Sanitized syllogistic reasoning

Given a universe X and predicates a, b, we put

$a \vDash b \quad \longleftrightarrow \quad \forall x \in X.a(x) \to b(x).$

A categorical proposition $\Phi(a, b)$ is one of $a \vDash b$, $a \vDash \neg b$, $a \nvDash \neg b$, $a \nvDash b$, or $\Phi(b, a)$.

All Greeks are human
All humans are mortal
$\therefore$ All Greeks are mortal

All lions are animals
Some lion is fierce
$\therefore$ Some animal is fierce

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A syllogism is a rule of the form $\dfrac{\Phi_1(a, b) \qquad \Phi_2(b, c)}{\Phi_3(a, c)}$

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Given a universe X and predicates a, b, we put

$a \models b \quad \longleftrightarrow \quad \forall x \in X . a(x) \to b(x)$.

A categorical proposition $\Phi(a, b)$ is one of $a \models b$, $a \models \neg b$, $a \not\models \neg b$, $a \not\models b$, or $\Phi(b, a)$.

A syllogism is a rule of the form $\dfrac{\Phi_1(a, b) \qquad \Phi_2(b, c)}{\Phi_3(a, c)}$

All sound syllogisms can be derived from $\dfrac{a \models b \quad b \models c}{a \models c}$ by applying

- renaming of predicates by (negated) predicates
- double negation cancellation $\neg\neg a \quad \longleftrightarrow \quad a$
- contraposition of a sequent: $\Phi \models \Psi \quad \longleftrightarrow \quad \neg\Psi \models \neg\Phi$
- contraposition of the rule: $\dfrac{\Phi \quad \Psi}{\Upsilon} \quad \longleftrightarrow \quad \dfrac{\Phi \quad \neg\Upsilon}{\neg\Psi}$

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Aristotle's syllogisms

Categorical propositions (with mediaeval abbreviations) are:

Aab universal affirmative: *a* holds of every *b* (every *b* is *a*)

Eab universal negative: *a* holds of no *b* (no *b* is *a*)

Iab particular affirmative: *a* holds of some *b* (some *b* is *a*)

Oab particular negative: *a* fails of some *b* (some *b* is not *a*)

The *a*, *b* are called terms. *a* is the predicate and *b* the subject of the proposition.

Handy mnemonic for abbreviations:

AffIrmo 'I affirm'

nEgO 'I deny'

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The *a*, *b* are called terms. *a* is the predicate and *b* the subject of the proposition.

Aristotle was not interested in unicorns: mentioning a term *a* implies that some *a* exists. All terms are inhabited! (The existential assumption.)

Note that this means *Aab* and *Oab* are **not** negatives of each other – something that caused 2000 years of argument.

Figures

A figure is an argument comprising two premise propositions and a conclusion proposition, such that one premise (the major) contains the predicate of the conclusion (the major term) and another middle term, and the other premise (the minor) contains the subject of the conclusion (the minor term) and the middle term.

All humans$^{\text{subj,mid}}$ are mortal$^{\text{pred,maj}}$ *major*
All Greeks$^{\text{subj,min}}$ are human$^{\text{pred,mid}}$ *minor*
∴ All Greeks$^{\text{subj,minor}}$ are mortal$^{\text{pred,major}}$

Amh, Ahg, $\therefore Amg$

Figures

A figure is an argument comprising two premise propositions and a conclusion proposition, such that one premise (the major) contains the predicate of the conclusion (the major term) and another middle term, and the other premise (the minor) contains the subject of the conclusion (the minor term) and the middle term.

Figures are of three(four) kinds:

First	$?ab$, $?bc$, $\therefore$ $?ac$
Second	$?ab$, $?ac$, $\therefore$ $?bc$
Third	$?ac$, $?bc$, $\therefore$ $?ab$
Fourth	$?ba$, $?cb$, $\therefore$ $?ac$, but Aristotle treated these under the First.

All humans$^{\text{subj,mid}}$ are mortal$^{\text{pred,maj}}$ *major*
All Greeks$^{\text{subj,min}}$ are human$^{\text{pred,mid}}$ *minor*
$\therefore$ All Greeks$^{\text{subj,minor}}$ are mortal$^{\text{pred,major}}$
Amh, Ahg, $\therefore$ Amg

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A sound figure is a syllogism. Aristotle took the First Figures to be self-evidently sound or unsound. The others were proved by conversions ($Aab \rightarrow Iba$, $Iab \leftrightarrow Iba$, $Eab \leftrightarrow Eba$), contradiction, and a dodgy argument called *ekthesis*, or disproved by counter-example.

Mediaeval syllogisms

Mediaeval logicians (Avicenna, Boethius, Peter Abelard, William of Ockham, John Buridan et al.) refined, developed and extended the theory (including flipping the order from 'Pred belongs to Subj' to 'Subj is Pred').

Buridan in particular developed Aristotle's modal logic (syllogisms with necessity and possibility) from something almost entirely incoherent to something coherent, and probably $S5$.

Mediaeval mnemonics

Mediaeval logic students understandably found it difficult to learn this stuff, and used mnemonics:

Barbara celarent darii ferio baralipton
Celantes dabitis fapesmo frisesomorum
Cesare camestres festino baroco
Darapti felapton disamis datisi bocardo ferison

Each word names a syllogism and reminds you what it is and how it is derived.

Unpacking *barbara celarent*

A univ affirm
E univ neg
I part affirm
O part neg

The first three vowels tell you the proposition forms.

Unpacking *barbara celarent*

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The first three vowels tell you the proposition forms.

The first letter labels the four sound First Figures:

Barbara	$Aab, Abc, \therefore\ Aac$
Celarent	$Eab, Abc, \therefore\ Eac$
Darii	$Aab, Ibc, \therefore\ Iac$
Ferio	$Eab, Ibc, \therefore\ Oac$

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Ferio	$Eab, Ibc, \therefore\ Oac$

Some letters show conversions of the preceding proposition:

P instantiate Abc to Icb

daraPti $Aac, Abc \rightarrow Aac, Icb, \therefore\ Iab$ (*darii*)

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S swap subj/pred in E or I

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Some letters show conversions of the preceding proposition:

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daraPti Aac, $Abc \rightarrow Aac$, Icb, $\therefore$ Iab (*darii*)

S swap subj/pred in E or I

datiSi Aac, $Ibc \rightarrow Aac$, Icb, $\therefore$ Iab (*darii*)

M swap premises

caMestres Aab, $Eac \rightarrow_s Aab$, $Eca =_m Eca$, Aab, $\therefore$ Ecb (*celarent*) $\rightarrow_s Ebc$

Unpacking *barbara celarent*

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The first three vowels tell you the proposition forms.

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Some letters show conversions of the preceding proposition:

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daraPti $Aac, Abc \to Aac, Icb, \therefore\ Iab$ (*darii*)

S swap subj/pred in E or I
datiSi $Aac, Ibc \to Aac, Icb, \therefore\ Iab$ (*darii*)

M swap premises
caMestres $Aab, Eac \to_s Aab, Eca =_m Eca, Aab, \therefore Ecb$ (*celarent*) $\to_s Ebc$

C contrapose premise and conclusion
baroCo $Aab, Oac \therefore Obc \leftrightarrow_c Aab, Abc, \therefore Aac$ (*barbara*)

Syllogistic music

Jacobus Gallus (1550–1591) was a Slovene composer and organist. As well as hundreds of religious motets, he wrote many secular madrigals.

Here is Gallus' madrigal *Barbara celarent* sung by the Czech early music group Societas Incognitorum.

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