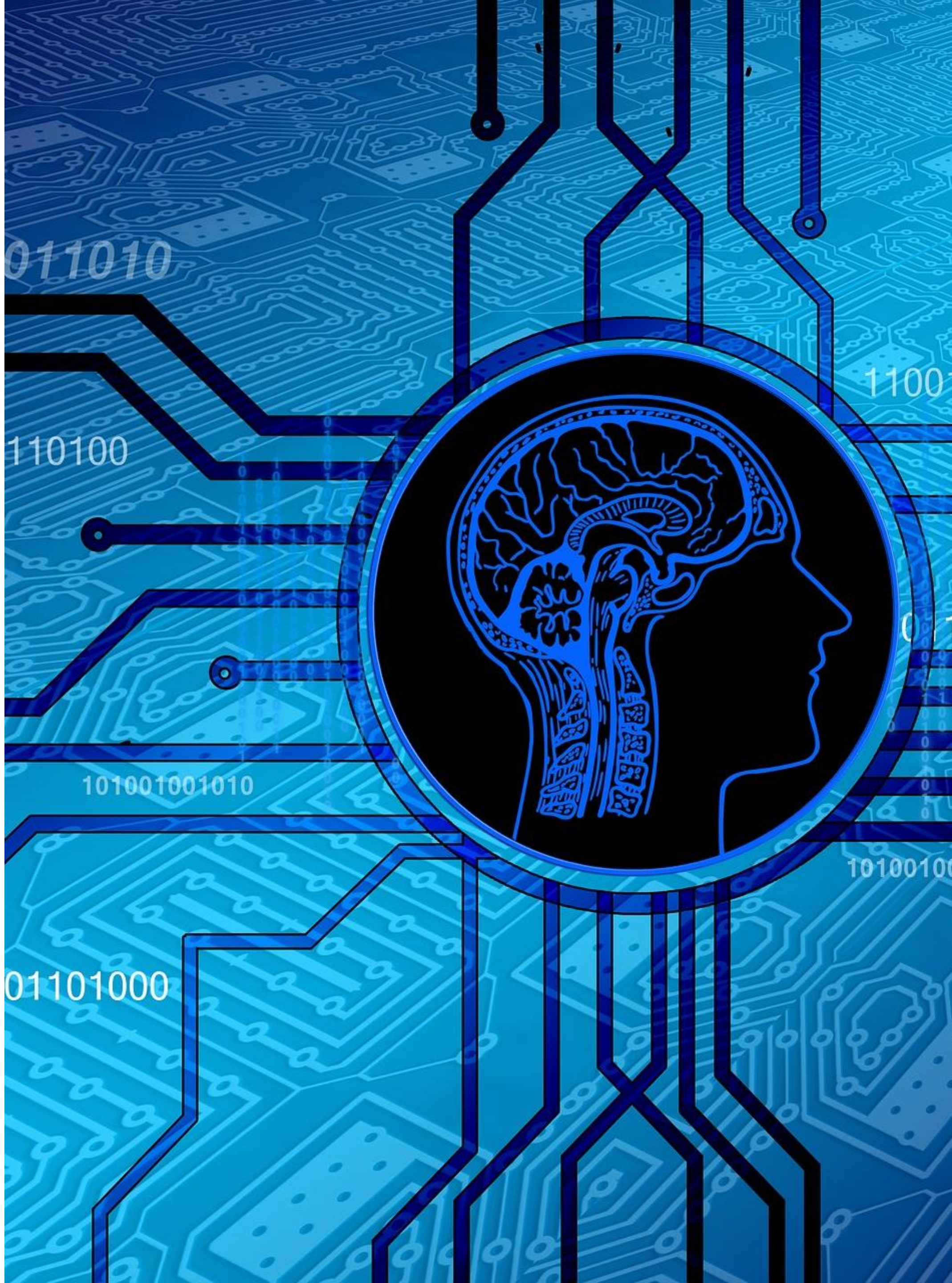
Effective Propositional Inference

Informatics 2D: Reasoning and Agents

Adapted from slides provided by Dr Petros Papapanagiotou



Outline

Two families of efficient algorithms for propositional inference:

Complete backtracking search algorithms

- DPLL algorithm (Davis, Putnam, Logemann, Loveland)

Incomplete local search algorithms

- WalkSAT algorithm

Clausal Form (CNF)

DPLL and WalkSAT manipulate formulae in conjunctive normal form (CNF).

