Informatics 1
Introduction to Computation
Lecture 16

Search in Trees

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Binary trees

```
data Tree a = Nil | Node (Tree a) a (Tree a)

t :: Tree Int
t = Node (Node (Node Nil 4 Nil)
               2
               (Node Nil 5 Nil))
         1
         (Node (Node (Node Nil 8 Nil)
                     6
                     (Node Nil 9 Nil))
               3
               (Node Nil 7 Nil))
```

Binary trees

```
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t = Node (Node (Node Nil 4 Nil)
               2
               (Node Nil 5 Nil))
         1
         (Node (Node (Node Nil 8 Nil)
                     6
                     (Node Nil 9 Nil))
               3
               (Node Nil 7 Nil))
```

```
        1
      /   \
    /      \
   2         3
 /   \     /   \
4     5   6     7
         / \
        8   9
```

Binary trees

```
inf :: Tree Int
inf = inffrom 0
    where
        inffrom x = Node (inffrom (x-1)) x (inffrom (x+1))
```

Binary trees

```
inf :: Tree Int
inf = inffrom 0
    where
        inffrom x = Node (inffrom (x-1)) x (inffrom (x+1))
```

```
                   0
                /     \
             /           \
          -1               1
        /    \           /   \
      -2      0         0     2
      / \    / \       / \   / \
    -3  -1 -1   1    -1   1 1   3
    /\  /\ /\  /\    /\  /\ /\  /\
   ... ... ... ...  ... ... ... ...
```

Depth-First Search

```
depthFirst :: Eq a => (a -> Bool) -> Tree a -> Maybe a
depthFirst p Nil = Nothing
depthFirst p (Node t1 x t2)
             | p x                          = Just x
             | depthFirst p t1 == Nothing = depthFirst p t2
             | otherwise                    = depthFirst p t1
```

Depth-First Search

```
depthFirst :: Eq a => (a -> Bool) -> Tree a -> Maybe a
depthFirst p Nil = Nothing
depthFirst p (Node t1 x t2)
              | p x                        = Just x
              | depthFirst p t1 == Nothing = depthFirst p t2
              | otherwise                  = depthFirst p t1

depthFirst (>4)  1
                / \
               2   ...
              / \
             4   5

= depthFirst (>4)  2
                  / \
                 4   5

= depthFirst (>4)  5    [since depthFirst (>4) 4 = Nothing]

= Just 5
```

Depth-First Search

```
depthFirst :: Eq a => (a -> Bool) -> Tree a -> Maybe a
depthFirst p Nil = Nothing
depthFirst p (Node t1 x t2)
              | p x                         = Just x
              | depthFirst p t1 == Nothing  = depthFirst p t2
              | otherwise                   = depthFirst p t1
```

```
df_traverse :: Tree a -> [a]
df_traverse Nil = []
df_traverse (Node t1 x t2)
    = x : (df_traverse t1) ++ (df_traverse t2)

depthFirst’ :: Eq a => (a -> Bool) -> Tree a -> Maybe a
depthFirst’ p t
    = head( [Just x | x <- df_traverse t, p x] ++ [Nothing] )
```

Depth-First Traverse

```
df_traverse :: Tree a -> [a]
df_traverse Nil = []
df_traverse (Node t1 x t2)
    = x : (df_traverse t1) ++ (df_traverse t2)
```

```
df_traverse   1
            /   \
          2      3
         / \    / \
        4   5 ... ...

= 1 : df_traverse   2   ++   df_traverse   3
                   / \                    / \
                  4   5                 ... ...

= 1 : 2 : df_traverse 4 ++ df_traverse 5  ++   df_traverse   3
                                                            / \
                                                          ... ...

= 1 : 2 : 4 : [] ++ [] ++ 5 : [] ++ []   ++   df_traverse   3
                                                           / \
                                                         ... ...

= [1, 2, 4, 5, ...]
```

Depth-First Search

```
depthFirst' :: Eq a => (a -> Bool) -> Tree a -> Maybe a
depthFirst' p t
    = head( [Just x | x <- df_traverse t, p x] ++ [Nothing] )


depthFirst' (>4)   1
                 /   \
               2       3
              / \     / \
             4   5  ... ...

= head( [Just x | x <- df_traverse   1   , x>4] ++ [Nothing] )
                                   /   \
                                 2       3
                                / \     / \
                               4   5  ... ...

= head( [Just x | x <- [1, 2, 4, 5, ...], x>4] ++ [Nothing] )

= head( [Just 5, ...] ++ [Nothing] )

= Just 5
```

