Informatics 2D: Reasoning and Agents

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Lecture 26b: Time and Uncertainty: Inference I

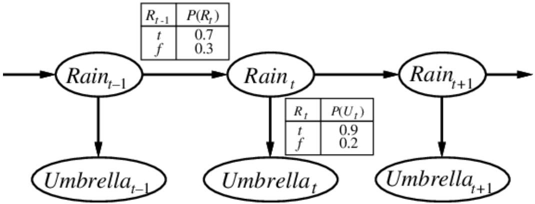
Where are we?

Last time. . .

- DBNs represent uncertainty in dynamic environments
- Two assumptions
  - Change is a stationary process
  - Markov assumption
- justify treating each time slice as a BN
  (with links from X_t to X_{t+1})
- Today: **DBN Inference I**

Reminder

- Bayesian network structure and conditional distributions
- Transition model $P(Rain_t | Rain_{t-1})$, sensor model $P(Umbrella_t | Rain_t)$



- Rain depends only on rainfall on previous day, whether this is reasonable depends on domain!

Inference tasks in temporal models

- Now that we have described general model, we need inference methods for a number of tasks
- **Filtering/monitoring**: compute **belief state** given evidence to date, i.e. $\mathbf{P}(\mathbf{X}_t|\mathbf{e}_{1:t})$
- Interestingly, an almost identical calculation yields the **likelihood** of the evidence sequence $P(\mathbf{e}_{1:t})$
- **Prediction**: computing posterior distribution over a future state given evidence to date: $\mathbf{P}(\mathbf{X}_{t+k}|\mathbf{e}_{1:t})$
- **Smoothing/hindsight**: compute posterior distribution of past state, $\mathbf{P}(\mathbf{X}_k|\mathbf{e}_{1:t})$, $0 \leq k < t$
- **Most likely explanation**: compute $\arg\max_{\mathbf{x}_{1:t}} P(\mathbf{x}_{1:t}|\mathbf{e}_{1:t})$ i.e. the most likely sequence of states given evidence

Filtering and prediction

- Done by recursive estimation: compute result for $t+1$ by doing it for t and then updating with new evidence $\mathbf{e}_{t+1}$. That is, for some function f:

$$\mathbf{P}(\mathbf{X}_{t+1}|\mathbf{e}_{1:t+1}) = f(\mathbf{e}_{t+1}, \mathbf{P}(\mathbf{X}_t|\mathbf{e}_{1:t}))$$

Why recursion works

$$\mathbf{P}(\mathbf{X}_{t+1}|\mathbf{e}_{1:t+1}) = \mathbf{P}(\mathbf{X}_{t+1}|\mathbf{e}_{1:t}, \mathbf{e}_{t+1}) \quad \text{(split notation)}$$

$$= \alpha \mathbf{P}(\mathbf{X}_{t+1}, \mathbf{e}_{1:t}, \mathbf{e}_{t+1}) \quad \text{(Bayes)}$$

$$= \alpha \mathbf{P}(\mathbf{e}_{t+1}|\mathbf{X}_{t+1}, \mathbf{e}_{1:t})\mathbf{P}(\mathbf{X}_{t+1}, \mathbf{e}_{1:t}) \quad \text{(Bayes)}$$

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Derivation continued...

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- $\mathbf{P}(\mathbf{e}_{t+1}|\mathbf{X}_{t+1})$ is sensor model; $\mathbf{P}(\mathbf{X}_{t+1}|\mathbf{x}_t)$ is transition model, $P(\mathbf{x}_t|\mathbf{e}_{1:t})$ is recursive bit.

Derivation continued...

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Filtering and prediction

- We can view estimate $\mathbf{P}(\mathbf{X}_t|\mathbf{e}_{1:t})$ as "message" $\mathbf{f}_{1:t}$ propagated and updated through sequence
- We write this process as $\mathbf{f}_{1:t+1} = \alpha \text{Forward}(\mathbf{f}_{1:t}, \mathbf{e}_{t+1})$
- Time and space requirements for this are constant regardless of length of sequence
- This is extremely important for agent design!
- All this is very abstract, let's look at an example

Example — Compute $\mathbf{P}(R_2|u_{1:2})$, $U_1 = true$, $U_2 = true$

- Suppose $\mathbf{P}(R_0) = \langle 0.5, 0.5\rangle$
- Recursive equations:

$$\mathbf{P}(R_2|u_1,u_2) = \alpha \mathbf{P}(u_2|R_2)\sum_{r_1} \mathbf{P}(R_2|r_1)P(r_1|u_1)$$

$$\begin{aligned}
\mathbf{P}(R_1|u_1) &= \alpha' \mathbf{P}(u_1|R_1)\sum_{r_0} \mathbf{P}(R_1|r_0)P(r_0) \\
&= \alpha'\langle 0.9, 0.2\rangle(\langle 0.7, 0.3\rangle \times 0.5 + \langle 0.3, 0.7\rangle \times 0.5) \\
&= \alpha'\langle 0.9, 0.2\rangle\langle 0.5, 0.5\rangle \\
&= \langle 0.818, 0.182\rangle
\end{aligned}$$

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\mathbf{P}(R_2|u_1,u_2) &= \alpha\langle 0.9, 0.2\rangle(\langle 0.7, 0.3\rangle \times 0.818 + \langle 0.3, 0.7\rangle \times 0.182) \\
&= \alpha\langle 0.9, 0.2\rangle\langle 0.627, 0.373\rangle \\
&= \alpha\langle 0.565, 0.075\rangle \\
&= \langle 0.883, 0.117\rangle
\end{aligned}$$

Filtering and prediction

- Prediction works like filtering without new evidence
- Computation involves only transition model and not sensor model:

$$\mathbf{P}(\mathbf{X}_{t+k+1}|\mathbf{e}_{1:t}) = \sum_{\mathbf{x}_{t+k}} \mathbf{P}(\mathbf{X}_{t+k+1}|\mathbf{x}_{t+k})P(\mathbf{x}_{t+k}|\mathbf{e}_{1:t})$$

- As we predict further and further into the future, distribution of rain converges to $\langle 0.5, 0.5 \rangle$
- This is called the **stationary distribution** of the Markov process (the more uncertainty, the quicker it will converge)

Filtering and prediction

- We can use the above method to compute **likelihood** of evidence sequence $P(\mathbf{e}_{1:t})$
- Useful to compare different temporal models
- Use a likelihood message $\mathbf{l}_{1:t} = \mathbf{P}(\mathbf{X}_t, \mathbf{e}_{1:t})$ and compute

$$\mathbf{l}_{1:t+1} = \alpha \text{Forward}(\mathbf{l}_{1:t}, \mathbf{e}_{t+1})$$

- Once we compute $\mathbf{l}_{1:t}$, summing out yields likelihood

$$L_{1:t} = P(\mathbf{e}_{1:t}) = \sum_{\mathbf{x}_t} \mathbf{l}_{1:t}(\mathbf{x}_t, \mathbf{e}_{1:t})$$

Summary

- DBNs for reasoning about Time and uncertainty
- Inference: Filtering and prediction
- Recursion
- Next time: **Time and Uncertainty: Inference II**