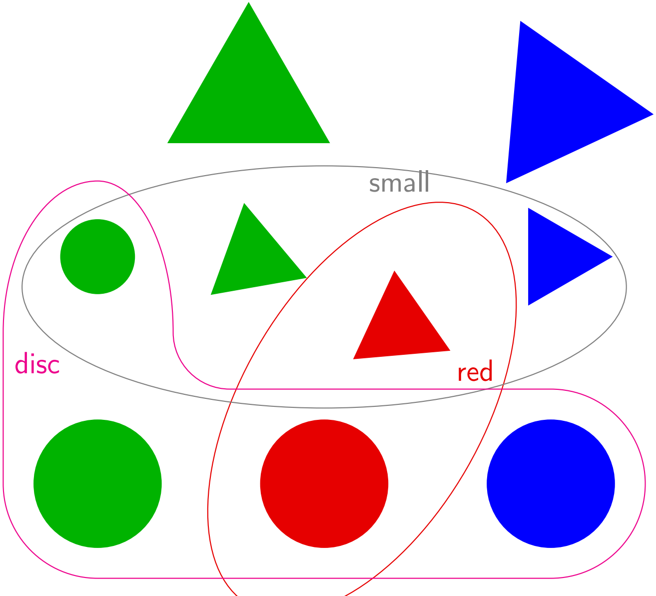

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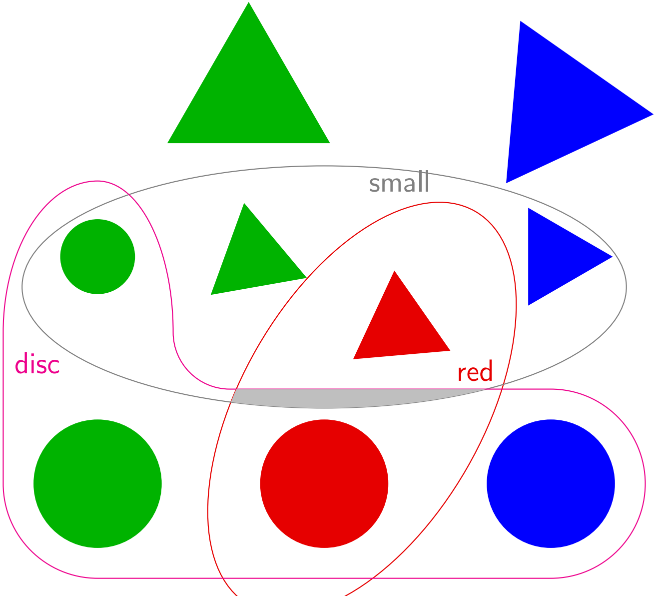

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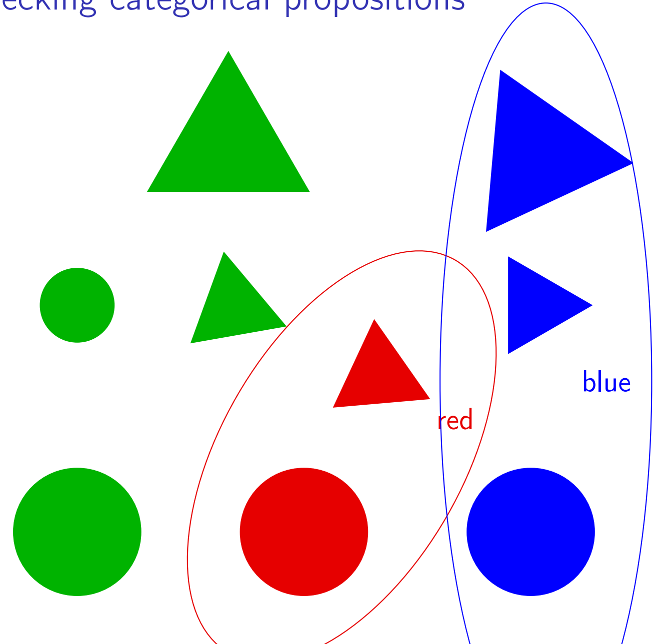

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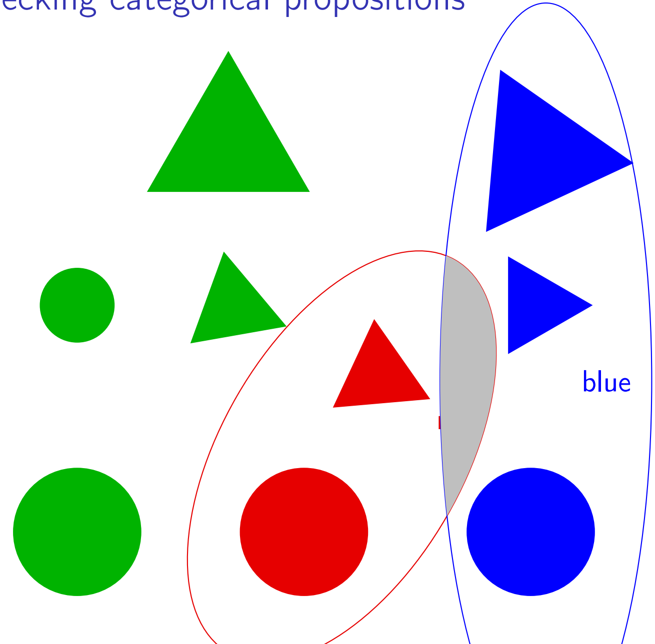

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Categorical propositions as predicate logic

Categorical propositions are a very restricted form of predicate logic:

- Every red thing is small
  $\forall x. \mathit{isRed}(x) \rightarrow \mathit{isSmall}(x)$
- Every small triangle is red
  $\forall x. (\mathit{isSmall}(x) \wedge \mathit{isTriangle}(x)) \rightarrow \mathit{isRed}(x)$
- Some small disc is red
  $\exists x. (\mathit{isSmall}(x) \wedge \mathit{isDisc}(x)) \wedge \mathit{isRed}(x)$

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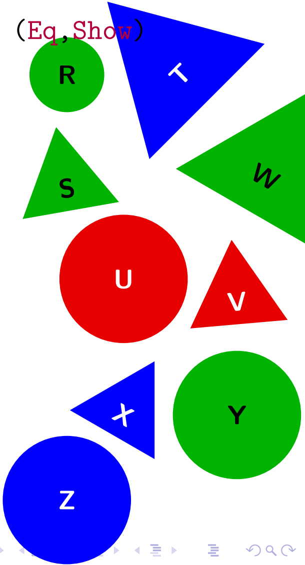
Can you write the general form of a categorical proposition?

