Informatics 1 – Introduction to Computation
Computation and Logic

Julian Bradfield
based on materials by
Michael P. Fourman

More Syllogisms
and more about syllogisms

Reprise

So far, we have seen (and hopefully agreed on) these **sound** rules about predicates and $\vDash$:

- $\neg\neg a = a$ *or* $\dfrac{a}{\neg\neg a}$ (double negation), where the rule is drawn with a double line
- $\dfrac{a \vDash b \quad b \vDash c}{a \vDash c}$ (*barbara*)
- $\dfrac{a \vDash b}{\neg b \vDash \neg a}$ (contraposition), where the rule is drawn with a double line

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- $\dfrac{a \vDash b}{\neg b \vDash \neg a}$ (contraposition)

We also saw a 'different' (for Aristotle) syllogism with negatives got from *barbara* by putting $\neg c$ for c:

$$\frac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *No snake has fur*

More syllogisms

By using (un)negated predicates in *barbara*, we get 8 syllogisms:

$$\frac{a \models b \quad b \models c}{a \models c} \qquad \frac{\neg a \models b \quad b \models c}{\neg a \models c}$$

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Aristotle only considered negative predicates on the right of $\models$ ($a \models \neg b$ means 'no a is b', so he viewed it as a negative statement about positive predicates). This leaves . . .

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barbara and *celarent*

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Even more syllogisms

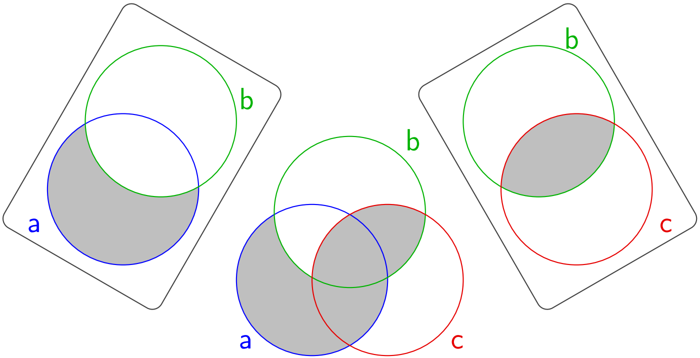
Contraposition lets us generate three more (Aristotelian) syllogisms from *celarent*:

$$\frac{a \vDash b \quad c \vDash \neg b}{a \vDash \neg c} \qquad \frac{a \vDash b \quad b \vDash \neg c}{c \vDash \neg a} \qquad \frac{a \vDash b \quad c \vDash \neg b}{c \vDash \neg a}$$

cesare, camenes, camestres

That brings us to 5 sound universal syllogisms. That's all!

Unsound syllogisms



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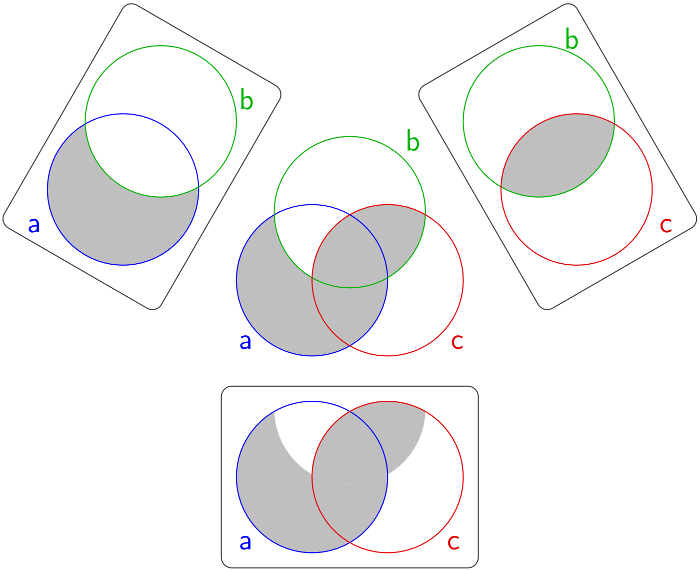