Sentence	• Formula whose satisfiability is to be determined • Conjunction of clauses
Clause	• Disjunction of literals
Literal	• Proposition symbol or negated proposition symbol

e.g. $(A, \neg B), (B, \neg C)$ represents $(A \vee \neg B) \wedge (B \vee \neg C)$

Conversion to CNF

$$\left(B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}\right)$$

Eliminate $\Leftrightarrow$: replace $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$

- $\left(B_{1,1} \Rightarrow \left(P_{1,2} \vee P_{2,1}\right)\right) \wedge \left(\left(P_{1,2} \vee P_{2,1}\right) \Rightarrow B_{1,1}\right)$

Eliminate $\Rightarrow$: replace $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$

- $\left(\neg B_{1,1} \vee \left(P_{1,2} \vee P_{2,1}\right)\right) \wedge \left(\neg\left(P_{1,2} \vee P_{2,1}\right) \vee B_{1,1}\right)$

Move $\neg$ inwards : use de Morgan's rules and double negation $\neg\neg\alpha = \alpha$

- $\left(\neg B_{1,1} \vee \left(P_{1,2} \vee P_{2,1}\right)\right) \wedge \left(\left(\neg P_{1,2} \wedge \neg P_{2,1}\right) \vee B_{1,1}\right)$

Create clauses: apply distributivity law ($\vee$ over $\wedge$) and flatten

- $\left(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}\right) \wedge \left(\neg P_{1,2} \vee B_{1,1}\right) \wedge \left(\neg P_{2,1} \vee B_{1,1}\right)$

DPLL

The DPLL algorithm

Determine if an input propositional logic sentence (in CNF) is satisfiable.

Improvements over truth table enumeration:

1. Early termination
2. Pure symbol heuristic
3. Unit clause heuristic

DPLL

1. Early termination

- A *clause* is true if **one** of its literals is true,
  - e.g., if A is true then $(A \vee \neg B)$ is true.
- A *sentence* is false if **any** of its clauses is false,
  - e.g., if A is false and B is true then
  - $(A \vee \neg B)$ is false, so any sentence containing it is false.

2. Pure symbol heuristic

- **Pure symbol**: **always** appears with the same *"sign"/polarity* in **all** clauses.
  - e.g., In the three clauses $(A \vee \neg B)$, $(\neg B \vee \neg C)$, $(C \vee A)$:
    - A and B are pure, C is impure.
- Make literal containing a pure symbol true.
  - e.g., Let A and $\neg B$ both be true.

3. Unit clause heuristic

- **Unit clause**: only one literal in the clause
  - e.g. (A)
- The only literal in a unit clause must be true.
  - e.g., A must be true.
- Also includes clauses where **all but one** literal is false,
  - e.g. (A,B,C) where B and C are false since it is equivalent to (A, false, false) i.e. (A).

DPLL

function DPLL-SATISFIABLE?(s) **returns** *true* or *false*
  inputs: s, a sentence in propositional logic

  $clauses \leftarrow$ the set of clauses in the CNF representation of s
  $symbols \leftarrow$ a list of the proposition symbols in s
  return DPLL($clauses, symbols, \{\ \}$)

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function DPLL($clauses, symbols, model$) **returns** *true* or *false*

  if every clause in $clauses$ is true in $model$ **then return** *true*
  if some clause in $clauses$ is false in $model$ **then return** *false*
  $P, value \leftarrow$ FIND-PURE-SYMBOL($symbols, clauses, model$)
  if P is non-null **then return** DPLL($clauses, symbols - P, model \cup \{P{=}value\}$)
  $P, value \leftarrow$ FIND-UNIT-CLAUSE($clauses, model$)
  if P is non-null **then return** DPLL($clauses, symbols - P, model \cup \{P{=}value\}$)
  $P \leftarrow$ FIRST($symbols$); $rest \leftarrow$ REST($symbols$)
  return DPLL($clauses, rest, model \cup \{P{=}true\}$) **or**
      DPLL($clauses, rest, model \cup \{P{=}false\}$))

The DPLL algorithm

Tautology Deletion (Optional)

- **Tautology**: both a proposition and its negation in a clause.
  - e.g. $(A, B, \neg A)$
- Clause bound to be true.
  - e.g., whether A is true or false.
  - Therefore, can be deleted.