A universe in Haskell (1)

We need names for the *things* in the universe:

```
data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq,Show)
things = [ R, S, T, U, V, W, X, Y, Z ]
```



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```

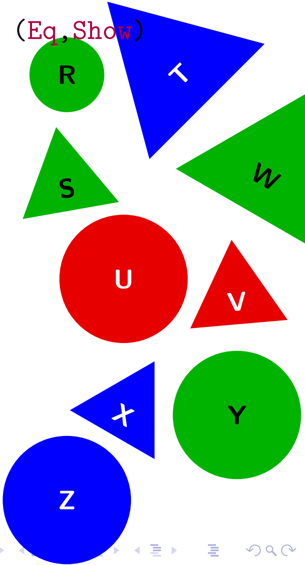
It's tempting to define types for the *features* that things have:

```
data Colour = Red | Blue | Green
data Shape = Disc | Triangle
data Size = Big | Small
```

and then define functions for the features:

```
colour :: Thing -> Colour
shape :: Thing -> Shape
size :: Thing -> Size
colour R = Green
```

etc. etc.



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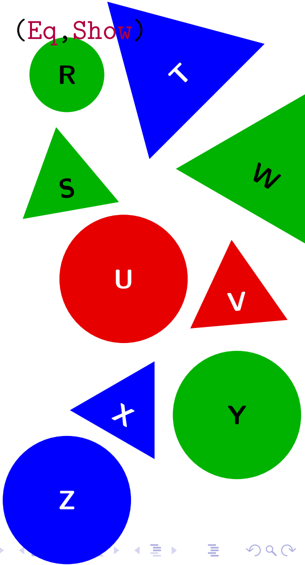
etc. etc.

However, because of all the types, this ends up being hard to work with.



A universe in Haskell (2)


```
data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq,Show)
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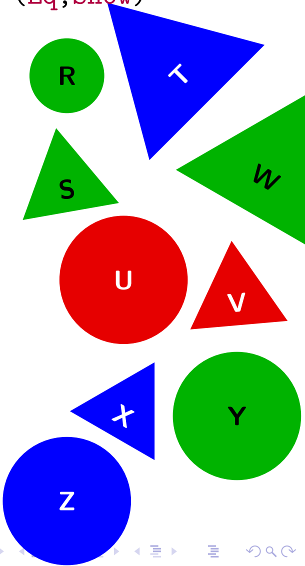
```

Instead of features, we define predicates, the basic propositions of logic. Every feature has a predicate, e.g. `isGreen`.



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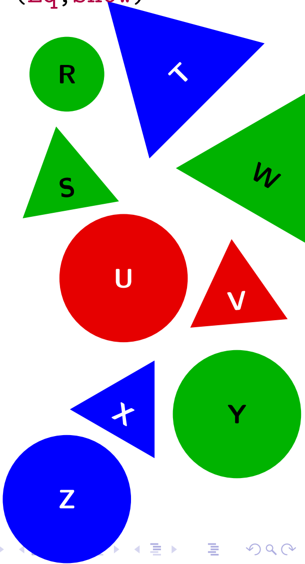
We could define the type of predicates on things:

```
type ThingPredicate = Thing -> Bool
isGreen :: ThingPredicate
```



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We could define the type of predicates on things:

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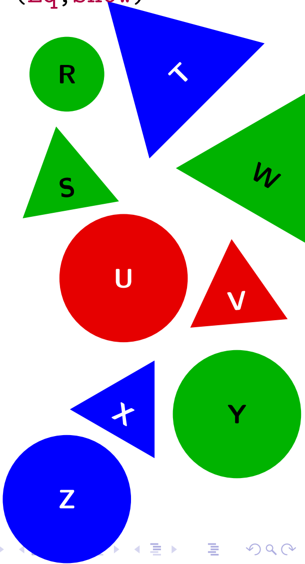
type ThingPredicate = Thing -> Bool
isGreen :: ThingPredicate
```

but it's more general and convenient to do:

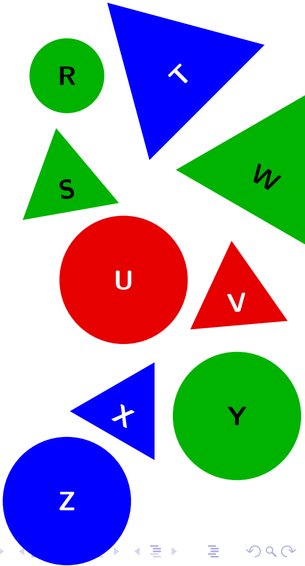
```
type Predicate u = u -> Bool
isGreen :: Predicate Thing
```



Defining the predicates

This is the simplest way to establish the predicates:

```
isGreen R = True
isGreen S = True
isGreen T = False
```



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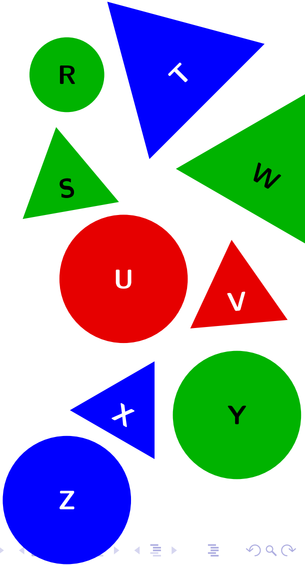
A lazier[1] way is:

```
isGreen x = x `elem` [ R, S, W, Y ]
isRed x = x `elem` [ U, V ]
```



[1] *The three chief virtues of a programmer are laziness, impatience, and hubris*
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isGreen x = x `elem` [ R, S, W, Y ]
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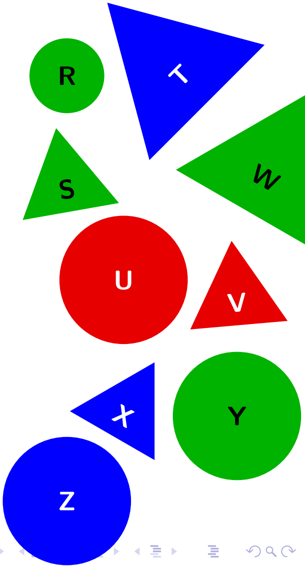
```

