# Informatics 2D: Reasoning and Agents

Alex Lascarides

School of informatics
The University of Edinburgh

Lecture 29a: Decision Making under Uncertainty: Preferences and Rationality

# Where are we?

Last time ...

- Looked at Dynamic Bayesian Networks
- General, powerful method for describing temporal probabilistic problems
- Unfortunately exact inference computationally too hard

Today ...

- **Decision Making under Uncertainty**

# Combining beliefs and desires

- Rational agents do things that are an optimal tradeoff between:
  - the likelihood of reaching a particular resultant state (given one's actions) and
  - The desirability of that state
- So far we have done the 'likelihood' bit: we know how to evaluate the probability of being in a particular state at a particular time.
- But we've not looked at an agent's preferences or desires
- Now we will discuss **utility theory** in more detail to obtain a full picture of decision-theoretic agent design

# Utility theory & utility functions

- Agent's preferences between world states are described using a **utility function**
- UF assigns some numerical value $U(S)$ to each state $S$ to express its desirability for the agent
- Nondeterministic action $a$ has results $Result(a)$ and probabilities $P(Result(a) = s'|a, \mathbf{e})$ summarise agent's knowledge about its effects given evidence observations $\mathbf{e}$.
- Can be combined with probabilities for outcomes to obtain **expected utility** of action:

$$EU(A|E) = \sum_{s'} P(Result(a) = s'|a, \mathbf{e})U(s')$$

# Utility theory & utility functions

- Principle of **maximum expected utility** (MEU) says agent should use action that maximises expected utility
- In a sense, this summarises the whole endeavour of AI:

  *If agent maximises utility function that correctly reflects the performance measure applied to it, then optimal performance will be achieved by averaging over all environments in which agent could be placed*

- Of course, this doesn't tell us how to define utility function or how to determine probabilities for any sequence of actions in a complex environment
- For now we will only look at **one-shot decisions**, not **sequential decisions** (next lecture)

# Constraints on rational preferences

- MEU sounds reasonable, but why should this be the best quantity to maximise? Why are numerical utilities sensible? Why single number?
- Questions can be answered by looking at constraints on preferences
- Notation:

| | |
|---|---|
| $A \succ B$ | $A$ is preferred to $B$ |
| $A \sim B$ | the agent is indifferent between $A$ and $B$ |
| $A \succsim B$ | the agent prefers $A$ to $B$ or is indifferent between them |

- But what are $A$ and $B$? Introduce **lotteries** with outcomes $C_1 \ldots C_n$ and accompanying probabilities $L = [p_1, C_1; p_2, C_2; \ldots; p_n, C_n]$

# Constraints on rational preferences

- Outcome of a lottery can be state or another lottery
- Can be used to understand how preferences between complex lotteries are defined in terms of preferences among their (outcome) states
- The following are considered reasonable **axioms of utility theory**
- **Orderability**: $(A \succ B) \vee (B \succ A) \vee (A \sim B)$
- **Transitivity**: If agent prefers $A$ over $B$ and $B$ over $C$ then he must prefer $A$ over $C$: $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$
- Example: Assume $A \succ B \succ C \succ A$ and $A$, $B$, $C$ are goods
    - Agent might trade $A$ and some money for $C$ if he has $A$
    - We then offer $B$ for $C$ and some cash and then trade $A$ for $B$
    - Agent would lose all his money over time

# Constraints on rational preferences

- **Continuity**: If $B$ is between $A$ and $C$ in preference, then with some probability agent will be indifferent between getting $B$ for sure and a lottery over $A$ and $C$

$$A \succ B \succ C \Rightarrow \exists p\,[p, A; 1-p, C] \sim B$$

- **Substitutability**: Indifference between lotteries leads to indifference between complex lotteries built from them

$$A \sim B \Rightarrow [p, A; 1-p, C] \sim [p, B; 1-p, C]$$

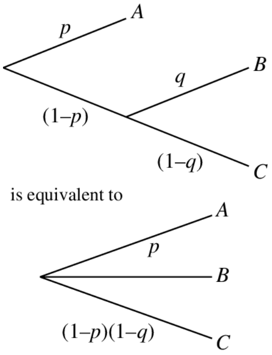
- **Monotonicity**: Preferring $A$ to $B$ implies preference for any lottery that assigns higher probability to $A$

$$A \succ B \Rightarrow (p \geq q \Leftrightarrow [p, A; 1-p, B] \succsim [q, A; 1-q, B]$$

