Inf2 - Foundations of Data Science: Logistic regression

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Announcements

- Week 4 workshop - we'll look at the paper that we'll be refer to in the exam

- Uses concepts from today's lecture!

- Solutions for Week 3 WS

A new unit:
The Maximum Likelihood Principle and Regression

Week 4: Logistic regression

Week 5: The maximum likelihood principle, and how we can use it to derive linear, logistic and other types of regression

Today

- Recap of Linear Regression

- Principle of Logistic Regression

- Interpretation of Logistic Regression coefficients

Wednesday:

- Multiple Logistic Regression

- Logistic Regression as a classifier

Inf2 - Foundations of Data Science: Recap of linear regression and classification

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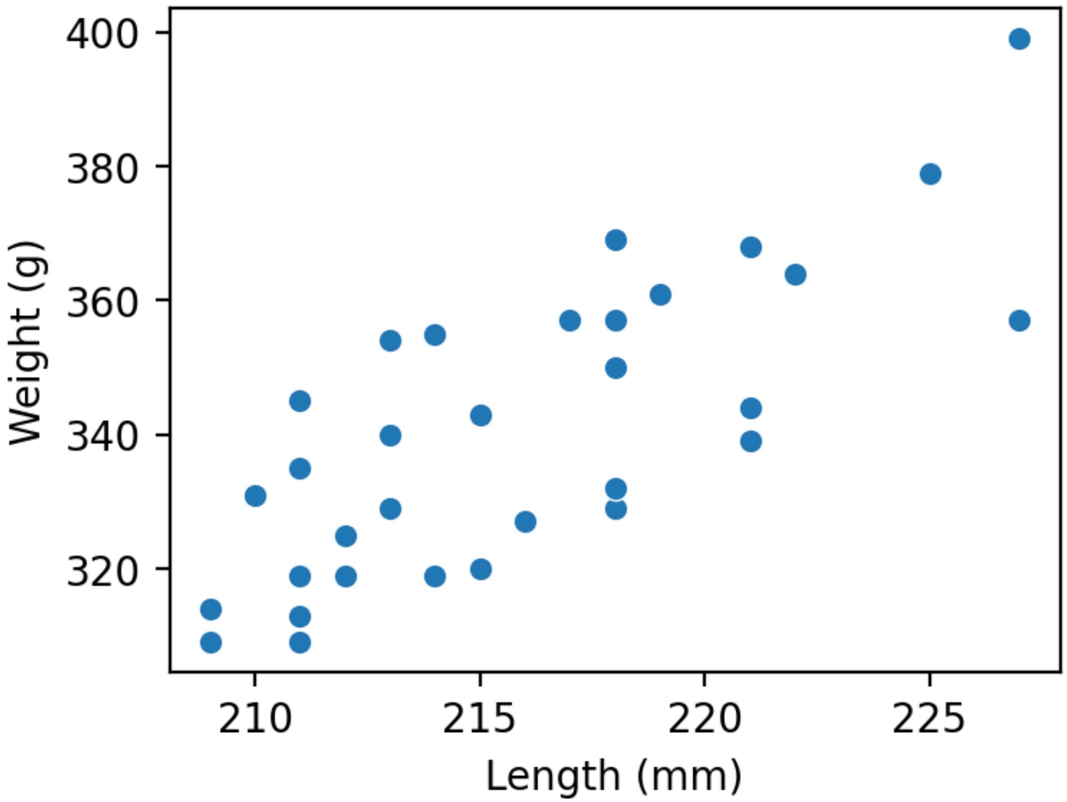
(Simple) Linear Regression

- Given numeric predictor variable, predict a numeric response variable
- Regression coefficients set by minimising sum of squared errors between data and model (can be done analytically)