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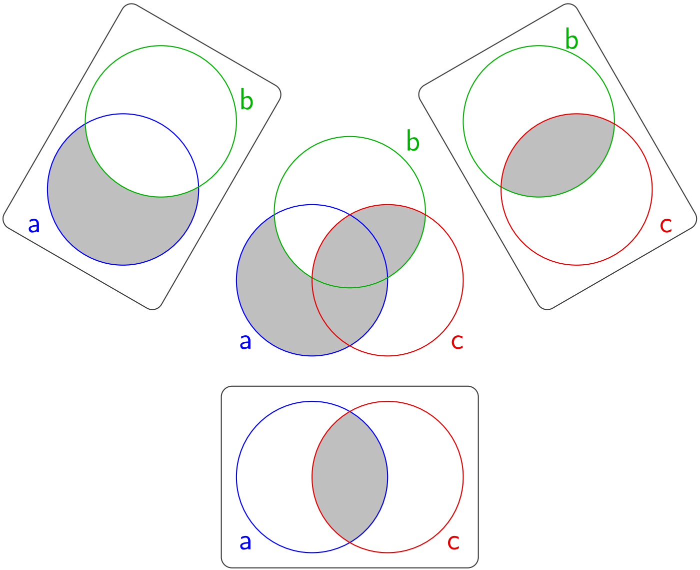

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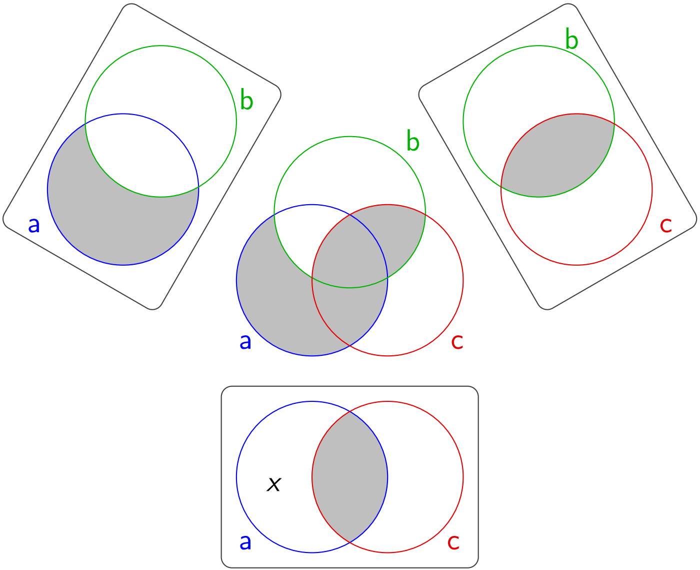

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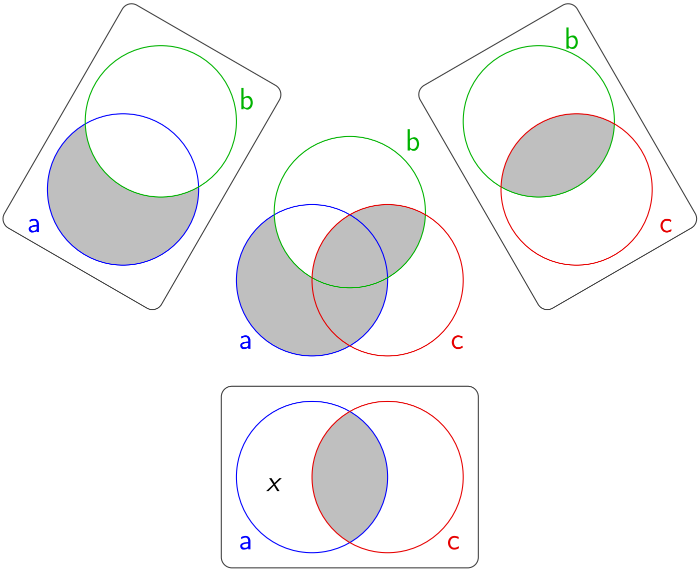

