Sanitized syllogistic reasoning

Given a universe $X$ and predicates $a, b$, we put $a \models b \iff \forall x \in X. a(x) \to b(x)$.

A categorical proposition $\Phi(a, b)$ is one of $a \models b$, $a \models \neg b$, $a \not\models \neg b$, $a \not\models b$, or $\Phi(b, a)$.

All Greeks are human
All humans are mortal
∴ All Greeks are mortal
All lions are animals
Some lion is fierce
∴ Some animal is fierce
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A syllogism is a rule of the form

$$\frac{\Phi_1(a, b) \quad \Phi_2(b, c)}{\Phi_3(a, c)}$$

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A syllogism is a rule of the form

$$\begin{array}{c}
\Phi_1(a, b) \\
\Phi_2(b, c) \\
\Phi_3(a, c)
\end{array} \leadsto \begin{array}{c}
a \models b \\
b \models c \\
a \models c
\end{array}$$

All sound syllogisms can be derived from $a \models b$, $b \models c$ by applying $a \models c$ by applying

- renaming of predicates by (negated) predicates
- double negation cancellation $\neg \neg a \iff a$
- contraposition of a sequent: $\Phi \models \psi \iff \neg \psi \models \neg \Phi$
- contraposition of the rule: $\begin{array}{c}
\Phi \\
\psi
\end{array} \leadsto \begin{array}{c}
\gamma \\
\neg \psi
\end{array}$

All Greeks are human
All humans are mortal
∴ All Greeks are mortal
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∴ Some animal is fierce
Aristotle’s syllogisms

Categorical propositions (with mediaeval abbreviations) are:

- **Aab** universal affirmative: *a* holds of every *b* (every *b* is *a*)
- **Eab** universal negative: *a* holds of no *b* (no *b* is *a*)
- **Iab** particular affirmative: *a* holds of some *b* (some *b* is *a*)
- **Oab** particular negative: *a* fails of some *b* (some *b* is not *a*)

The *a*, *b* are called terms. *a* is the predicate and *b* the subject of the proposition.

Handy mnemonic for abbreviations:

- **Affrmo** ‘I affirm’
- **nEgO** ‘I deny’
Aristotle’s syllogisms

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The *a*, *b* are called terms. *a* is the predicate and *b* the subject of the proposition.

Aristotle was not interested in unicorns: mentioning a term *a* implies that some *a* exists. All terms are inhabited! (The existential assumption.)

Note that this means Aab and Oab are not negatives of each other – something that caused 2000 years of argument.
Figures

A figure is an argument comprising two premise propositions and a conclusion proposition, such that one premise (the major) contains the predicate of the conclusion (the major term) and another middle term, and the other premise (the minor) contains the subject of the conclusion (the minor term) and the middle term.

All humans[^1] are mortal[^2].

All Greeks[^3] are human[^4].

∴ All Greeks[^5] are mortal[^6].

Amh, Ahg, ∴ Amg
A figure is an argument comprising two premise propositions and a conclusion proposition, such that one premise (the major) contains the predicate of the conclusion (the major term) and another middle term, and the other premise (the minor) contains the subject of the conclusion (the minor term) and the middle term.

Figures are of three(four) kinds:

First  \( ?ab, ?bc, \therefore ?ac \)
Second \( ?ab, ?ac, \therefore ?bc \)
Third \( ?ac, ?bc, \therefore ?ab \)
Fourth \( ?ba, ?cb, \therefore ?ac \), but Aristotle treated these under the First. \( Amh, Ahg, \therefore Amg \)

All humans\textsuperscript{subj,mid} are mortal\textsuperscript{pred,maj}

**major**

All Greeks\textsuperscript{subj,min} are human\textsuperscript{pred,mid} **minor**

\( \therefore \) All Greeks\textsuperscript{subj,minor} are mortal\textsuperscript{pred,major} \( Amh, Ahg, \therefore Amg \)
A figure is an argument comprising two premise propositions and a conclusion proposition, such that one premise (the major) contains the predicate of the conclusion (the major term) and another middle term, and the other premise (the minor) contains the subject of the conclusion (the minor term) and the middle term. Figures are of three(four) kinds:

First $\sim ab, \sim bc, \therefore \sim ac$

Second $\sim ab, \sim ac, \therefore \sim bc$

Third $\sim ac, \sim bc, \therefore \sim ab$

Fourth $\sim ba, \sim cb, \therefore \sim ac$, but Aristotle treated these under the First. $\therefore$ All Greeks$^\text{subj,minor}$ are mortal$^\text{pred,major}$

A sound figure is a syllogism. Aristotle took the First Figures to be self-evidently sound or unsound. The others were proved by conversions ($Aab \rightarrow Iba, lab \leftrightarrow lba, Eab \leftrightarrow Eba$), contradiction, and a dodgy argument called ekthesis, or disproved by counter-example.
Mediaeval syllogisms

Mediaeval logicians (Avicenna, Boethius, Peter Abelard, William of Ockham, John Buridan et al.) refined, developed and extended the theory (including flipping the order from 'Pred belongs to Subj' to 'Subj is Pred').

Buridan in particular developed Aristotle's modal logic (syllogisms with necessity and possibility) from something almost entirely incoherent to something coherent, and probably S5.
Mediaeval mnemonics

Mediaeval logic students understandably found it difficult to learn this stuff, and used mnemonics:

*Barbara celarent darii ferio baralipton*
*Celantes dabitis fapesmo frisesomorum*
*Cesare camestres festino baroco*
*Darapti felapton disamis datisi bocardo ferison*

Each word names a syllogism and reminds you what it is and how it is derived.
Unpacking *barbara* *celarent*

The first three vowels tell you the proposition forms.

- **A** univ affirm
- **E** univ neg
- **I** part affirm
- **O** part neg
The first three vowels tell you the proposition forms.
The first letter labels the four sound First Figures:

- **Barbara**  \( Aab, Abc, \therefore Aac \)
- **Celarent**  \( Eab, Abc, \therefore Eac \)
- **Darii**  \( Aab, Ibc, \therefore Iac \)
- **Ferio**  \( Eab, Ibc, \therefore Oac \)
Unpacking *barbara* *celarent*

The first three vowels tell you the proposition forms.
The first letter labels the four sound First Figures:

- **Barbara**  \(Aab, Abc, \therefore Aac\)
- **Celarent**  \(Eab, Abc, \therefore Eac\)
- **Darii**  \(Aab, lbc, \therefore lac\)
- **Ferio**  \(Eab, lbc, \therefore Oac\)

Some letters show conversions of the preceding proposition:

- **P**  instantiate \(Abc\) to \(lcb\)
  
  *daraP*  \(Aac, Abc \rightarrow Aac, lcb, \therefore lab\)  *(darii)*
Unpacking *barbara* and *celarent*

The first three vowels tell you the proposition forms.
The first letter labels the four sound First Figures:

- **Barbara**: $Aab, Abc, \therefore Aac$
- **Celarent**: $Eab, Abc, \therefore Eac$
- **Darii**: $Aab, lbc, \therefore lac$
- **Ferio**: $Eab, lbc, \therefore Oac$

Some letters show conversions of the preceding proposition:

- **P**: instantiate $Abc$ to $lcb$
  
  \[\text{daraPti } Aac, Abc \rightarrow Aac, lcb, \therefore lab \ (\text{darii})\]

- **S**: swap subj/pred in $E$ or $I$
  
  \[\text{datiSi } Aac, lbc \rightarrow Aac, lcb, \therefore lab \ (\text{darii})\]
Unpacking *barbara celarent*

The first three vowels tell you the proposition forms.
The first letter labels the four sound First Figures:

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**S** swap subj/pred in $E$ or $I$
\[
\text{datiSi } Aac, lbc \to Aac, lcb, \therefore lab \ (\text{darii})
\]

**M** swap premises
\[
\text{caMestres } Aab, Eac \to_s Aab, Eca =_m Eca, Aab, \therefore Ecb \ (\text{celarent}) \to_s Ebc
\]
Unpacking *barbara* *celarent*

The first three vowels tell you the proposition forms.

The first letter labels the four sound First Figures:

- **Barbara**  $Aab, Abc, \therefore Aac$
- **Celarent**  $Eab, Abc, \therefore Eac$
- **Darii**  $Aab, Ibc, \therefore Iac$
- **Ferio**  $Eab, Ibc, \therefore Oac$

Some letters show conversions of the preceding proposition:

- **P** instantiate $Abc$ to $lcb$
  
  *daraPti*  $Aac, Abc \to Aac, lcb, \therefore lab$ (darii)

- **S** swap subj/pred in $E$ or $I$
  
  *datiSi*  $Aac, Ibc \to Aac, lcb, \therefore lab$ (darii)

- **M** swap premises
  
  *caMestres*  $Aab, Eac \to_s Aab, Eca =_m Eca, Aab, \therefore Ecb$ (celarent)  $\to_s Ebc$

- **C** contrapose premise and conclusion
  
  *baroCo*  $Aab, Oac \therefore Obc \leftrightarrow_c Aab, Abc, \therefore Aac$ (barbara)
Jacobus Gallus (1550–1591) was a Slovene composer and organist. As well as hundreds of religious motets, he wrote many secular madrigals.
Here is Gallus’ madrigal *Barbara celarent* sung by the Czech early music group Societas Incognitorum.
Sources