Multiple regression

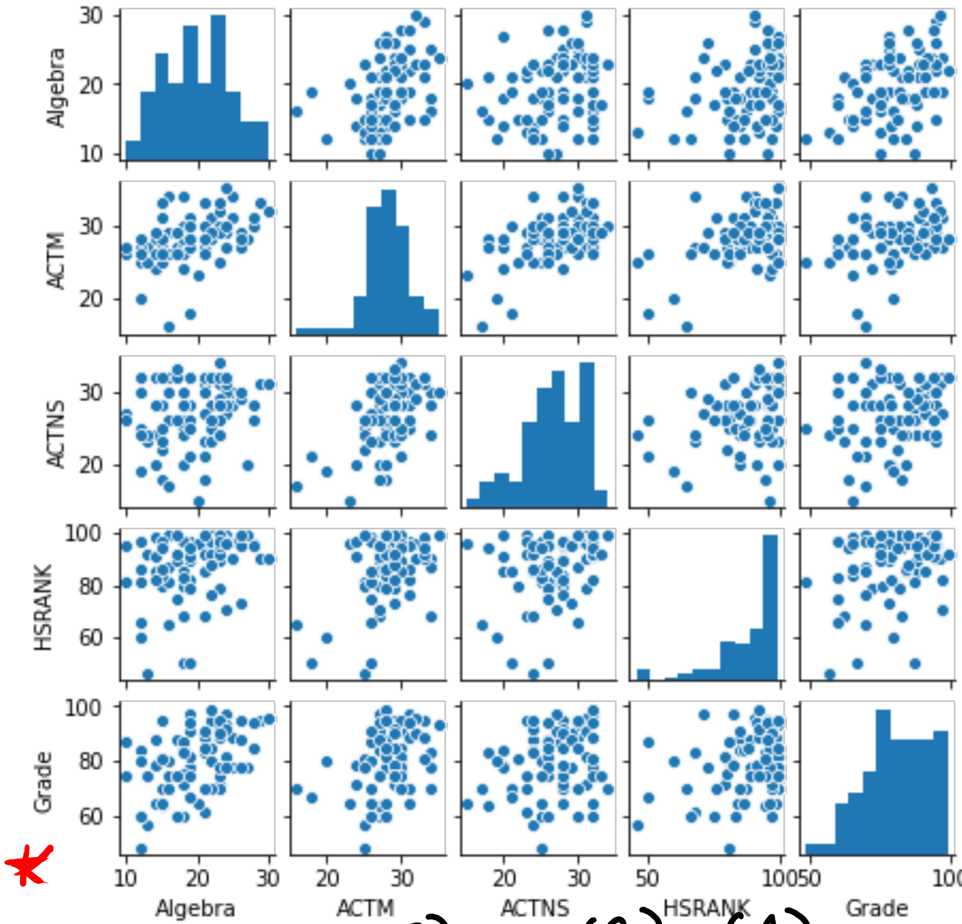
- Multiple predictors
- Categorical predictors via one-hot encoding or indicator variables
- Can be used for prediction...
- ... or explanation, including"controlling for" variables not of interest
  => can use observational data to assess effect of treatment
  on outcome

Confidence intervals and p-values of regression coefficients

Dep. Variable:	Grade	**R-squared:**	0.289
Model:	OLS	**Adj. R-squared:**	0.251
Method:	Least Squares	**F-statistic:**	7.622
Date:	Wed, 26 Oct 2022	**Prob (F-statistic):**	3.30e-05
Time:	09:42:47	**Log-Likelihood:**	-294.31
No. Observations:	80	**AIC:**	598.6
Df Residuals:	75	**BIC:**	610.5
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	36.1215	10.752	3.360	0.001	14.703	57.540
Algebra	0.9610	0.264	3.640	0.000	0.435	1.487
ACTM	0.2718	0.454	0.599	0.551	-0.632	1.175
ACTNS	0.2161	0.313	0.690	0.492	-0.408	0.840
HSRANK	0.1353	0.104	1.306	0.196	-0.071	0.342

β_1 β_2 β_1 $\hat{\sigma}_{\hat{\beta}_1}$ ***



$$T = \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}_{\hat{\beta}_1}}$$

Edge and Friedberg (1984)