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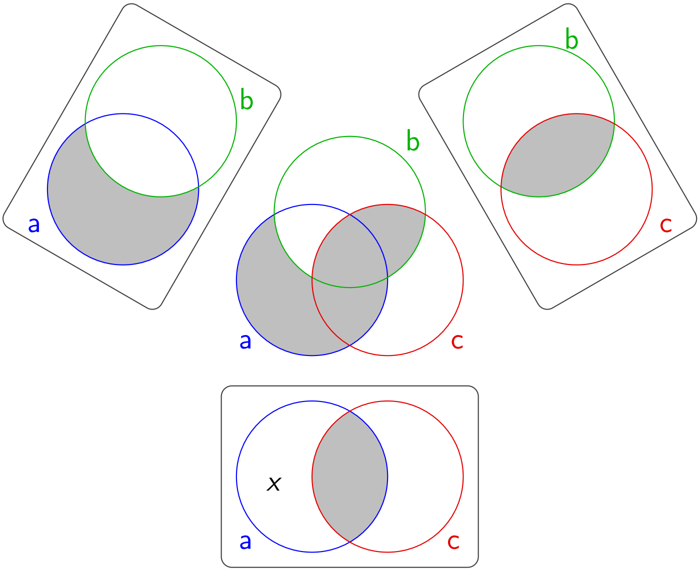

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Is there a universe where this syllogism *is* valid?

(Aristotle said 'no'; we moderns differ. Hint: St Patrick.)

Five sound universal syllogisms

From *barbara*, contraposition, and double negation, we have five sound syllogisms about universal statements:

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Note that the conclusion is negative iff exactly one of the premises is negative – compare the unsound syllogism on the previous slide.

Reprise

From the fundamental rule *barbara*

$$\frac{a \models b \quad b \models c}{a \models c}$$

together with *contraposition* and *double negation*, we got five sound syllogisms about *universal categorical statements*.

Contraposition is negating and swapping the two parts of a sequent:

$$a \models b \quad \longrightarrow \quad \neg b \models \neg a$$

Barbara is the feminine form of the Greek βάρβαρος (barbaros) 'foreign'. Taken into Latin, it was used as the name of a mythical early Christian martyr, daughter of a pagan (barbarian).

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$$\frac{\text{In Scotland} \qquad \text{Time between 10h and 22h}}{\text{Can legally buy alcohol}}$$

In England you can buy alcohol at any time. In some countries you can't buy it (legally) at all.

The predicates have an implicit argument combining an adult, a time, and a place.

This rule is not *sound*, it just holds in some universes – it's a rule of law, not logic!

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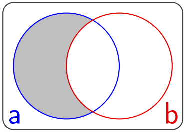
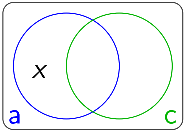
every *a* is *b*



$a \vDash b$

some *a* is not *c*



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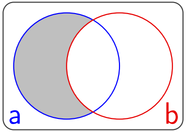

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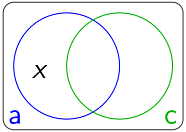


$a \nvDash c$

Why does contraposition work between the conclusion and *one* premise at a time?

What is the relation between the two premises?

Can we combine them into one? What happens with contraposition then?

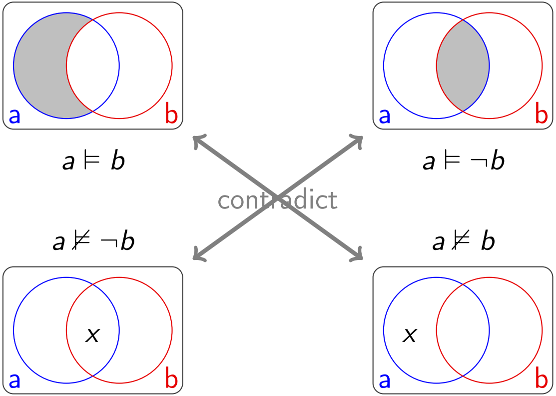
What is the difference between $a \vDash b$ and $a \to b$?

These two syllogisms are *bocardo* and *baroco*.