Is this too lazy? (What happens when we extend the universe?)

```
isBlue x = not (isGreen x || isRed x)
```



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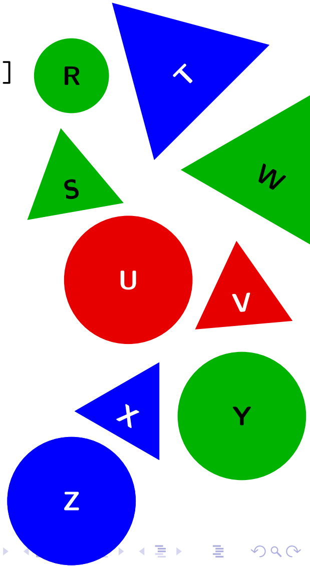
Representing statements with list comprehension

Haskell's *list comprehension* gives a powerful way of representing statements:

```
[ x | x <- things, isBlue x || (isBig x && isDisc x) ]
```



Representing statements with list comprehension

Haskell's *list comprehension* gives a powerful way of representing statements:

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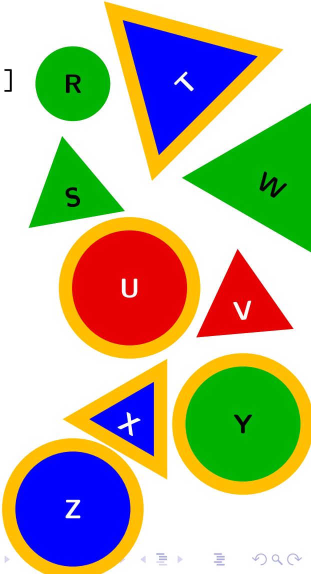
[ x | x <- things, isBlue x || (isBig x && isDisc x) ]
```

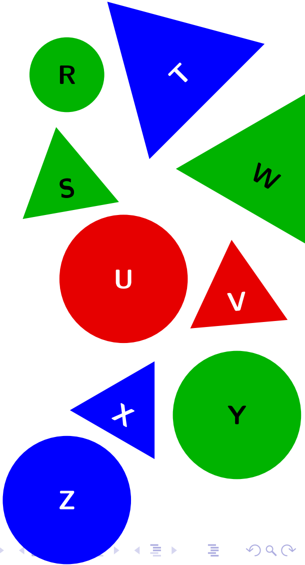
'the set (list) of things that are either blue or are big discs'



Categorical statements with Haskell

Combining list comprehension with boolean operators on lists lets us express categorical statements.

Every small triangle is red. ✗



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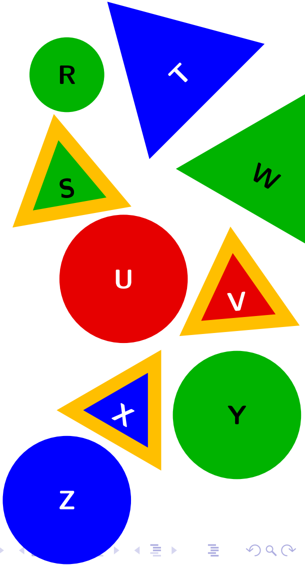
```
[ x | x <- things, isTriangle(x) && isSmall(x) ]
[S,V,X]
```

‘The set of things that are small triangles.’



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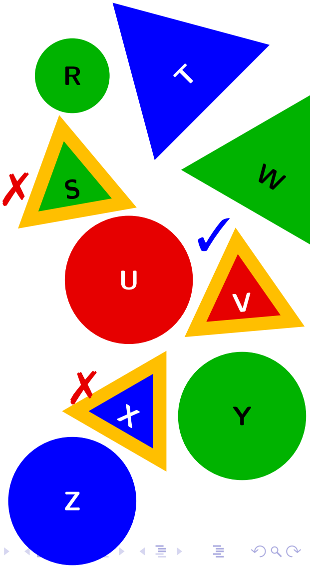
```
[ isRed(x) | x <- things, isTriangle(x) && isSmall(x) ]
[False,True,False]
```

'Whether each small triangle is red.'



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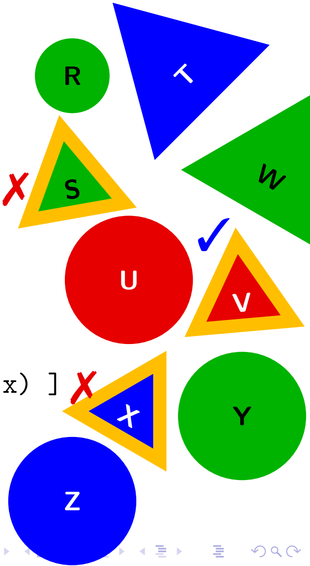
[False,True,False]
```

'Whether each small triangle is red.'

```
and [ isRed(x) | x <- things, isTriangle(x) && isSmall(x) ] ✗
False
```

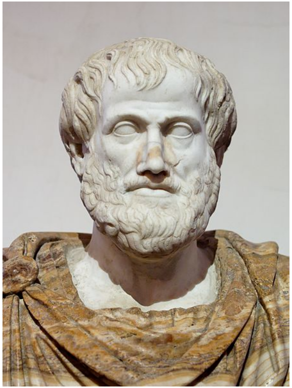
'Every small triangle is red.'



Aristotle's Syllogistic Reasoning

A syllogism is discourse (logos) in which, certain things being stated, something other than what is stated follows of necessity from those things.