Depth-First vs Breadth-First

```
df_traverse :: Tree a -> [a]
df_traverse Nil = []
df_traverse (Node t1 x t2)
    = x : (df_traverse t1) ++ (df_traverse t2)
```

```
bf_traverse :: Tree a -> [a]
bf_traverse t = bft [t]
  where
   bft [] = []
   bft xs = [x | Node _ x _ <- xs]
               ++ bft (concat [ [t1,t2] | Node t1 _ t2 <- xs ])
```

Breadth-First Search

```
bf_traverse :: Tree a -> [a]
bf_traverse t = bft [t]
  where
   bft [] = []
   bft xs = [x | Node _ x _ <- xs]
                 ++ bft (concat [ [t1,t2] | Node t1 _ t2 <- xs ])

breadthFirst :: (a -> Bool) -> Tree a -> Maybe a
breadthFirst p t
    = head( [Just x | x <- bf_traverse t, p x] ++ [Nothing] )
```

Breadth-First Traverse

```
bf_traverse :: Tree a -> [a]
bf_traverse t = bft [t]
  where
   bft [] = []
   bft xs = [x | Node _ x _ <- xs]
                 ++ bft (concat [ [t1,t2] | Node t1 _ t2 <- xs ])
```

```
bf_traverse   1                          = bft  [        1        ]
            /   \                                      /   \
          2       3                                  2       3
         / \     / \                                / \     / \
        4   5   6   7                              4   5   6   7
               / \                                        / \
             ... ...                                    ... ...

= [1] ++ bft (concat [[  2   ,   3   ]]  =  [1] ++ bft [  2   ,   3   ]
                        / \     / \                      / \     / \
                       4   5   6   7                    4   5   6   7
                              / \                              / \
                            ... ...                          ... ...
```

Breadth-First Traverse

```
bf_traverse :: Tree a -> [a]
bf_traverse t = bft [t]
  where
   bft [] = []
   bft xs = [x | Node _ x _ <- xs]
                 ++ bft (concat [ [t1,t2] | Node t1 _ t2 <- xs ])
```

```
...  =  [1] ++ bft [  2   ,   3   ]
                     / \     / \
                    4   5   6   7
                           / \
                         ... ...

= [1] ++ [2,3] ++ bft (concat [[ 4 , 5 ], [ 6 , 7 ]])
                                           / \
                                         ... ...

= [1] ++ [2,3] ++ bft [ 4 , 5 , 6 , 7 ]
                               / \
                             ... ...

= [1] ++ [2,3] ++ [4,5,6,7] ++ ...

= [1, 2, 3, 4, 5, 6, 7, ...]
```

Breadth-First Search

```
breadthFirst :: (a -> Bool) -> Tree a -> Maybe a
breadthFirst p t
    = head( [Just x | x <- bf_traverse t, p x] ++ [Nothing] )
```

```
breadthFirst (>4)    1
                   /   \
                 2       3
                / \     / \
               4   5  ... ...

= head( [Just x | x <- bf_traverse   1   , x>4] ++ [Nothing] )
                                   /   \
                                 2       3
                                / \     / \
                               4   5  ... ...

= head( [Just x | x <- [1, 2, 3, 4, 5, 6, 7, ...], x>4] ++ [Nothing] )

= head( [Just 5, Just 6, Just 7, ...] ++ [Nothing] )

= Just 5
```

Depth-First Search vs Breadth-First Search

```
depthFirst (>4)    1
                 /   \
               2       3
              / \     / \
             4   5   ... ...

= Just 5


breadthFirst (>4)    1
                   /   \
                 2       3
                / \     / \
               4   5   ... ...

= Just 5
```

Depth-First Search vs Breadth-First Search

```
depthFirst (>2)     1
                  /   \
                2       3
               / \     / \
              4   5  ... ...

= Just 4
```

```
breadthFirst (>2)     1
                    /   \
                  2       3
                 / \     / \
                4   5  ... ...

= Just 3
```

Infinite case

```
> depthFirst (>0) inf
*** Exception: stack overflow

> depthFirst' (>0) inf
[runs forever]

> breadthFirst (>0) inf
Just 1
(0.01 secs, 93,200 bytes)
```

```
                0
             /     \
          /           \
       -1               1
     /    \           /   \
   -2      0         0     2
   / \    / \       / \   / \
 -3  -1 -1   1    -1   1 1   3
 /\  /\ /\  /\    /\  /\ /\  /\
... ... ... ...  ... ... ... ...
```