Aristotle's categorical propositions

universal affirmative
every *a* is *b*

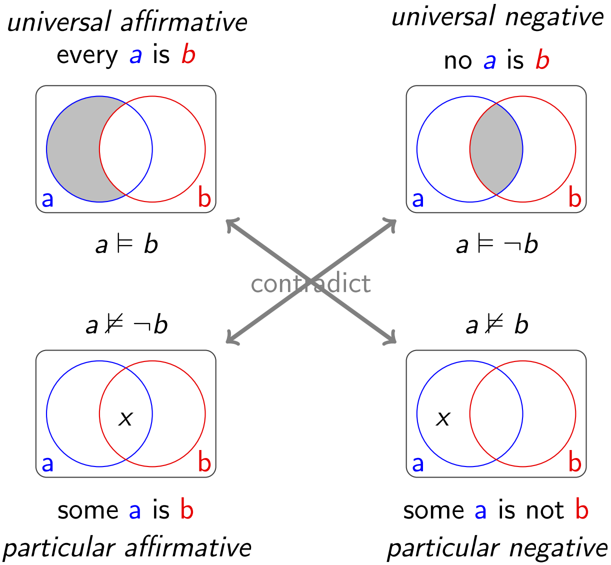
universal negative
no *a* is *b*



some a is b
particular affirmative

some a is not b
particular negative

(not exactly) Aristotle's categorical propositions



Why not exactly?

Aristotle made the *existential assumption*: if you say 'all *a* are *b*', or 'no *a* is *b*', that means that some *a* exists. So for him, universal affirmative implies particular affirmative, and universal negative implies particular negative.

Checking syllogisms (again)

This syllogism comes from *barbara* via contrapositions (do that!), so it should be sound. Check with Venn diagrams:

$$\frac{c \vDash \neg b \qquad a \nvDash \neg c}{a \nvDash b}$$

No mathematician is infallible
Some programmers are mathematicians
$\therefore$ *Some programmers are fallible*

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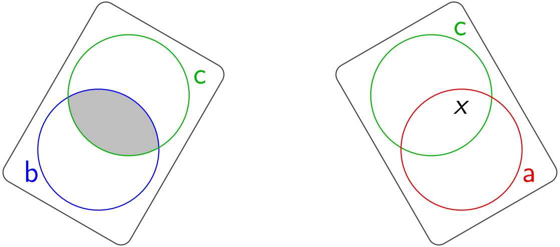

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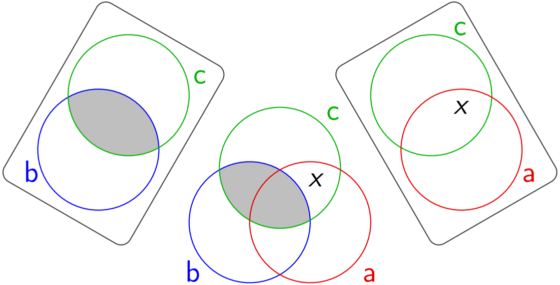

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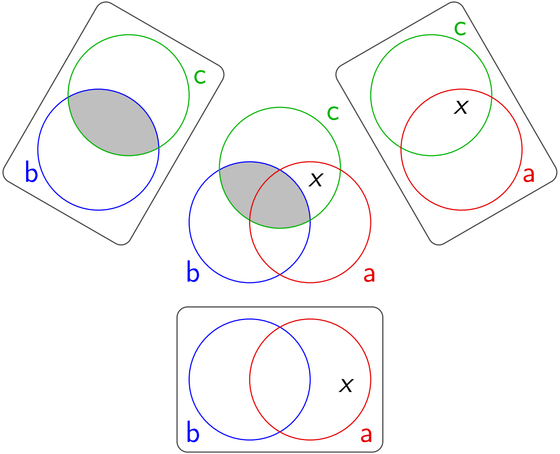