Mid-Lecture Exercise

- Apply DPLL heuristics to the following sentence:

$$(S_{2,1}),\ (\neg S_{1,1}),\ (\neg S_{1,2}),$$
$$(\neg S_{2,1}, W_{2,2}),\ (\neg S_{1,1}, W_{2,2}),\ (\neg S_{1,2}, W_{2,2}),$$
$$(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$$

- Use **case splits** if model not found by the heuristics.
- Symbols: $S_{1,1}$, $S_{1,2}$, $S_{2,1}$, $W_{2,2}$

Solution

Pure symbol heuristic:

$(S_{2,1})$

$(\neg S_{1,1})$

$(\neg S_{1,2})$

$(\neg S_{2,1}, W_{2,2})$

$(\neg S_{1,1}, W_{2,2})$

$(\neg S_{1,2}, W_{2,2})$

$(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

$(S_{2,1})$

$(\neg S_{1,1})$

$(\neg S_{1,2})$

$(\neg S_{2,1}, W_{2,2})$

$(\neg S_{1,1}, W_{2,2})$

$(\neg S_{1,2}, W_{2,2})$

$(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

- $S_{2,1}$ is true

$\mathbf{T}$

$(\neg S_{1,1})$

$(\neg S_{1,2})$

$(\mathbf{F}, W_{2,2})$

$(\neg S_{1,1}, W_{2,2})$

$(\neg S_{1,2}, W_{2,2})$

$(\neg W_{2,2}, \mathbf{T}, S_{1,1}, S_{1,2})$

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

- $S_{2,1}$ is true

Early termination heuristic:

- $(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$ is true

T

$(\neg S_{1,1})$

$(\neg S_{1,2})$

$(\mathbf{F}, W_{2,2})$

$(\neg S_{1,1}, W_{2,2})$

$(\neg S_{1,2}, W_{2,2})$

T

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

- $S_{2,1}$ is true
- $S_{1,1}$ is false

Early termination heuristic:

- $(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$ is true

T

T

$(\neg S_{1,2})$

$(\mathbf{F}, W_{2,2})$

$(\mathbf{T}, W_{2,2})$

$(\neg S_{1,2}, W_{2,2})$

T

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

- $S_{2,1}$ is true
- $S_{1,1}$ is false
- $S_{1,2}$ is false

Early termination heuristic:

- $(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$ is true
- $(\neg S_{1,1}, W_{2,2})$ is true

T

T

T

(**F**, $W_{2,2}$)

T

(**T**, $W_{2,2}$)

T

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

- $S_{2,1}$ is true
- $S_{1,1}$ is false
- $S_{1,2}$ is false

Early termination heuristic:

- $(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$ is true
- $(\neg S_{1,1}, W_{2,2})$ is true
- $(\neg S_{2,1}, W_{2,2})$ is true

T

T

T

(**F**, $W_{2,2}$)

T

T

T

Solution

Pure symbol heuristic:
- No literal is pure.