- All Greeks are human
- All humans are mortal
- $\therefore$ All Greeks are mortal



Syllogisms

Can be expressed in many forms:

- $\{ x \mid \text{isGreek}(x) \} \subseteq \{ x \mid \text{isHuman}(x) \}$
- $\{ x \mid \text{isHuman}(x) \} \subseteq \{ x \mid \text{isMortal}(x) \}$
- $\therefore \{ x \mid \text{isGreek}(x) \} \subseteq \{ x \mid \text{isMortal}(x) \}$

All Greeks are human
All humans are mortal
∴ all humans are mortal

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- $\therefore \{ x \mid \text{isGreek}(x) \} \subseteq \{ x \mid \text{isMortal}(x) \}$

In modern logic, we write it as:

$$\frac{\text{isGreek} \vDash \text{isHuman} \qquad \text{isHuman} \vDash \text{isMortal}}{\text{isGreek} \vDash \text{isMortal}}$$

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The general form of this syllogism is

$$\frac{a \vDash b \qquad b \vDash c}{a \vDash c}$$

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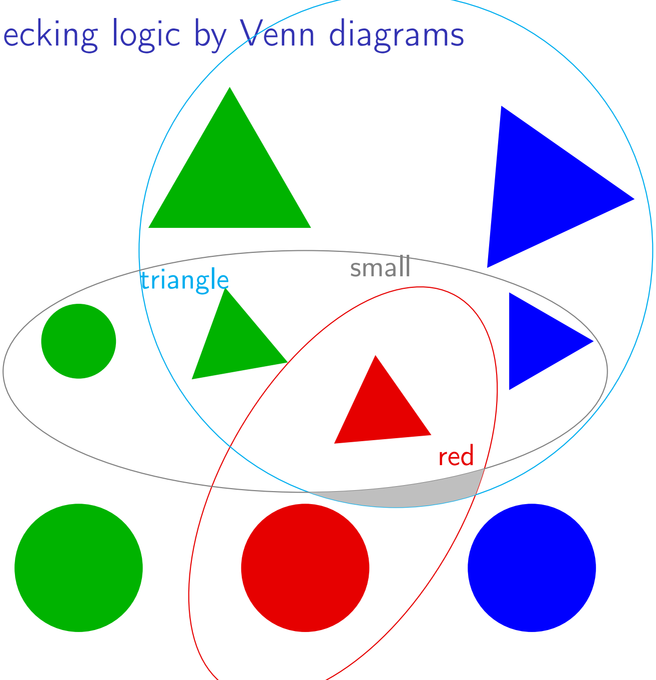
The general form of this syllogism is

$$\frac{a \vDash b \qquad b \vDash c}{a \vDash c}$$

Is this syllogism sound? I.e. valid in *every* universe?

All Greeks are human
All humans are mortal
$\therefore$ all humans are mortal

Checking logic by Venn diagrams



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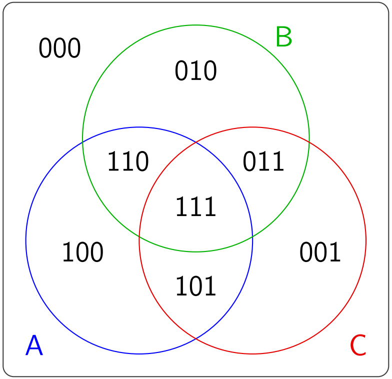
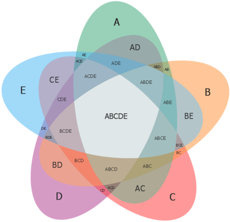
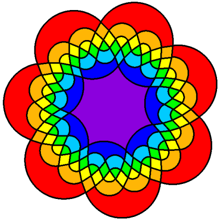
Venn diagrams

Venn diagrams show every possible combination



000	$\bar{A} \cap \bar{B} \cap \bar{C}$
001	$\bar{A} \cap \bar{B} \cap C$
010	$\bar{A} \cap B \cap \bar{C}$
011	$\bar{A} \cap B \cap C$
100	$A \cap \bar{B} \cap \bar{C}$
101	$A \cap \bar{B} \cap C$
110	$A \cap B \cap \bar{C}$
111	$A \cap B \cap C$

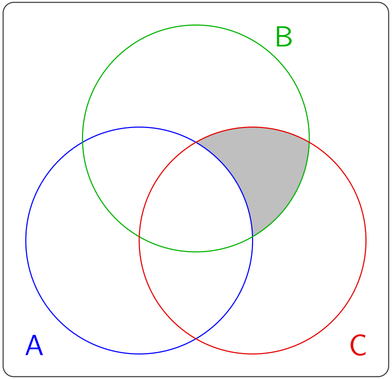




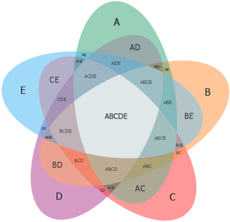
A rotationally symmetric Venn diagram for $n > 1$ sets exists iff n is prime

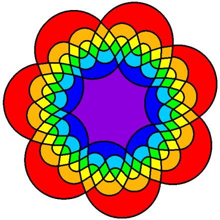
Venn diagrams

We use light shading to show **emptiness** of a region



$$\bar{A} \cap B \cap C = \varnothing$$

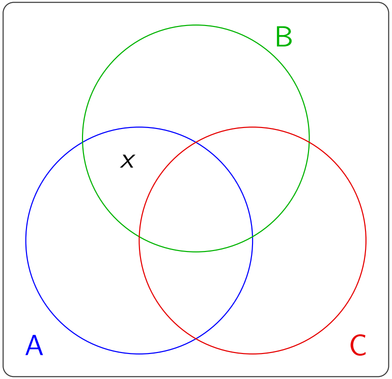




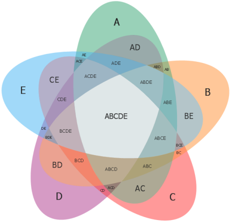
A rotationally symmetric Venn diagram for $n > 1$ sets exists iff n is prime

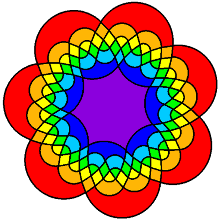
Venn diagrams

We may write a *variable* to show **non-emptiness** of a region



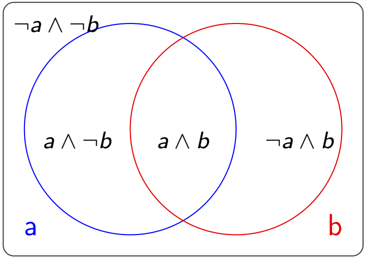
$$x \in A \cap B \cap \bar{C}$$





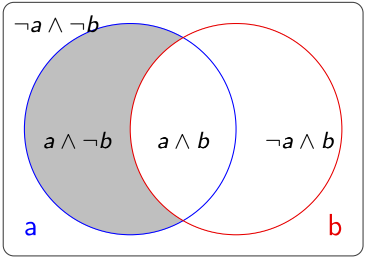
A rotationally symmetric Venn diagram for $n > 1$ sets exists iff n is prime

Venn interpretation of $a \vDash b$



$a \vDash b$	'every a is b'
$a \vDash b$	'no a is not b'
$a \vDash b$	'nothing is a and not b'
$a \vDash b$	$a \cap \bar{b} = \varnothing$
$a \vDash b$	$\neg(a \wedge \neg b)$
$a \vDash b$	$b \vee \neg a$

Venn interpretation of $a \models b$



$a \models b$ 'every a is b'

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$a \models b$ $a \cap \bar{b} = \varnothing$

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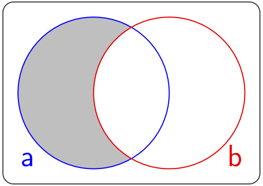
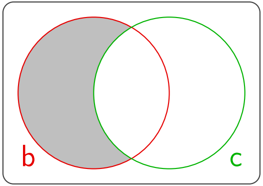
$a \models b$ $b \vee \neg a$

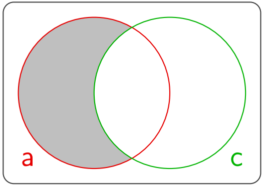
Venn syllogism



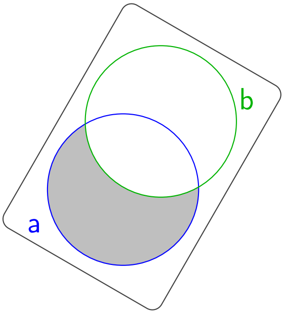


$$\frac{a \models b \quad b \models c}{a \models c}$$

$$\frac{\text{every } a \text{ is } b \quad \text{every } b \text{ is } c}{\text{every } a \text{ is } c}$$

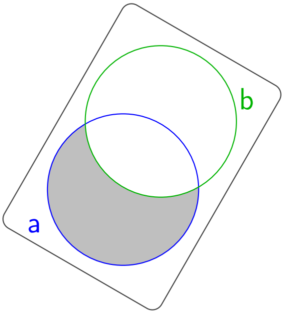


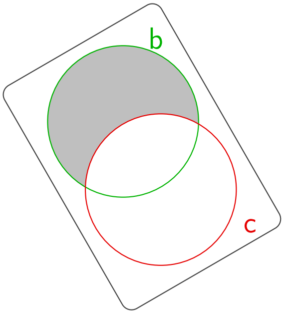
Combining diagrams



$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

Combining diagrams





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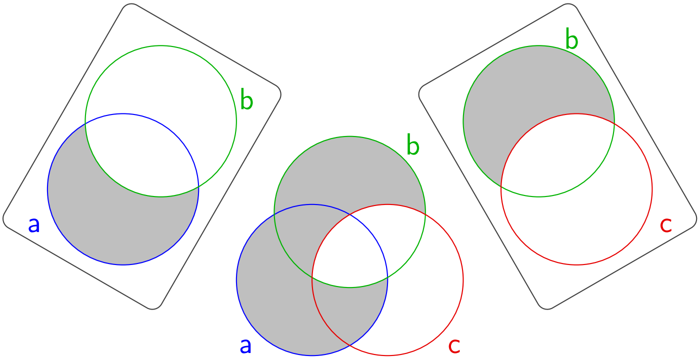

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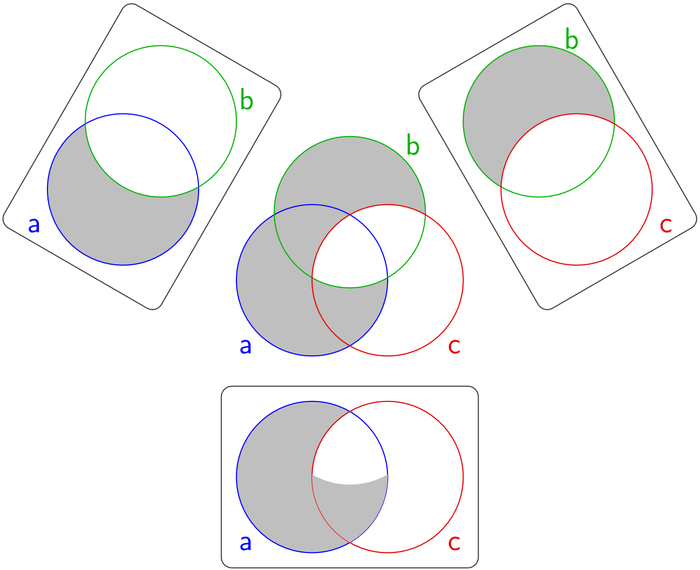

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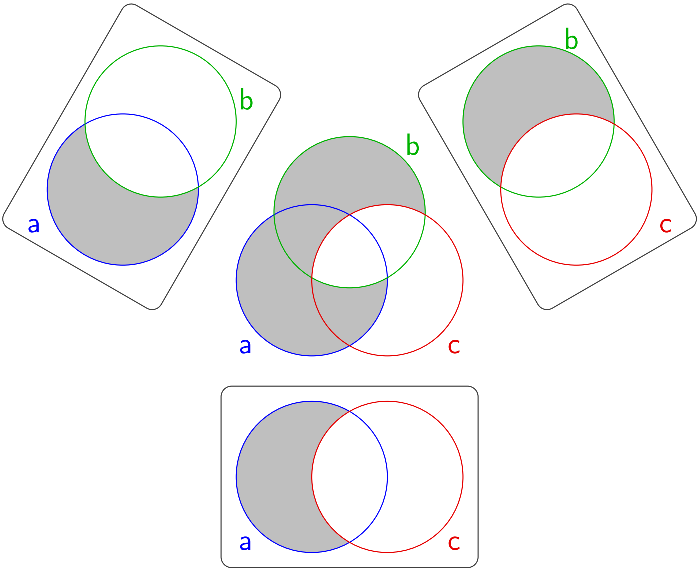
Combining diagrams



$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

Barbara is sound!

$$\text{barbara} \quad \frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

This rule, as we've seen, is sound:

for *any predicates* a, b, c in *any universe*, we have:

if the premises (above the line) are valid
then the conclusion (below the line) is valid.