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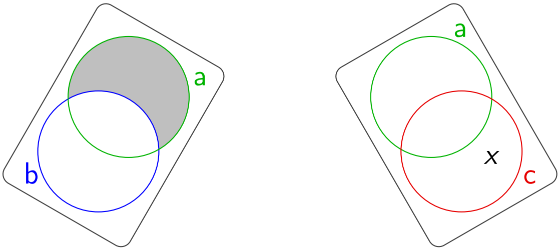

$$\frac{a \vDash b \qquad c \nvDash a}{c \nvDash b}$$

All plants are fungi
Some flowers are not plants
∴ Some flowers are not fungi

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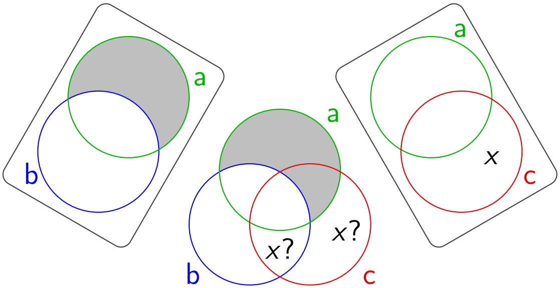

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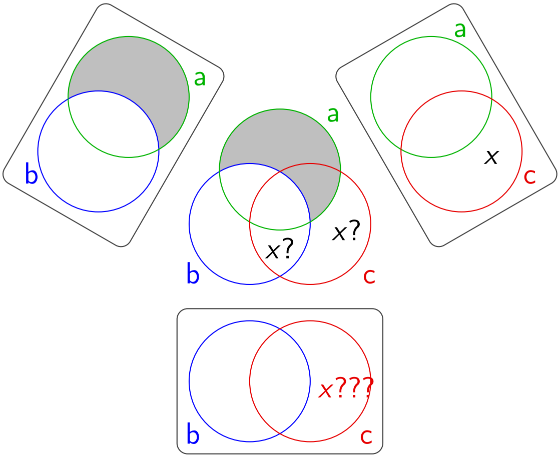

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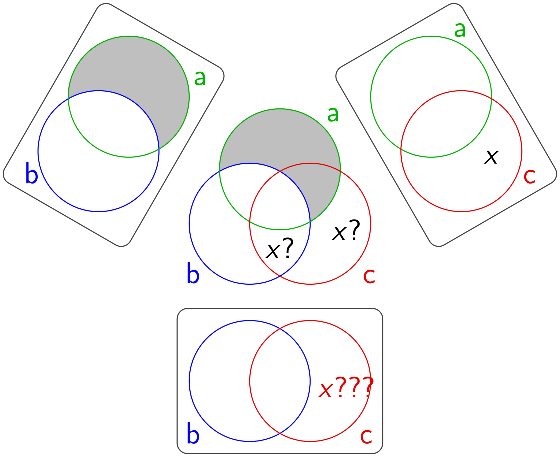

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In the usual meanings, no plant is a fungus, and all flowers are plants. That doesn't matter: the argument doesn't depend on the truth or falsity of the premises in a particular universe.

Suppose a *quonce* is a fungus flower, but not a plant, and nothing else exists. This disproves the syllogism.

Deriving rules

We have used the following to derive sound rules from *barbara*:

- substitution (e.g. q for a, $\neg b$ for b)
- double negation cancellation ($\neg\neg a \longrightarrow a$)
- contraposition within sequents ($a \vDash b \longrightarrow \neg b \vDash \neg a$)
- contraposition between conclusion and a premise

$$\frac{a \vDash b \qquad b \vDash c}{a \vDash c} \longrightarrow \frac{a \vDash b \qquad a \nvDash c}{b \nvDash c}$$

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Because these processes are symmetrical, they also derive unsound rules from unsound rules.

All the sound syllogisms

$$\frac{a \models b \quad b \models c}{a \models c} \qquad \frac{a \models b \quad a \not\models c}{b \not\models c} \qquad \frac{b \models c \quad a \not\models c}{a \not\models b}$$

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You can derive all these. Mediaeval students learned them, with the help of this verse:

Barbara celarent darii ferio baralipton
Celantes dabitis fapesmo frisesomorum
Cesare camestres festino baroco
Darapti felapton disamis datisi bocardo ferison

Reprise

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- All sound syllogisms come from *barbara* via contraposition etc.

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It's all much murkier than this. This existential assumption contradicts other aspects of Aristotle's system. In short, he was most likely confused.

D. W. Mulder, The existential assumptions of traditional logic, *Hist. & Phil. Logic*, 17:1-2, 141–154