Informatics 1 – Introduction to Computation
Computation and Logic
Julian Bradfield
based on materials by
Michael P. Fourman

More Syllogisms
and more about syllogisms
So far, we have seen (and hopefully agreed on) these **sound** rules about predicates and $\models$:

- $\neg\neg a = a$ or $\dfrac{a}{\neg\neg a}$ (double negation)

- $a \models b \quad b \models c \quad \therefore \quad a \models c$ (**barbara**)

- $\dfrac{a \models b}{\neg b \models \neg a}$ (contraposition)
So far, we have seen (and hopefully agreed on) these sound rules about predicates and $\models$:

- $\neg\neg a = a$ or $\frac{a}{\neg\neg a}$ (double negation)

- $a \models b \quad b \models c \quad \frac{a \models c}{barbara}$

- $a \models b \quad \frac{\neg b \models \neg a}{contraposition}$

We also saw a ‘different’ (for Aristotle) syllogism with negatives got from $barbara$ by putting $\neg c$ for $c$:

$$a \models b \quad b \models \neg c \quad \frac{a \models \neg c}{All\ snakes\ are\ reptiles}$$

$$No\ reptile\ has\ fur \quad \therefore\ No\ snake\ has\ fur$$
By using (un)negated predicates in *barbara*, we get 8 syllogisms:

\[
\begin{array}{c}
\frac{a \models b \quad b \models c}{a \models c} & \quad \frac{\neg a \models b \quad b \models c}{\neg a \models c} \\
\frac{a \models b \quad b \models \neg c}{a \models \neg c} & \quad \frac{\neg a \models b \quad b \models \neg c}{\neg a \models \neg c} \\
\frac{a \models \neg b \quad \neg b \models c}{a \models c} & \quad \frac{\neg a \models \neg b \quad \neg b \models c}{\neg a \models c} \\
\frac{a \models \neg b \quad \neg b \models \neg c}{a \models \neg c} & \quad \frac{\neg a \models \neg b \quad \neg b \models \neg c}{\neg a \models \neg c}
\end{array}
\]
By using (un)negated predicates in *barbara*, we get 8 syllogisms:

\[
\begin{align*}
    a &\models b & b &\models c & a &\models c \\
    a &\models b & b &\models \neg c & a &\models \neg c \\
    a &\models \neg b & \neg b &\models c & a &\models c \\
    a &\models \neg b & \neg b &\models \neg c & a &\models \neg c
\end{align*}
\]

Aristotle only considered negative predicates on the right of \(\models\) (\(a \models \neg b\) means ‘no a is b’, so he viewed it as a negative statement about positive predicates). This leaves . . .
More syllogisms

By using (un)negated predicates in *barbara*, we get 8 syllogisms:

\[
\begin{align*}
  a \models b & \quad b \models c \\
  \quad a \models c \\
  a \models b & \quad b \models \neg c \\
  \quad a \models \neg c \\
\end{align*}
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*barbara* and *celarent*

Aristotle only considered negative predicates on the right of $\models$:

\((a \models \neg b)\) means ‘no a is b’, so he viewed it as a negative statement about positive predicates). This leaves . . .
Even more syllogisms

*Contraposition* lets us generate three more (Aristotelian) syllogisms from *celarent*:

\[
\begin{align*}
 a & \models b & c & \models \neg b & \quad a & \models b & b & \models \neg c \\
 a & \models \neg c & c & \models \neg a & \quad a & \models b & c & \models \neg b \\
 c & \models \neg a & c & \models \neg a
\end{align*}
\]

That brings us to 5 sound universal syllogisms. That’s all!
Unsound syllogisms

\[
\begin{align*}
\text{All snakes are reptiles} & \\
\text{No reptile has fur} & \\
\therefore \text{All snakes have fur}
\end{align*}
\]

To disprove a syllogism, we need just one universe where it's invalid (e.g. Edinburgh zoo). Is there a universe where this syllogism is valid? (Aristotle said ‘no’; we moderns differ. Hint: St Patrick.)