Mediaeval logicians gave mnemonic names to syllogisms. This one is *barbara*. Consult Wikipedia to find out what that means – but only if you don't value your sanity!

Aristotle's *universal* propositions

make statements about *all* of something: 'all a are b'.

We can make universal *negative* statements: 'no a is b'.

$$\text{'no } a \text{ is } b\text{' iff 'every } a \text{ is } \neg b\text{' iff } a \vDash \neg b$$

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Here is a syllogism involving universal negatives:

$$\frac{s \vDash r \quad r \vDash \neg f}{s \vDash \neg f}$$

All snakes are reptiles
No reptile has fur
∴ *No snake has fur*

Is this an instance of *barbara* (and so valid)?

Aristotle's *universal* propositions

make statements about *all* of something: 'all a are b'.

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'no a is b' iff 'every a is $\neg b$' iff $a \vDash \neg b$

Here is a syllogism involving universal negatives:

$$\frac{s \vDash r \quad r \vDash \neg f}{s \vDash \neg f}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *No snake has fur*

Is this an instance of *barbara* (and so valid)?

For us modern logicians, it is: $a \equiv s, b \equiv r, c \equiv \neg f$.

A negated predicate is also a predicate.

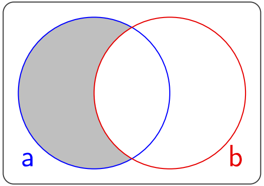
Aristotle differed from us moderns on the relation between 'all' and 'no'. For him, this syllogism contained a universal affirmative and two universal negatives. The mediaeval logicians called it *celarent*.

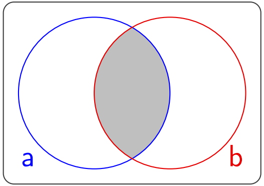
The key difference was the 'existential assumption' – see later.

Universal statements

whether affirmative or negative, say that some region is *empty*:

all a are b	no a is b
$a \models b$	$a \models \neg b$





Universal statements

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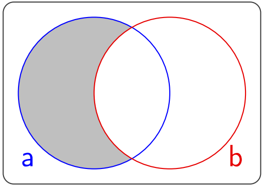
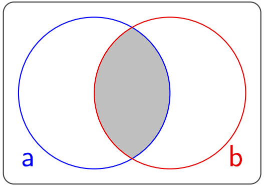
all *a* are *b*

$a \models b$



no *a* is *b*

$a \models \neg b$



What about $\neg a \models b$ and $\neg a \models \neg b$?

Universal statements

whether affirmative or negative, say that some region is *empty*:

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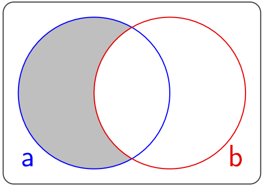
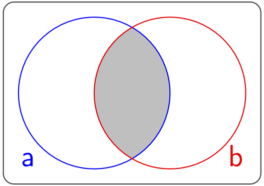
$a \vDash b$



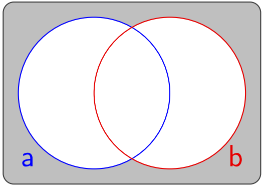
no *a* is *b*

$a \vDash \neg b$



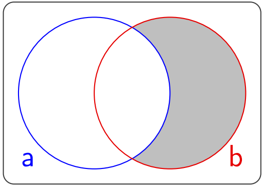
???

$\neg a \vDash b$



???

$\neg a \vDash \neg b$



Universal statements

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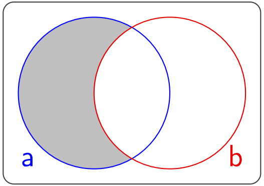

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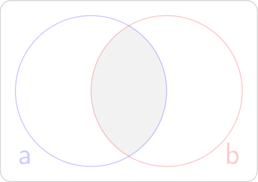
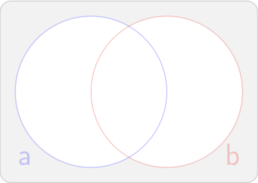


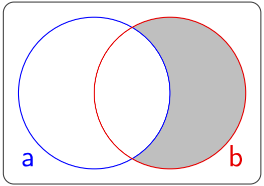
???

$\neg a \vDash b$



???

$\neg a \vDash \neg b$



We can observe:

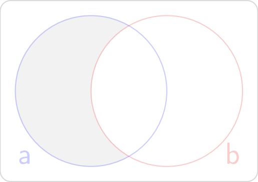
- $a \vDash b$ and $\neg a \vDash \neg b$ are reflections of each other: so $\neg a \vDash \neg b$ is the same as $b \vDash a$. $\neg a \vDash \neg b$ is the contrapositive of $b \vDash a$.

Universal statements

whether affirmative or negative, say that some region is *empty*:

all a are b

$a \vDash b$



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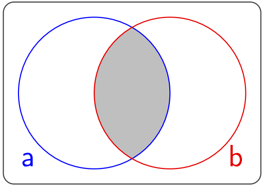
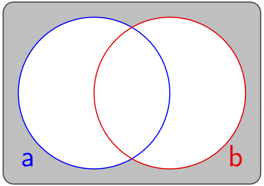


???

$\neg a \vDash b$



???

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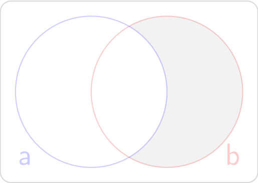

We can observe:

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- $a \vDash \neg b$ is symmetrical, so is the same as $b \vDash \neg a$ – they are contrapositives. Likewise $\neg a \vDash b$ and $\neg b \vDash a$.

Negation and contraposition

Negation can be tricky – modern classical logic makes it simple.

Natural languages differ, within and between themselves, on how they treat multiple negatives:

'I didn't never do nothing to nobody!'.

How does your native language/ dialect treat multiple negatives?