Best-First Search

Idea: Decide the order of nodes to visit using an evaluation function.

```
bestFirst :: (a -> Bool) -> (Tree a -> Int)
                                 -> Tree a -> Maybe a
bestFirst p f t = bfs p (insert t (empty f))

bfs :: (a -> Bool) -> PQ (Tree a) -> Maybe a
bfs p pq | isempty pq = Nothing
         | otherwise = if p x then Just x
                       else bfs p (insertnode t1
                                    (insertnode t2 pq'))
               where Node t1 x t2 = top pq
                     pq'          = pop pq

insertnode :: Tree a -> PQ (Tree a) -> PQ (Tree a)
insertnode Nil pq = pq
insertnode t pq   = insert t pq
```

Uses a priority queue with functions empty, insert, top, pop, isempty

Priority Queue

```
data PQ a = MkPQ (a->Int) [a]

invariant :: PQ a -> Bool
invariant (MkPQ f xs)   =
  and [ f x >= f y | (x,y) <- zip xs (tail xs) ]
     -- in descending order of evaluation function results

empty :: (a->Int) -> PQ a
empty f =  MkPQ f []

insert :: a -> PQ a -> PQ a
insert x (MkPQ f ys)   =  MkPQ f (ins x ys)
  where ins x []                        =   [x]
        ins x (y:ys) | f x >= f y  =   x : y : ys
                     | f x < f y   =   y : ins x ys

top :: PQ a -> a      -- return item with highest priority
top (MkPQ _ (x:xs)) = x

pop :: PQ a -> PQ a  -- remove item with highest priority
pop (MkPQ f (x:xs)) = MkPQ f xs

isempty :: PQ a -> Bool
isempty (MkPQ f xs) = null xs
```

Best-First Search

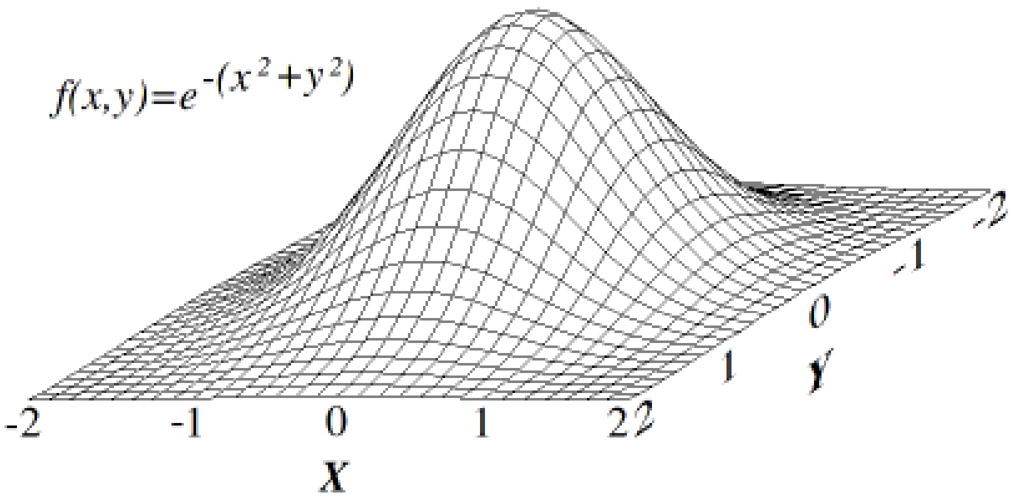
```
eval :: Tree Int -> Int
eval Nil = 0
eval (Node t1 x t2) = x

BreadthFirst> breadthFirst (>19) inf
Just 20
(9.72 secs, 1,031,895,824 bytes)
required examining 2097151 nodes

BestFirst> bestFirst (>19) eval inf
Just 20
(0.02 secs, 156,592 bytes)
required examining 21 nodes
```

Best-First Search

```
eval :: Tree Int -> Int
eval Nil = 0
eval (Node t1 x t2) = x

BreadthFirst> breadthFirst (>19) inf
Just 20
(9.72 secs, 1,031,895,824 bytes)
required examining 2097151 nodes

BestFirst> bestFirst (>19) eval inf
Just 20
(0.02 secs, 156,592 bytes)
required examining 21 nodes

> breadthFirst (>100) inf
[requires examining 5070602400912917605986812821503 nodes]

BestFirst> bestFirst (>100) eval inf
Just 101
(0.01 secs, 433,256 bytes)
required examining 101 nodes
```

A Limitation of Best-First Search

Local maximum is global maximum:



Local maximum which is not global maximum: