Informatics 2D: Reasoning and Agents

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Lecture 22c: Combining Evidence

Where are we?

So far...

- JPDs can answer any query, but intractable
- Independence critical for practical inference
- Bayes Rule useful for inference from available evidence
- Today: **Combining Evidence**

Combining evidence

- Attempting to use additional evidence is easy in the JPD model

$$\mathbf{P}(Cavity|toothache \wedge catch) = \alpha\langle 0.108, 0.016\rangle \approx \langle 0.871, 0.129\rangle$$

but requires additional knowledge in Bayesian model:

$$\mathbf{P}(Cavity|toothache \wedge catch) = \alpha\mathbf{P}(toothache \wedge catch|Cavity)\mathbf{P}(Cavity)$$

- This is basically almost as hard as JPD calculation
- Refining idea of independence: *Toothache* and *Catch* are independent given presence/absence of *Cavity* (both caused by cavity, no effect on each other)

$$\mathbf{P}(toothache \wedge catch|Cavity) = \mathbf{P}(toothache|Cavity)\mathbf{P}(catch|Cavity)$$

Conditional independence

- Two variables X and Y are conditionally independent given Z if $\mathbf{P}(X, Y|Z) = \mathbf{P}(X|Z)\mathbf{P}(Y|Z)$
- Equivalent forms $\mathbf{P}(X|Y, Z) = \mathbf{P}(X|Z)$, $\mathbf{P}(Y|X, Z) = \mathbf{P}(Y|Z)$
- So in our example:

$$\mathbf{P}(Cavity|toothache \wedge catch) = \alpha\mathbf{P}(toothache|Cavity)\mathbf{P}(catch|Cavity)\mathbf{P}(Cavity)$$

- As before, this allows us to decompose large JPD tables into smaller ones, grows as $O(n)$ instead of $O(2^n)$
- This is what makes probabilistic reasoning methods scalable at all!

Conditional independence

- Conditional independence assumptions much more often reasonable than absolute independence assumptions
- **Naive Bayes model**:

$$\mathbf{P}(Cause, Effect_1, \ldots, Effect_n) = \mathbf{P}(Cause) \prod_i \mathbf{P}(Effect_i | Cause)$$

- Based on the idea that all effects are conditionally independent given the cause variable
- Also called **Bayesian classifier** or (by some) even "**idiot Bayes** model"
- Works surprisingly well in many domains despite its simplicity!

Summary

- Probabilistic inference using JPDs is impractical
- Independence and conditional independence help make inference tractable.
- Model design must balance independence assumptions vs. accuracy and complexity of inference
- Next time: **Probabilistic Reasoning with Bayesian Networks**