Unit clause heuristic:
- $S_{2,1}$ is true
- $S_{1,1}$ is false
- $S_{1,2}$ is false
- $W_{2,2}$ is true

Early termination heuristic:
- $(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$ is true
- $(\neg S_{1,1}, W_{2,2})$ is true
- $(\neg S_{2,1}, W_{2,2})$ is true

T

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WalkSAT

The WalkSAT algorithm

- Incomplete, local search algorithm
- Evaluation function:
  - The min-conflict heuristic of minimizing the number of unsatisfied clauses
- Algorithm checks for satisfiability by randomly flipping the values of variables
- Balance between greediness and randomness

function WALKSAT(*clauses*, *p*, *max_flips*) **returns** a satisfying model or *failure*
    inputs: *clauses*, a set of clauses in propositional logic
            p, the probability of choosing to do a "random walk" move, typically around 0.5
            max_flips, number of flips allowed before giving up

    model ← a random assignment of *true*/*false* to the symbols in *clauses*
    for $i = 1$ **to** *max_flips* **do**
        if *model* satisfies *clauses* **then return** *model*
        clause ← a randomly selected clause from *clauses* that is false in *model*
        with probability *p* flip the value in *model* of a randomly selected symbol from *clause*
        else flip whichever symbol in *clause* maximizes the number of satisfied clauses
    return *failure*