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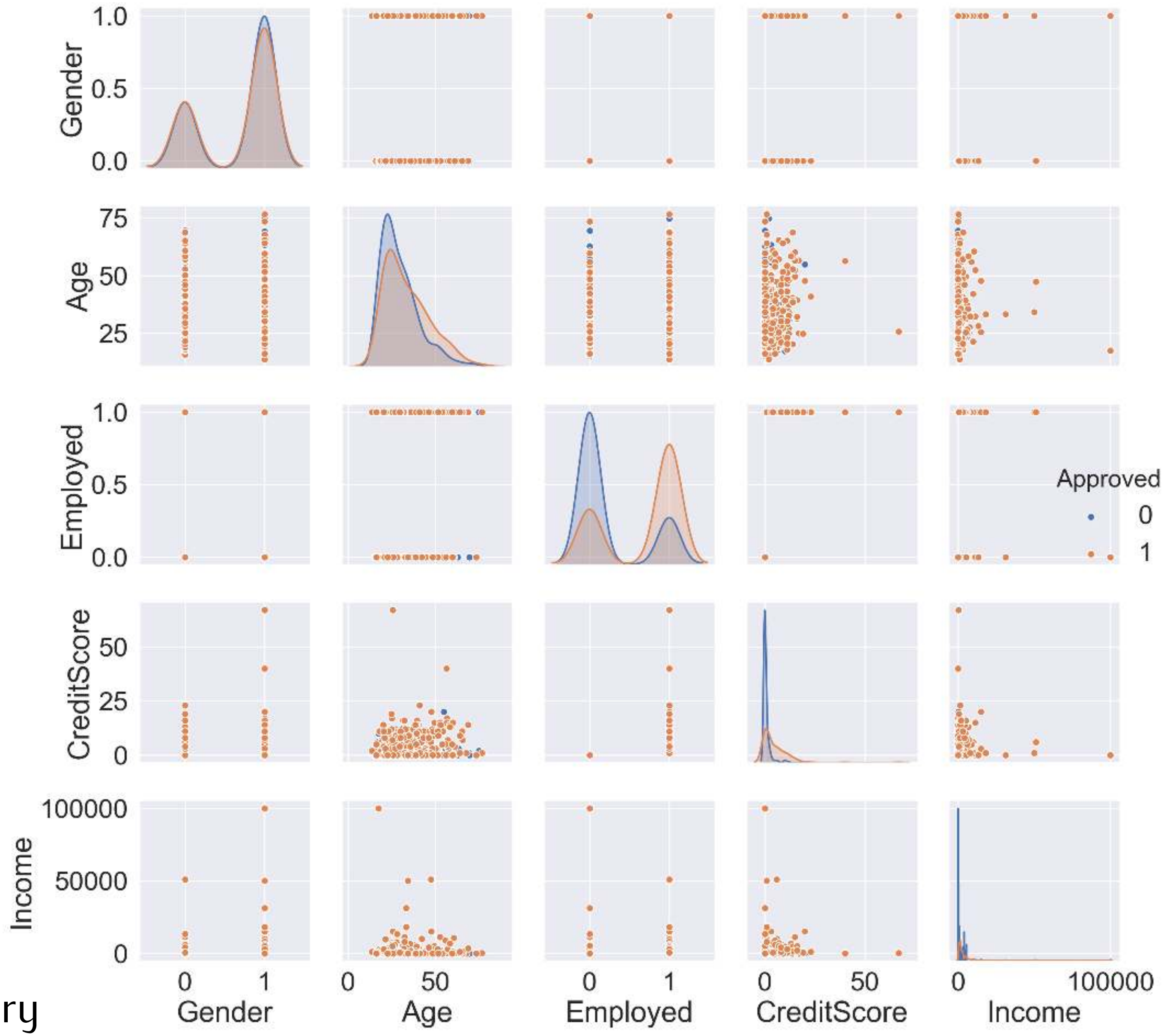
Supervised classification

Classifier predicts the label or class of an unseen point from features

Binary (or dichotomous) response variable: Credit

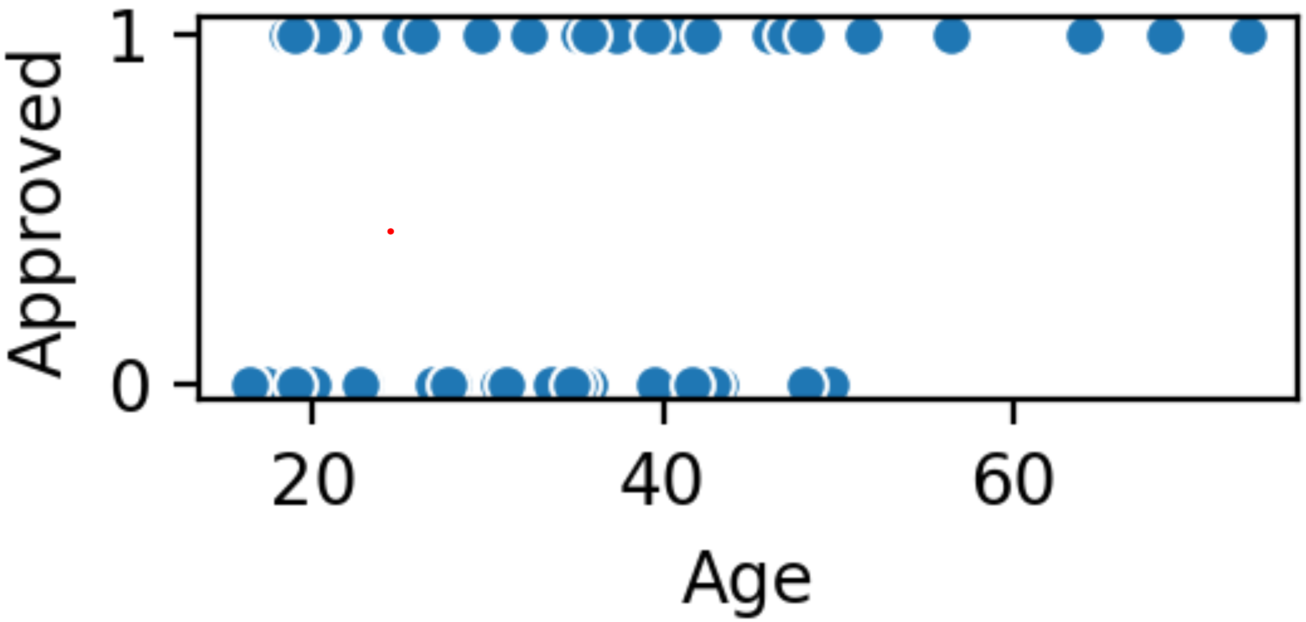
Approved

Not approved

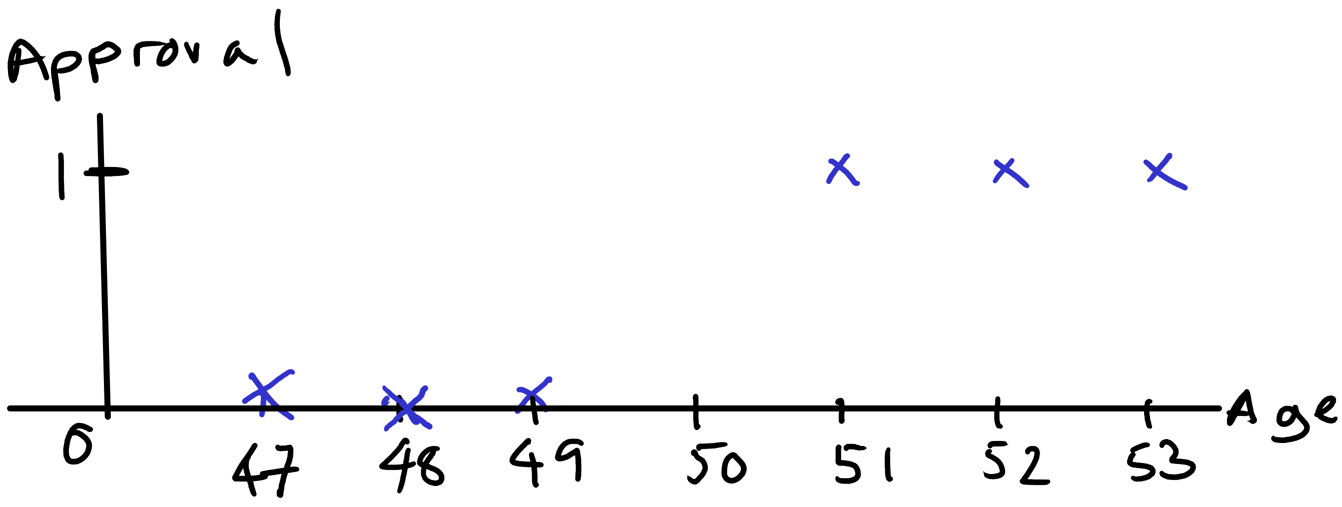


UCI Machine Learning Repository
https://archive.ics.uci.edu/dataset/27/credit+approval

Classification task on one continuous variable



Exercise :

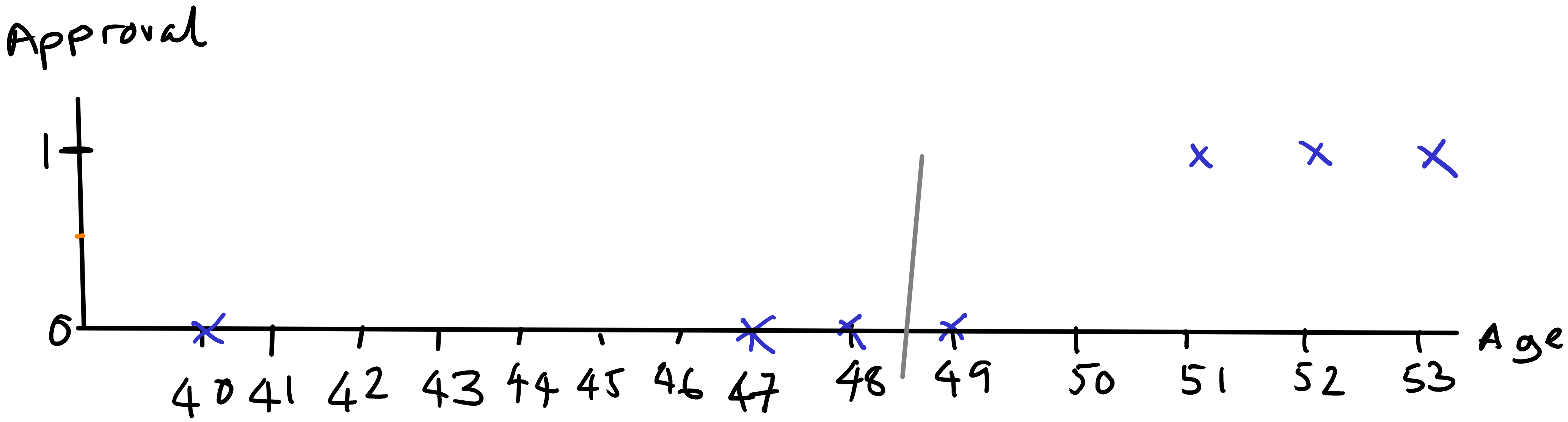


(a) Draw a Linear regression line through this toy data

(b) Convert the linear regression prediction to a predicted class label (0/1)

(c) Where is the decision boundary ?

(d) How many classification errors are there ?



(a) Draw Linear regression line through this toy data

(b) Convert the linear regression prediction to a predicted class label (0/1)

(c) Where is the decision boundary?

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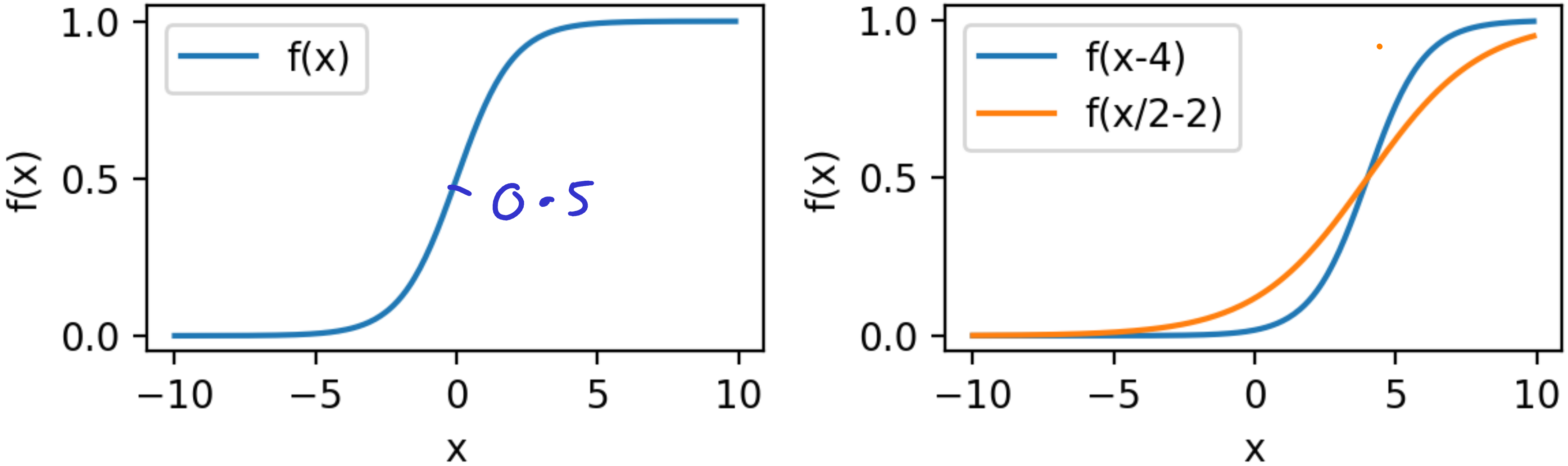
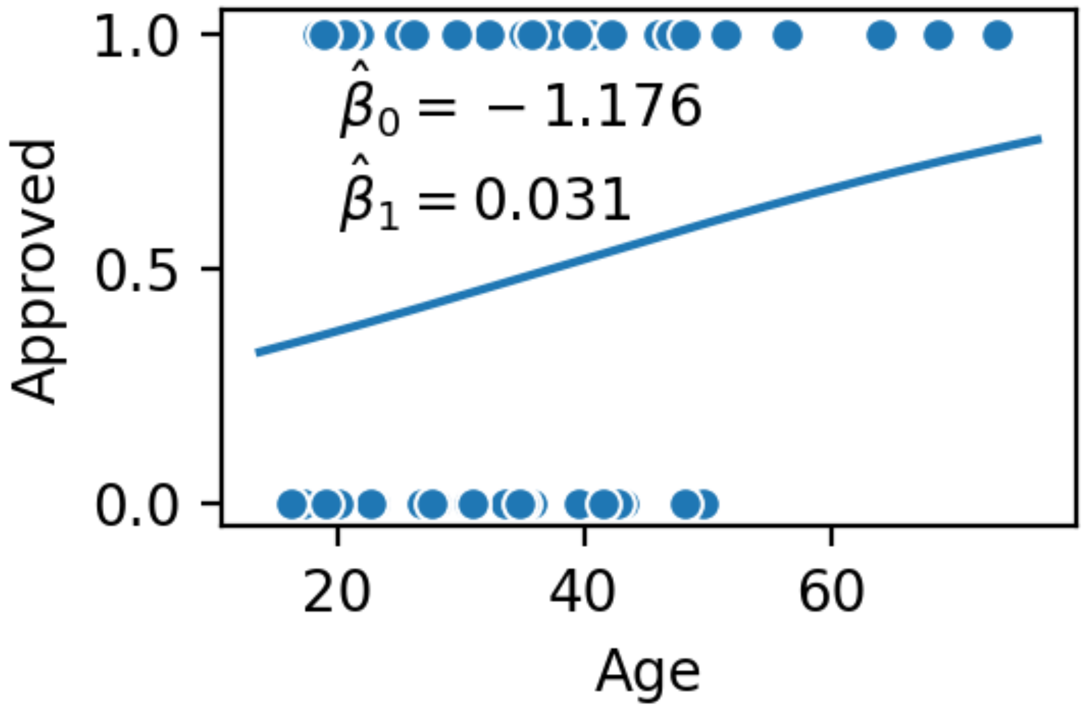
Logistic function



$$f(x) = \frac{e^{x}}{1 \; e^{x}} = \frac{1}{1 + e^{-x}}$$

Application to continuous variable in credit example



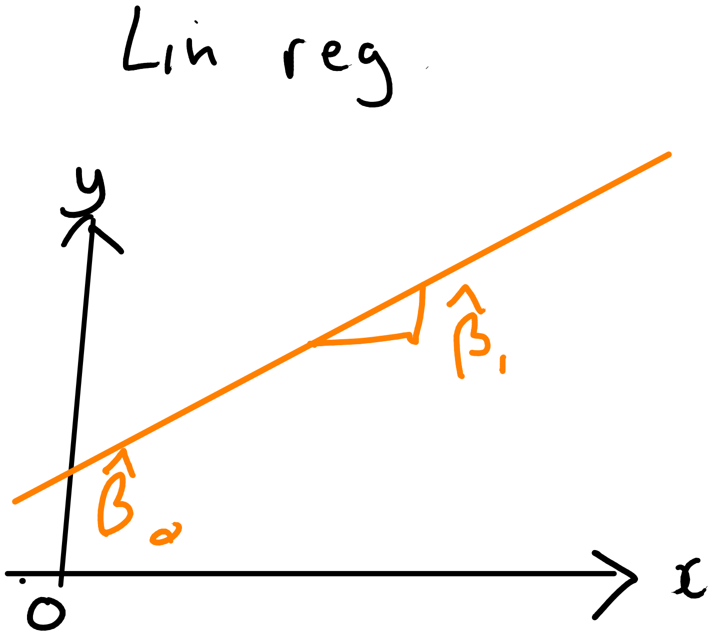
Binary variables: odds and odds ratios

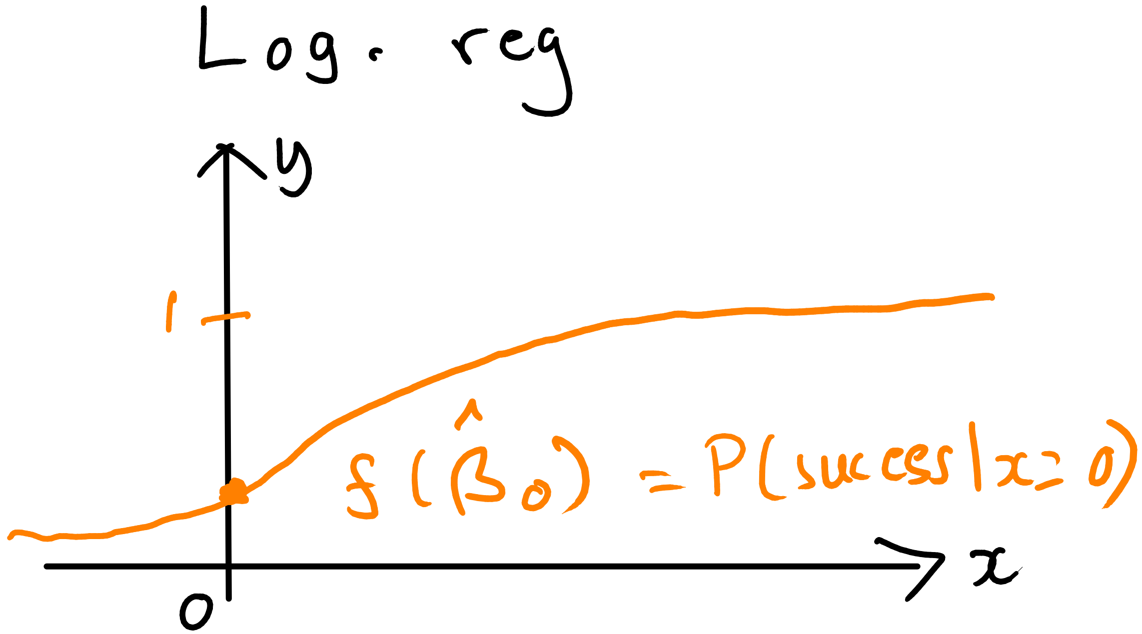
Employed	Approved	Not approved	Approval odds
0	0.25	0.75	0.34
1	0.71	0.29	2.42

$y \in \{$ "Not approved", "Approved" $\}$
$x \in \{$ "Not Emp.", "Emp." $\}$

Inf2 - Foundations of Data Science: Interpretation of logistic regression coefficients

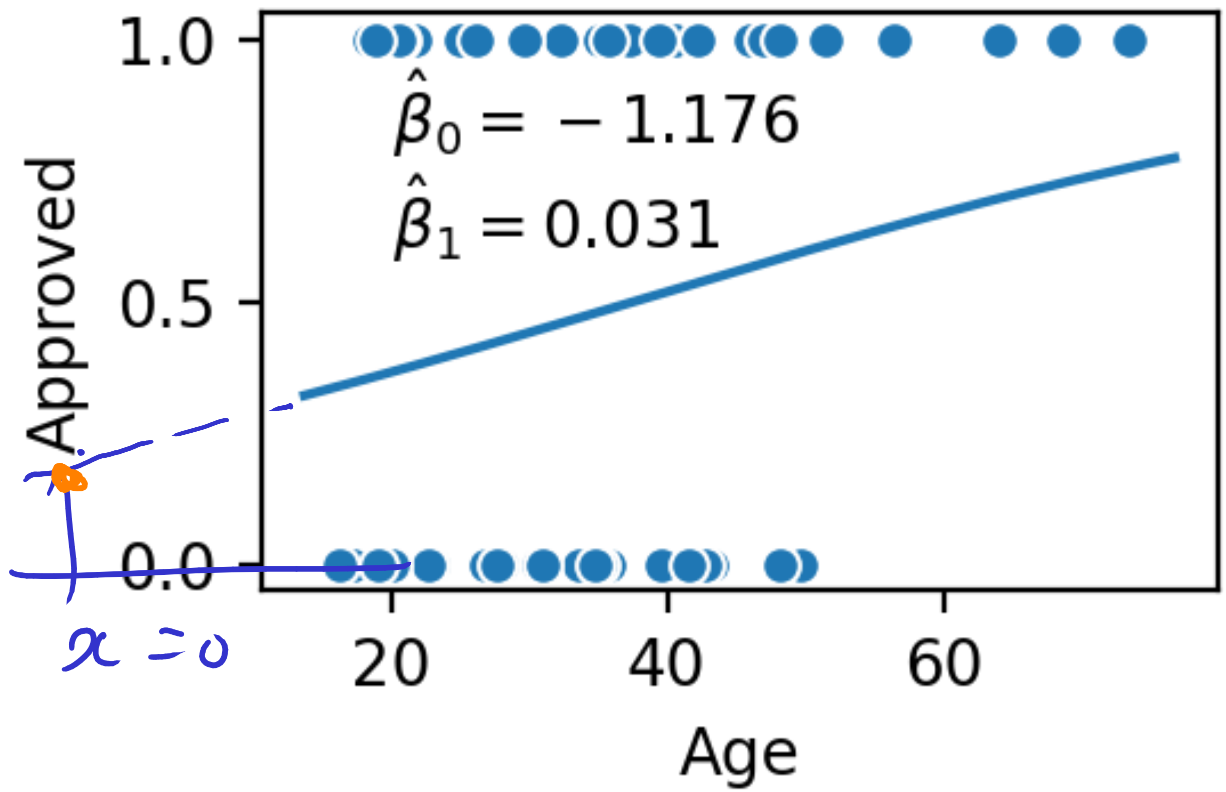
Interpretation of $\hat{\beta}_0$





$f(\hat{\beta}_0) = f(-1.176)$

$= 0.236$



Log odds

$$\text{Log Odds(Success)} = \ln \frac{P(\text{Success})}{P(\text{Failure})}$$

($\ln$ ← $\log_e$)

$$= \ln \frac{P(\text{Success})}{1 - P(\text{Success})} = \ln \frac{p}{1-p}$$

p	odds	Log odds
0.5		
>0.5		
<0.5		
1		
0		

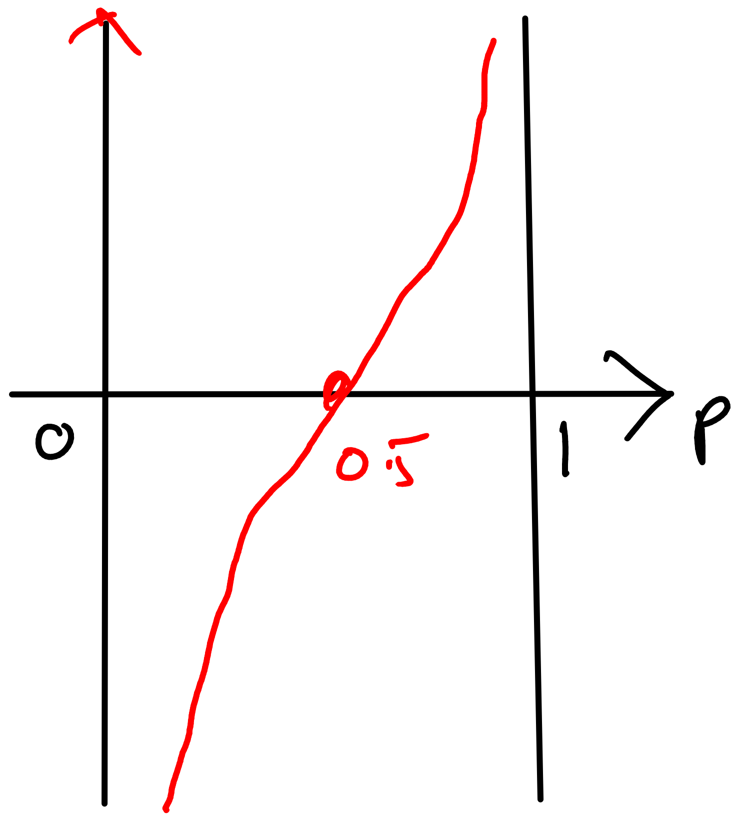
Logit scale

Log odds +1 $\Rightarrow$ odds increase by factor e

logistic unit = logit

$\hat{\beta}_0 = -1.176$ logits

$$\text{logit}(p) = \ln \frac{p}{1-p}$$



Logistic regression in terms of log odds

Success $P(Y=1 \mid x) = f(\beta_0 + \beta_1 x) = \dfrac{1}{1 + e^{-\beta_0 - \beta_1 x}}$ ①

Failure $P(Y=0 \mid x) = 1 - f(\beta_0 + \beta_1 x) = 1 - \dfrac{1}{1 + e^{-\beta_0 - \beta_1 x}}$

$$= \frac{e^{-\beta_0 - \beta_1 x}}{1 + e^{-\beta_0 - \beta_1 x}}$$ ②

① / ②

Odds $\dfrac{P(Y=1 \mid x)}{P(Y=0 \mid x)} = \dfrac{1}{e^{-\beta_0 - \beta_1 x}} = e^{\beta_0 + \beta_1 x}$

Log odds $\ln \dfrac{P(Y=1 \mid x)}{P(Y=0 \mid x)} = \beta_0 + \beta_1 x = \text{logit}(P(Y=1 \mid x))$

Interpretation of $\hat{\beta}_1$

$$\text{Odds}(x) = e^{\hat{\beta}_0 + \hat{\beta}_1 x}$$

$$= e^{\hat{\beta}_0} e^{\hat{\beta}_1 x}$$

$x = \{0, 1\}$

Preview

- Almost all pieces now in place to understand most of target paper

- Indicator variables

- Odds ratios

- Confidence intervals

- Last piece: Multiple logistic regression (next time)

Summary

- Logistic regression as a classification task

- Transforming linear regresssion into
  logistic regression: The sigmod function

- Odds, log odds, and odds ratios