# Decomposability example

- **Decomposability**: Compound lotteries can be reduced to simpler one

$$[p, A; 1-p, [q, B; 1-q, C]] \sim [p, A; (1-p)q, B; (1-p)(1-q), C]$$



# From preferences to utility

- The following **axioms of utility** ensure that utility functions follow the above axioms on preference:
  - Utility principle: there exists a function such that

$$U(A) > U(B) \Leftrightarrow A \succ B \quad U(A) = U(B) \Leftrightarrow A \sim B$$

  - MEU principle: utility of lottery is sum of probability of outcomes times their utilities

$$U([p_1, S_1; \ldots; p_n, S_n]) = \sum_i p_i U(S_i)$$

- But an agent might not know even his own utilities!
- But you can work out his (or even your own!) utilities by observing his (your) behaviour and assuming that he (you) chooses to MEU.

# Utility functions

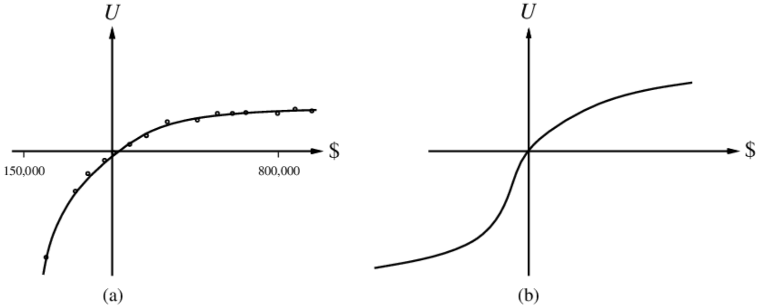
- According to the above axioms, arbitrary preferences can be expressed by utility functions
  - I prefer to have a prime number of £in my bank account; when I have £10 I will give away £3.
- But usually preferences are more systematic, a typical example being money (roughly, we like to maximise our money)
- Agents exhibit **monotonic preference** toward money, but how about lotteries involving money?
- "Who wants to be a millionaire"-type problem, is pocketing a smaller amount irrational?
- **Expected monetary value** (EMV) is actual expectation of outcome

# Utility of money

- Assume you can keep 1 million or risk it with the prospect of getting three millions at the toss of a (fair) coin
- EMV of accepting gamble is $0.5 \times 0 + 0.5 \times 3{,}000{,}000$ which is greater than $1{,}000{,}000$
- Use $S_n$ to denote state of possessing wealth "$n$ dollars", current wealth $S_{k+1M}$
- Expected utilities become:
  - $EU(Accept) = \frac{1}{2}U(S_k) + \frac{1}{2}U(S_{k+3M})$
  - $EU(Decline) = U(S_{k+1M})$
- But it all depends on utility values you assign to levels of monetary wealth (is first million more valuable than second?)

# Utility of money (empirical study)

- It turns out that for most people this is usually concave (curve (a)), showing that going into debt is considered disastrous relative to small gains in money—**risk averse**.




- But if you're already \$10M in debt, your utility curve is more like (b)—**risk seeking** when desperate!

# Utility scales

- Axioms don't say anything about scales
- For example transformation of $U(S)$ into $U'(S) = k_1 + k_2 U(S)$ ($k_2$ positive) doesn't affect behaviour
- In deterministic contexts behaviour is unchanged by any monotonic transformation (utility function is **value function/ordinal function**)
- One procedure for assessing utilities is to use **normalised utility** between "best possible prize" ($u^{\top} = 1$) and "worst possible catastrophe" ($u^{\perp} = 0$)
- Ask agent to indicate preference between $S$ and the standard lottery $[p, u^{\top} : (1-p), u^{\perp}]$, adjust $p$ until agent is indifferent between $S$ and standard lottery, set $U(S) = p$

# Summary

- Foundations of rational decision making: Maximise Expected Utility (MEU)
- Utility theory and its axioms, utility functions