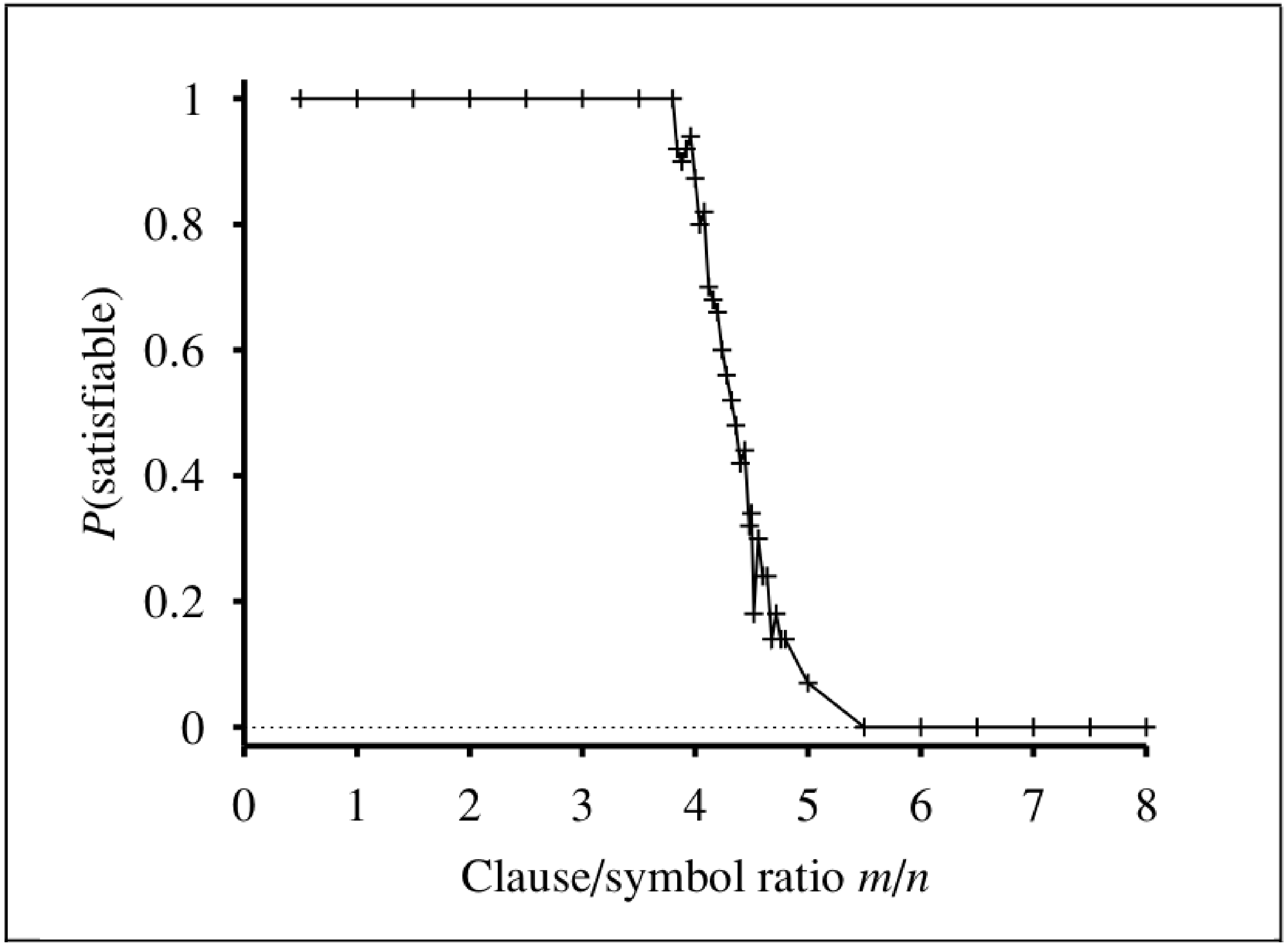
The WalkSAT algorithm

Hard satisfiability problems

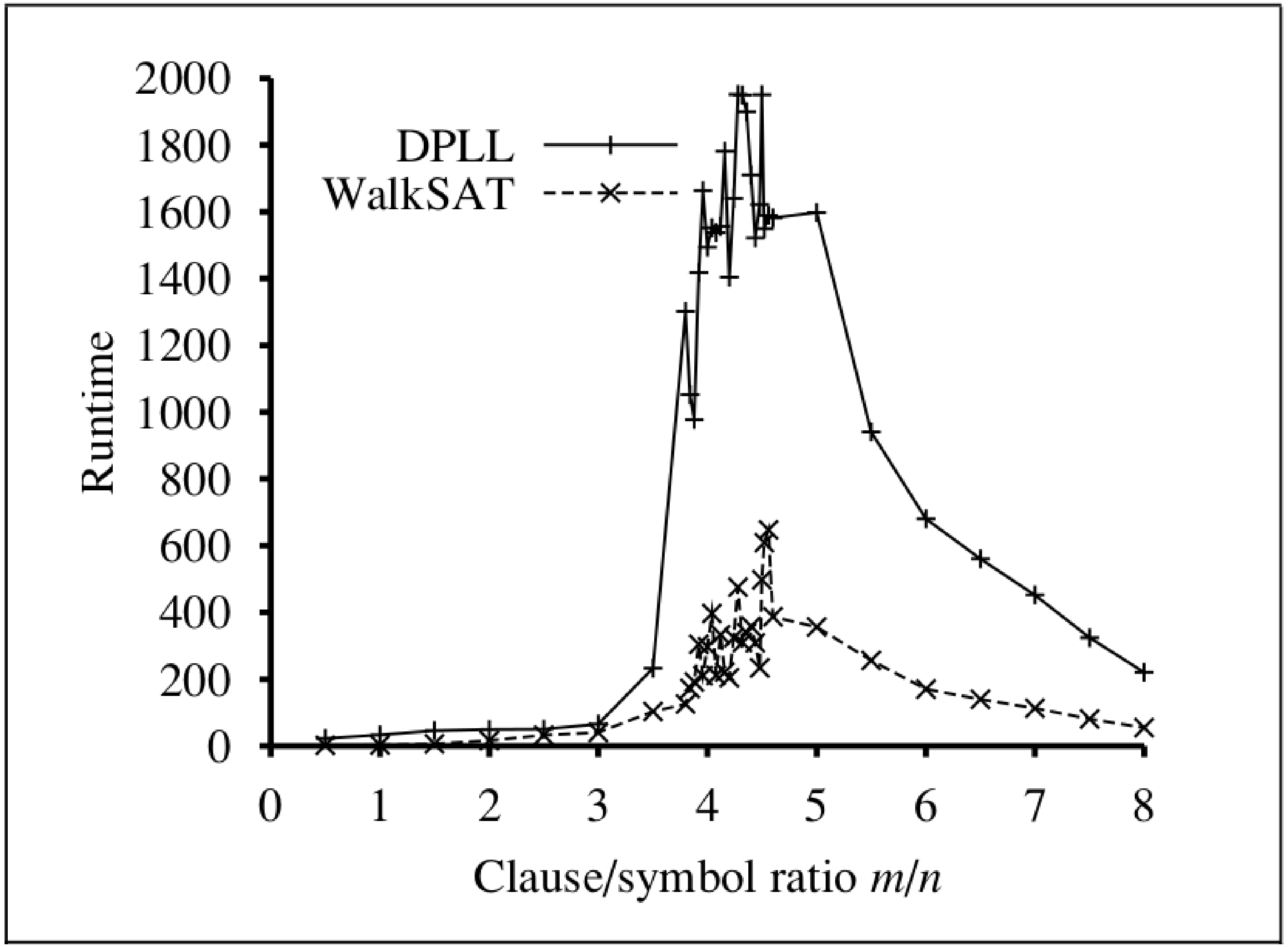
- Consider random 3-CNF sentences.
  - Example:

$$(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E) \land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$$

  - m = number of clauses
  - n = number of symbols
- Hard problems seem to cluster near m/n = 4.3 (critical point)

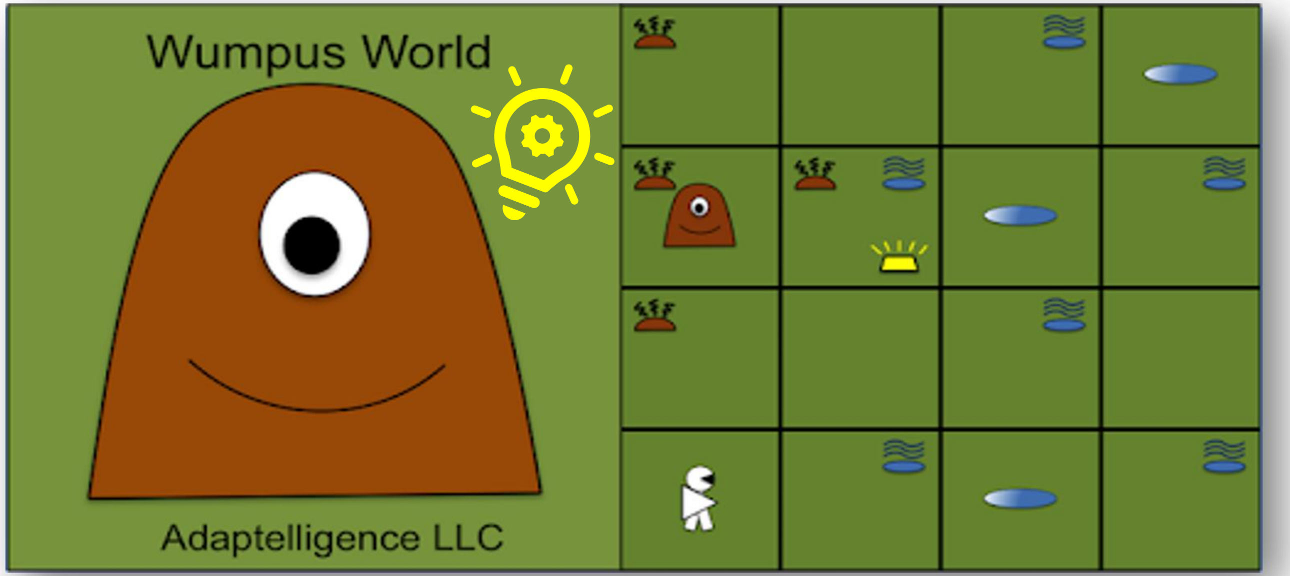


Hard satisfiability problems



Hard satisfiability problems

Median runtime for 100 satisfiable random 3-CNF sentences, n = 50



Inference in the Wumpus World

Inference-based agents in the wumpus world

- A wumpus-world agent using propositional logic:
  - $\neg P_{1,1}$
  - $\neg W_{1,1}$
  - $B_{x,y} \Leftrightarrow (P_{x,y+1} \vee P_{x,y-1} \vee P_{x+1,y} \vee P_{x-1,y})$
  - $S_{x,y} \Leftrightarrow (W_{x,y+1} \vee W_{x,y-1} \vee W_{x+1,y} \vee W_{x-1,y})$
  - $W_{1,1} \vee W_{1,2} \vee \ldots \vee W_{4,4}$
  - $\neg W_{1,1} \vee \neg W_{1,2}$
  - $\neg W_{1,1} \vee \neg W_{1,3}$
  - …
- 64 distinct proposition symbols, 155 sentences

function HYBRID-WUMPUS-AGENT($percept$) **returns** an $action$
inputs: $percept$, a list, $[stench, breeze, glitter, bump, scream]$
persistent: KB, a knowledge base, initially the atemporal "wumpus physics"
t, a counter, initially 0, indicating time
$plan$, an action sequence, initially empty