\[
\frac{\begin{align*}
\text{a} \models b \\
\text{b} \models \neg c
\end{align*}}{\text{a} \models c}
\]

*All snakes are reptiles
No reptile has fur
∴ All snakes have fur*
Unsound syllogisms

\[
a \models b \quad b \models \neg c \\
\therefore a \models c
\]

All snakes are reptiles
No reptile has fur
\therefore All snakes have fur
Unsound syllogisms

\[ a \vdash b \quad b \vdash \neg c \]
\[ a \vdash c \]

All snakes are reptiles
No reptile has fur
\[ \therefore \text{All snakes have fur} \]
Unsound syllogisms

To disprove a syllogism, we need just one universe where it's invalid (e.g. Edinburgh zoo).

Is there a universe where this syllogism is valid?

(Aristotle said 'no'; we moderns differ. Hint: St Patrick.)

\[ a \models b \quad b \models \neg c \quad a \models c \]

All snakes are reptiles
No reptile has fur
\[
\therefore \text{All snakes have fur} 
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Unsound syllogisms

All snakes are reptiles
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Is there a universe where this syllogism is valid?

(Aristotle said ‘no’; we moderns differ. Hint: St Patrick.)
From *barbara*, contraposition, and double negation, we have five sound syllogisms about universal statements:

\[
\begin{align*}
& a \models b \quad b \models c \quad \therefore a \models c \\
& a \models b \quad b \models \neg c \quad \therefore a \models \neg c \\
& a \models b \quad c \models \neg b \quad \therefore a \models \neg c \\
& a \models b \quad b \models \neg c \quad \text{equivalently} \quad c \models b \quad b \models \neg a \quad \therefore a \models \neg c \\
& a \models b \quad c \models \neg b \quad \text{equivalently} \quad c \models b \quad a \models \neg b \quad \therefore a \models \neg c
\end{align*}
\]

Note that the conclusion is negative iff exactly one of the premises is negative – compare the unsound syllogism on the previous slide.
From *barbara*, contraposition, and double negation, we have five sound syllogisms about universal statements:

\[
\begin{align*}
\frac{a \models b \quad b \models c}{a \models c} \\
\frac{a \models b \quad b \models \neg c}{a \models \neg c} \\
\frac{a \models b \quad c \models \neg b}{a \models \neg c} \\
\frac{a \models b \quad b \models \neg c}{c \models \neg a} & \text{ equivalently } \frac{c \models b \quad b \models \neg a}{a \models \neg c} \\
\frac{a \models b \quad c \models \neg b}{c \models \neg a} & \text{ equivalently } \frac{c \models b \quad a \models \neg b}{a \models \neg c}
\end{align*}
\]

Note that the conclusion is negative iff exactly one of the premises is negative – compare the unsound syllogism on the previous slide.
Reprise

From the fundamental rule \textit{barbara}

\begin{align*}
a \vDash b & \quad b \vDash c \\
\hline
a & \vDash c
\end{align*}

together with \textit{contraposition} and \textit{double negation}, we got five sound syllogisms about \textit{universal categorical statements}.

Contraposition is negating and swapping the two parts of a sequent:

\begin{align*}
a \vDash b & \quad \rightarrow \quad \neg b \vDash \neg a
\end{align*}

\textit{Barbara} is the feminine form of the Greek βάρβαρος (barbaros) ‘foreign’. Taken into Latin, it was used as the name of a mythical early Christian martyr, daughter of a pagan (barbarian).
Contraponing propositions

Contraposition is a powerful general reasoning technique. We can use it not only inside sequents, but on rules. For example:
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If you are over 18 (we assume this henceforth), you can legally buy alcohol from shops in Scotland between 10:00 and 22:00 each day.