Negation and contraposition

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The law of double negation: $\neg\neg a = a$ (two negatives make a positive).

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The law of contraposition: $a \vDash b$ iff $\neg b \vDash \neg a$.

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The law of double negation: $\neg\neg a = a$ (two negatives make a positive).

The law of contraposition: $a \vDash b$ iff $\neg b \vDash \neg a$.

Thus we get $a \vDash b$ iff $\neg b \vDash \neg a$ iff $\neg\neg a \vDash \neg\neg b$ iff $a \vDash b$.

$$\frac{a \vDash b}{\neg b \vDash \neg a}$$

The double line means the rule works both ways.

Natural languages differ, within and between themselves, on how they treat multiple negatives: 'I didn't never do nothing to nobody!'. How does your native language/ dialect treat multiple negatives?

Reprise

So far, we have seen (and hopefully agreed on) these **sound** rules about predicates and $\models$:

- $\neg\neg a = a$ *or* $\dfrac{a}{\neg\neg a}$ (double negation)
- $\dfrac{a \models b \quad b \models c}{a \models c}$ (*barbara*)
- $\dfrac{a \models b}{\neg b \models \neg a}$ (contraposition)

Reprise

So far, we have seen (and hopefully agreed on) these **sound** rules about predicates and $\vDash$:

- $\neg\neg a = a$ *or* $\dfrac{a}{\neg\neg a}$ (double negation)
- $\dfrac{a \vDash b \quad b \vDash c}{a \vDash c}$ (*barbara*)
- $\dfrac{a \vDash b}{\neg b \vDash \neg a}$ (contraposition)

We also saw a 'different' (for Aristotle) syllogism with negatives got from *barbara* by putting $\neg c$ for c:

$$\frac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *No snake has fur*

More syllogisms

By using (un)negated predicates in *barbara*, we get 8 syllogisms:

$$\frac{a \models b \quad b \models c}{a \models c} \qquad \frac{\neg a \models b \quad b \models c}{\neg a \models c}$$

$$\frac{a \models b \quad b \models \neg c}{a \models \neg c} \qquad \frac{\neg a \models b \quad b \models \neg c}{\neg a \models \neg c}$$

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Aristotle only considered negative predicates on the right of $\models$ ($a \models \neg b$ means 'no a is b', so he viewed it as a negative statement about positive predicates). This leaves ...

More syllogisms

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$$\frac{a \models b \quad b \models c}{a \models c}$$

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barbara and *celarent*

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Even more syllogisms

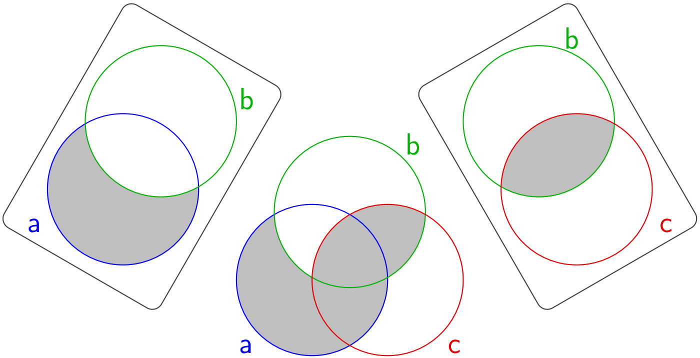
Contraposition lets us generate three more (Aristotelian) syllogisms from *celarent*:

$$\frac{a \vDash b \quad c \vDash \neg b}{a \vDash \neg c} \qquad \frac{a \vDash b \quad b \vDash \neg c}{c \vDash \neg a} \qquad \frac{a \vDash b \quad c \vDash \neg b}{c \vDash \neg a}$$

cesare, camenes, camestres

That brings us to 5 sound universal syllogisms. That's all!

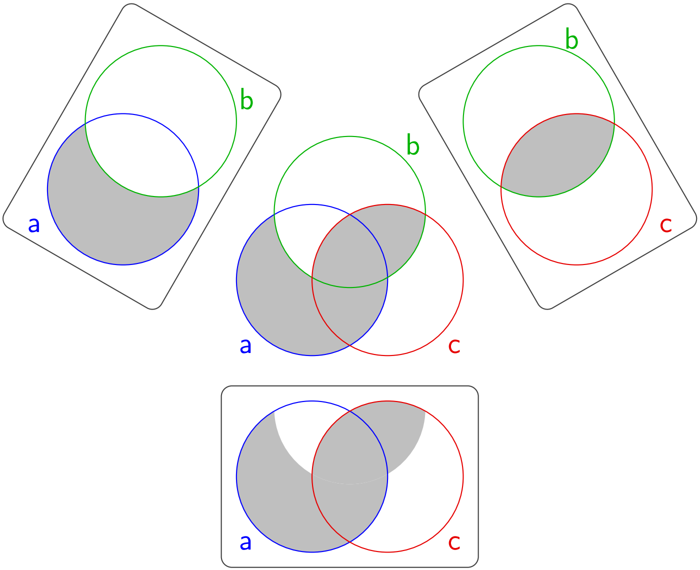
Unsound syllogisms



$$\frac{a \vDash b \qquad b \vDash \neg c}{a \vDash c}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *All snakes have fur*

Unsound syllogisms



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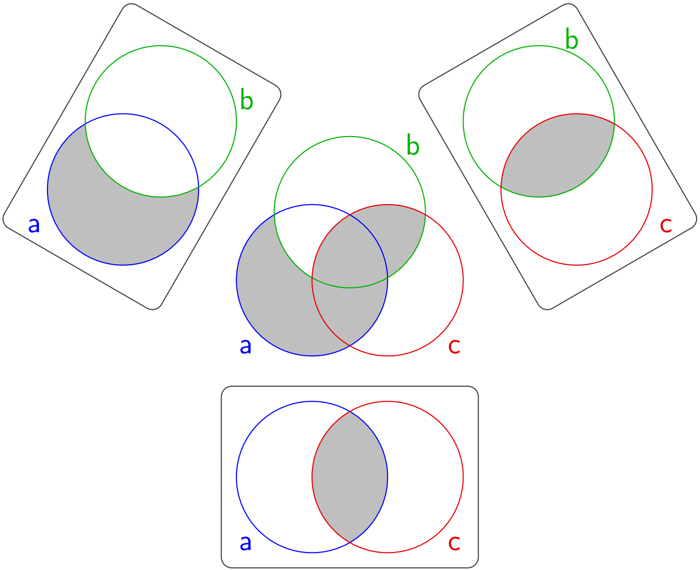