TELL(KB, MAKE-PERCEPT-SENTENCE($percept, t$))
TELL the KB the temporal "physics" sentences for time t
$safe \leftarrow \{[x, y] : \text{ASK}(KB, OK^t_{x,y}) = true\}$
if ASK($KB, Glitter^t$) $= true$ **then**
$plan \leftarrow [Grab]$ + PLAN-ROUTE($current$, $\{[1,1]\}$, $safe$) + $[Climb]$
if $plan$ is empty **then**
$unvisited \leftarrow \{[x, y] : \text{ASK}(KB, L^{t'}_{x,y}) = false$ for all $t' \leq t\}$
$plan \leftarrow$ PLAN-ROUTE($current$, $unvisited \cap safe$, $safe$)
if $plan$ is empty and ASK($KB, HaveArrow^t$) $= true$ **then**
$possible\_wumpus \leftarrow \{[x, y] : \text{ASK}(KB, \neg W_{x,y}) = false\}$
$plan \leftarrow$ PLAN-SHOT($current$, $possible\_wumpus$, $safe$)
if $plan$ is empty **then** // no choice but to take a risk
$not\_unsafe \leftarrow \{[x, y] : \text{ASK}(KB, \neg OK^t_{x,y}) = false\}$
$plan \leftarrow$ PLAN-ROUTE($current$, $unvisited \cap not\_unsafe$, $safe$)
if $plan$ is empty **then**
$plan \leftarrow$ PLAN-ROUTE($current$, $\{[1, 1]\}$, $safe$) + $[Climb]$
$action \leftarrow$ POP($plan$)
TELL(KB, MAKE-ACTION-SENTENCE($action, t$))
$t \leftarrow t + 1$
return $action$

The Wumpus Agent

The Wumpus Agent

function PLAN-ROUTE(*current*, *goals*, *allowed*) **returns** an action sequence
  inputs: *current*, the agent's current position
    goals, a set of squares; try to plan a route to one of them
    allowed, a set of squares that can form part of the route

  problem ← ROUTE-PROBLEM(*current*, *goals*, *allowed*)
  return A*-GRAPH-SEARCH(*problem*)

We need more!

Effect axioms

$$L^0_{1,1} \wedge FacingEast^0 \wedge Forward^0 \Rightarrow L^1_{2,1} \wedge \neg L^1_{1,1}$$

We need extra axioms about the world.

Frame problem! – representational & inferential

Frame axioms:

$$Forward^t \Rightarrow (HaveArrow^t \Leftrightarrow HaveArrow^{t+1})$$
$$Forward^t \Rightarrow (WumpusAlive^t \Leftrightarrow WumpusAlive^{t+1})$$

Successor-state axioms:

$$HaveArrow^{t+1} \Leftrightarrow (HaveArrow^t \wedge \neg Shoot^t)$$

Expressiveness limitation of propositional logic

- KB contains "physics" sentences for every single square.
- For every time t and every location [x,y],

$$L^{t}_{x,y} \wedge FacingRight^{t} \wedge Forward^{t} \Rightarrow L^{t+1}_{x+1,y}$$

- Rapid proliferation of clauses!

Why?

- Fundamentals behind *SAT/SMT solvers*.
- Highly specialised and optimised tools.
  - Capable of solving problems with thousands of propositions and millions of constraints, despite NP-completeness and exponential algorithms!
- Close relation to CSPs and optimization problems.
- Very large array of applications, e.g.:
  - Circuit routing and testing, automatic test generation, formal verification, planning & scheduling, configuration/customisation, etc.