In England you can buy alcohol at any time. In some countries you can't buy it (legally) at all.
Contraponing propositions

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If you are over 18 (we assume this henceforth), you can legally buy alcohol from shops in Scotland between 10:00 and 22:00 each day. So in our current universe, the following rule holds:

\[
\begin{array}{c}
\text{In Scotland} & \text{Time between 10h and 22h} \\
\text{Can legally buy alcohol}
\end{array}
\]

The predicates have an implicit argument combining an adult, a time, and a place. This rule is not sound, it just holds in some universes – it’s a rule of law, not logic!
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In Scotland Time between 10h and 22h
Can legally buy alcohol

What other rules can we infer from this?

In Scotland Cannot legally buy alcohol

Time between 10h and 22h Cannot legally buy alcohol

In England you can buy alcohol at any time. In some countries you can't buy it (legally) at all.

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& \text{Can legally buy alcohol}
\end{array}
\]

What other rules can we infer from this?

\[
\begin{array}{c}
\text{In Scotland} & \text{Cannot legally buy alcohol} \\
& \text{Time between 22h and 10h}
\end{array}
\]

\[
\begin{array}{c}
\text{Time between 10h and 22h} & \text{Cannot legally buy alcohol}
\end{array}
\]

In England you can buy alcohol at any time. In some countries you can’t buy it (legally) at all.

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\begin{align*}
\text{In Scotland} & \quad \text{Time between 10h and 22h} \\
\text{Can legally buy alcohol} \\
\end{align*}
\]

What other rules can we infer from this?

\[
\begin{align*}
\text{In Scotland} & \quad \text{Cannot legally buy alcohol} \\
\text{Time between 22h and 10h} \\
\end{align*}
\]

\[
\begin{align*}
\text{Time between 10h and 22h} & \quad \text{Cannot legally buy alcohol} \\
\text{Not in Scotland} \\
\end{align*}
\]

In England you can buy alcohol at any time. In some countries you can't buy it (legally) at all.

The predicates have an implicit argument combining an adult, a time, and a place. This rule is not sound, it just holds in some universes – it’s a rule of law, not logic!
Contraposing *barbara*

Why does contraposition work between the conclusion and one premise at a time? What is the relation between the two premises? Can we combine them into one? What happens with contraposition then? What is the difference between $\vdash a \rightarrow b$ and $a \vdash b$?

These two syllogisms are *bocardo* and *baroco*.

Sequents are themselves propositions. Applying the same principle, from $a \vdash b$ and $b \vdash c$, we get $a \vdash c$. Some $a$ are $b$, every $a$ is $b$, some $a$ is not $c$. $a \vdash b$ is $\not\vdash c$, $b \not\vdash c$, $b \vdash c$, $a \not\vdash c$. $a \not\vdash b$. $a \vdash b$.
Sequents are themselves propositions. Applying the same principle, from

\[
\frac{a \vdash b \quad b \vdash c}{a \vdash c}
\]

we get

\[
\begin{align*}
a \vdash b & \quad a \nLeftarrow c & \quad b \nLeftarrow c & \quad a \nLeftarrow c \\
& \quad b \nLeftarrow c & \quad a \nLeftarrow b
\end{align*}
\]
Contraponing *barbara*

Sequents are themselves propositions. Applying the same principle, from

\[
\begin{align*}
  a \vdash b & \quad b \vdash c \\
  \hline
  a \vdash c
\end{align*}
\]

we get

\[
\begin{align*}
  a \vdash b & \quad a \not\vdash c \\
  \hline
  b \not\vdash c \\

  b \vdash c & \quad a \not\vdash c \\
  \hline
  a \not\vdash b
\end{align*}
\]
Contraposing *barbara*

Sequents are themselves propositions. Applying the same principle, from

\[ a \models b \quad b \models c \]

we get

\[ a \models c \]

\[ a \nL b \quad a \nR c \]

\[ b \nL c \]

\[ b \nR c \quad a \nR b \]
Contraposing *barbara*

Sequents are themselves propositions. Applying the same principle, from

\[
\frac{a \vdash b \quad b \vdash c}{a \vdash c}
\]

we get

\[
\frac{a \vdash b \quad a \not\vdash c}{b \not\vdash c} \quad \frac{b \vdash c \quad a \not\vdash c}{a \not\vdash b}
\]

- every *a* is *b*
- some *a* is not *c*
Contraposing *barbara*

Sequents are themselves propositions. Applying the same principle, from

\[
\frac{a \models b \quad b \models c}{a \models c}
\]

we get

\[
\begin{align*}
\frac{a \models b \quad a \not\models c}{b \not\models c} & \quad \text{every } a \text{ is } b \\
\frac{b \models c \quad a \not\models c}{a \not\models b} & \quad \text{some } a \text{ is not } c
\end{align*}
\]

Why does contraposition work between the conclusion and *one* premise at a time?