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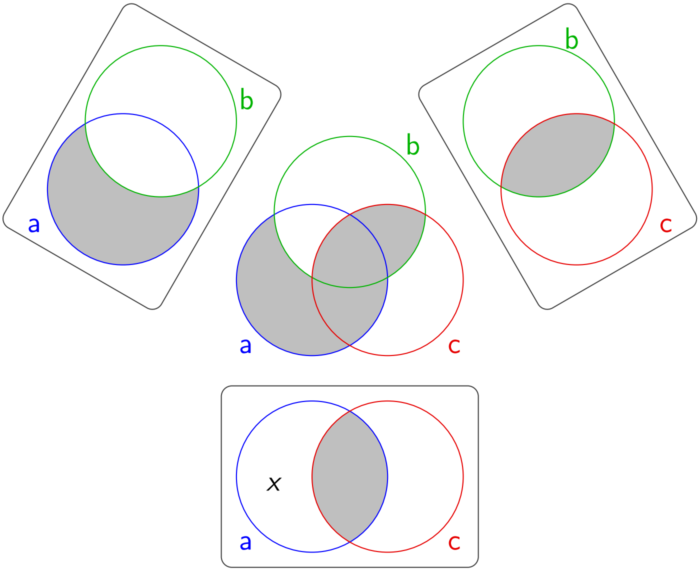

$a \models b \qquad b \models \neg c$

$a \models c$

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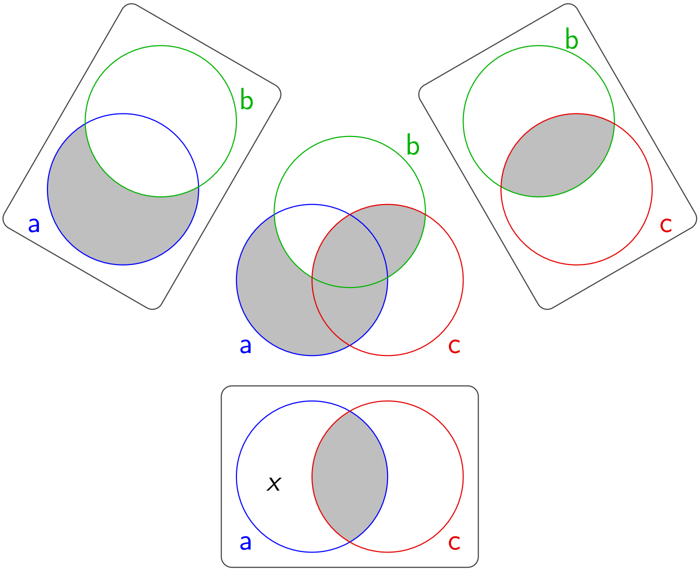
Unsound syllogisms



$$\frac{a \vDash b \qquad b \vDash \neg c}{a \vDash c}$$

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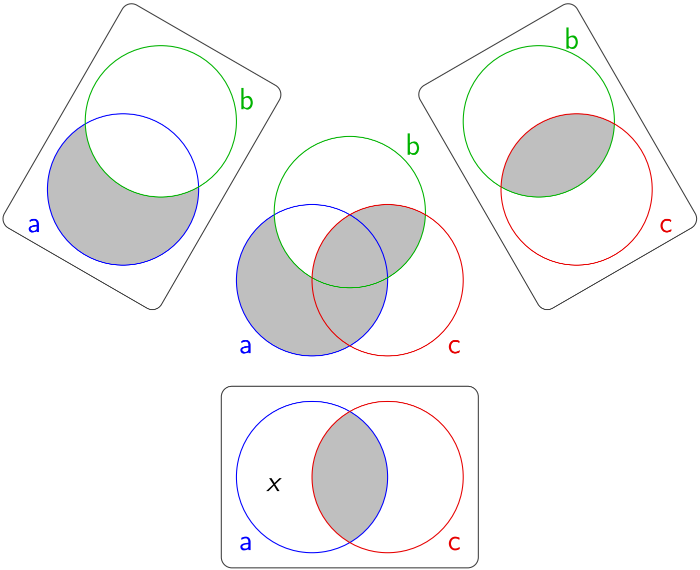
To disprove a syllogism, we need just one universe where it's invalid (e.g. Edinburgh zoo).

Unsound syllogisms



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Is there a universe where this syllogism *is* valid?

(Aristotle said 'no'; we moderns differ. Hint: St Patrick.)

Reprise: Sound universal syllogisms

From *barbara*, contraposition, and double negation, we have five sound syllogisms about universal statements:

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

$$\frac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c}$$

$$\frac{a \vDash b \quad c \vDash \neg b}{a \vDash \neg c}$$

$$\frac{a \vDash b \quad b \vDash \neg c}{c \vDash \neg a} \quad \text{equivalently} \quad \frac{c \vDash b \quad b \vDash \neg a}{a \vDash \neg c}$$

$$\frac{a \vDash b \quad c \vDash \neg b}{c \vDash \neg a} \quad \text{equivalently} \quad \frac{c \vDash b \quad a \vDash \neg b}{a \vDash \neg c}$$

Reprise: Sound universal syllogisms

From *barbara*, contraposition, and double negation, we have five sound syllogisms about universal statements:

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

$$\frac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c}$$

$$\frac{a \vDash b \quad c \vDash \neg b}{a \vDash \neg c}$$

$$\frac{a \vDash b \quad b \vDash \neg c}{c \vDash \neg a} \quad \text{equivalently} \quad \frac{c \vDash b \quad b \vDash \neg a}{a \vDash \neg c}$$

$$\frac{a \vDash b \quad c \vDash \neg b}{c \vDash \neg a} \quad \text{equivalently} \quad \frac{c \vDash b \quad a \vDash \neg b}{a \vDash \neg c}$$

Note that the conclusion is negative iff exactly one of the premises is negative – compare the unsound syllogism on the previous slide.