What is the relation between the two premises?

Can we combine them into one?

What happens with contraposition then?

What is the difference between \( a \models b \) and \( a \to b \)?

These two syllogisms are *bocardo* and *baroco*. 

Aristotle’s categorical propositions

**universal affirmative**

every $a$ is $b$

$a \models b$

$a \not\models \neg b$

some $a$ is $b$

**particular affirmative**

**universal negative**

no $a$ is $b$

$a \models \neg b$

$a \not\models b$

some $a$ is not $b$

**particular negative**
Aristotle’s categorical propositions

\[
\begin{align*}
\text{universal affirmative} & \quad \text{universal negative} \\
\text{every } a \text{ is } b & \quad \text{no } a \text{ is } b \\
a \models b & \quad a \models \neg b \\
a \nvdash \neg b & \quad a \nvdash b \\
\text{some } a \text{ is } b & \quad \text{some } a \text{ is not } b \\
\text{particular affirmative} & \quad \text{particular negative}
\end{align*}
\]

Why not exactly?
Aristotle made the existential assumption: if you say ‘all a are b’, or ‘no a is b’, that means that some a exists. So for him, universal affirmative implies particular affirmative, and universal negative implies particular negative.
Checking syllogisms (again)

This syllogism comes from *barbara* via contrapositions (do that!), so it should be sound. Check with Venn diagrams:

\[
\begin{align*}
    c &
    \models \neg b & a &
    \not\equiv \neg c \\
    \therefore & a &
    \not\equiv b
\end{align*}
\]

*No mathematician is infallible*

*Some programmers are mathematicians*

*Some programmers are fallible*
Checking syllogisms (again)

This syllogism comes from *barbara* via contrapositions (do that!), so it should be sound. Check with Venn diagrams:

No mathematician is infallible
Some programmers are mathematicians
∴ Some programmers are fallible
Checking syllogisms (again)

This syllogism comes from *barbara* via contrapositions (do that!), so it should be sound. Check with Venn diagrams:

\[
\begin{align*}
\text{c} & \models \neg b & \text{a} & \not\equiv \neg c \\
\hline
\text{a} & \not\equiv b
\end{align*}
\]

*No mathematician is infallible*

*Some programmers are mathematicians*

\[\therefore \text{Some programmers are fallible}\]
Checking syllogisms (again)

This syllogism comes from *barbara* via contrapositions (do that!), so it should be sound. Check with Venn diagrams:

\[ c \models \neg b \quad a \not\models \neg c \]
\[ a \not\models b \]

*No mathematician is infallible*

*Some programmers are mathematicians*

\[ \therefore \text{Some programmers are fallible} \]
What about this one?

\[ a \models b \quad c \not\models a \]
\[ \therefore c \not\models b \]

All plants are fungi
Some flowers are not plants
\[ \therefore \text{Some flowers are not fungi} \]
What about this one?

\[ a \models b \]
\[ c \not\models a \]
\[ \vdash c \not\models b \]

All plants are fungi
Some flowers are not plants
\[ \therefore \text{ Some flowers are not fungi} \]
What about this one?

All plants are fungi
Some flowers are not plants
∴ Some flowers are not fungi

In the usual meanings, no plant is a fungus, and all flowers are plants. That doesn't matter: the argument doesn't depend on the truth or falsity of the premises in a particular universe.
What about this one?

Suppose a *quonce* is a fungus flower, but not a plant, and nothing else exists. This disproves the syllogism.

All plants are fungi
Some flowers are not plants
∴ Some flowers are not fungi

In the usual meanings, no plant is a fungus, and all flowers are plants. That doesn’t matter: the argument doesn’t depend on the truth or falsity of the premises in a particular universe.
Deriving rules

We have used the following to derive sound rules from \textit{barbara}:

- substitution (e.g. \(q\) for \(a\), \(\neg b\) for \(b\))
- double negation cancellation (\(\neg\neg a \rightarrow a\))
- contraposition within sequents (\(a \vdash b \rightarrow \neg b \vdash \neg a\))
- contraposition between conclusion and a premise

\[
\begin{array}{c}
a \vdash b & b \vdash c \\
\hline
a \vdash c
\end{array} \quad \rightarrow \quad 
\begin{array}{c}
a \vdash b & a \not\vdash c \\
\hline
b \not\vdash c
\end{array}
\]

Because these processes are symmetrical, they also derive unsound rules from unsound rules.
Deriving rules

We have used the following to derive sound rules from *barbara*:

- substitution (e.g. $q$ for $a$, $\neg b$ for $b$)
- double negation cancellation ($\neg \neg a \rightarrow a$)
- contraposition within sequents ($a \vdash b \rightarrow \neg b \vdash \neg a$)
- contraposition between conclusion and a premise

\[
\begin{align*}
\frac{a \vdash b \quad b \vdash c}{a \vdash c} & \quad \frac{a \vdash b \quad a \nvdash c}{b \nvdash c}
\end{align*}
\]

Because these processes are symmetrical, they also derive unsound rules from unsound rules.
All the sound syllogisms

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<th><strong>a ⊨ b</strong></th>
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All the sound syllogisms

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You can derive all these.
Mediaeval students learned them, with the help of this verse:

Barbara celarent darii ferio baralipton
Celantes dabitis fapesmo frisesomorum
Cesare camestres festino baroco
Darapti felapton disamis datisi bocardo ferison
What have we done so far?

- *predicates* talk about things in a universe

- Categorical propositions relate two predicates, universally or particularly, affirmatively or negatively.
  - They can be concisely written as sequents $a \models b$ or $a \not\models b$.
  - They can be interpreted in Venn diagrams.

- A syllogism takes two premise categorical propositions and derives a conclusion categorical proposition.

- We can check syllogisms for soundness with Venn diagrams.

- All sound syllogisms come from *barbara* via contraposition etc.
What have we done so far?

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- *categorical propositions* relate two predicates, universally or particularly, affirmatively or negatively
What have we done so far?

- **predicates** talk about things in a universe
- **categorical propositions** relate two predicates, universally or particularly, affirmatively or negatively
- they can be concisely written as sequents $a \vdash b$ or $a \not\vdash b$
Reprise

What have we done so far?

- *predicates* talk about things in a universe
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What have we done so far?

- *predicates* talk about things in a universe
- *categorical propositions* relate two predicates, universally or particularly, affirmatively or negatively
- they can be concisely written as *sequents* \(a \vdash b\) or \(a \not\vdash b\)
- they can be interpreted in Venn diagrams.
- A *syllogism* takes two *premise* categorical propositions and derives a *conclusion* categorical proposition.
What have we done so far?

- *predicates* talk about things in a universe
- *categorical propositions* relate two predicates, universally or particularly, affirmatively or negatively
- they can be concisely written as sequents \( a \vdash b \) or \( a \not\vdash b \)
- they can be interpreted in Venn diagrams.
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- We can check syllogisms for *soundness* with Venn diagrams.
- All sound syllogisms come from *barbara* via contraposition etc.
As we’ve mentioned, Aristotle did not approve of talking about non-existent things. For him, ‘all/no $a$ are $b$’ also implies the existence of an $a$. 

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All snakes are reptiles
∴ Some snakes have no fur
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We can write the existential assumption as a rule with no premise:

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It’s all much murkier than this. This existential assumption contradicts other aspects of Aristotle’s system. In short, he was most likely confused.