Informatics 2D: Tutorial 3

Adversarial Search and Propositional Logic

Week 4

1 Adversarial Search

This exercise was taken from R&N Chapter 5.

Consider the two-player game shown in Figure 1

1. Draw the complete game tree, using the following conventions:
   - Write each state as (S_A, S_B) where S_A and S_B denote token locations
   - Put each terminal state in square boxes and write its game value in a circle
   - Put *loop states* (states that already appear on the path to the root) in double square boxes. Since it is not clear how to assign values to loop states, annotate each with a "?" in a circle.
2. Now mark each node with its backed-up minimax value (also in a circle). You will have to think of a way to assign values to the loop states.
3. Explain why the standard minimax algorithm would fail on this game tree and briefly sketch how you might fix it, drawing on your answer to item (2) above. Does your modified algorithm give optimal decisions for all games with loops?

Answers

1. The game tree, complete with annotations is shown in Figure 2
2. The "?" nodes can be assigned a value of 0, so choosing the loop state will not benefit either player. If there is a win for a player from a loop state then they will chose the moves that lead to the win, rather than end up in the loop state.
3. Standard minimax is depth-first and would go into an infinite loop. It can be fixed by comparing the current state against the stack; and if the state is repeated, then return a "?" value. Propagation of "?" is handled as above. Although it works in this case, it does not always

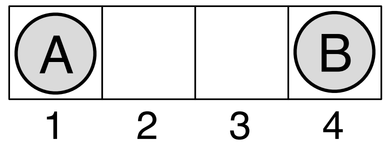

Figure 1: The starting position of a simple game. Player A moves first. The two players take turns moving, and each player must move their token to an open adjacent space in either direction. If the opponent occupies an adjacent space, then the player may jump over the opponent to the next available space. For example, if A is on 3 and B is on 2, then A may move back to 1. The game ends when one player reaches the opposite end of the board. If player A reaches space 4 first, then the value of the game to A is $+1$; if player B reaches space 1 first the value of the game to A is -1.

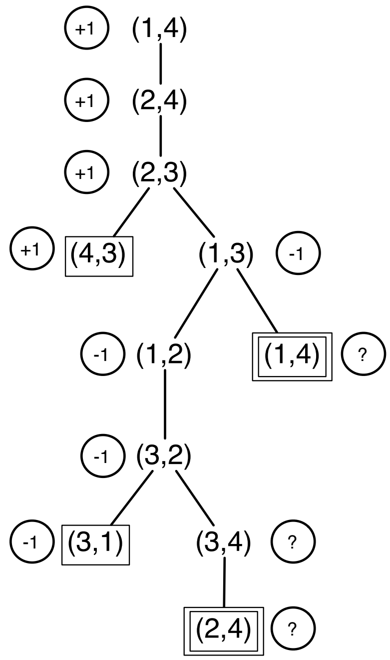

Figure 2: The game tree for the four-square game in Exercise 6.3. Terminal states are in single boxes, loop states in double boxes. Each state is annotated with its minimax value in a circle.

work. For example, it is not clear how to compare "?" with a drawn position: a drawn position is not better or worse for either player, but might at least bring an end to the game, so assigning 0 to "?" states will not work there.

2 The Wumpus World

2.1 Propositional Rules

Translate the following statements into propositional logic formulae. You can use a schematic representation for the location of a square, e.g. use a proposition $W_{i,j}$ to represent that there is a wumpus in the square in the ith row and jth column (don't worry about the edges of the grid when formalising your propositions).

1. A square cannot contain the wumpus and a pit.
2. If a square is breezy then one of the (not diagonally) adjacent squares contains a pit.
3. There is a stench in the square if and only if it contains the wumpus or is (not diagonally) adjacent to the square containing the wumpus.

2.2 Entailment

Using the above rules, and the assumed facts, show the following statements are entailed by the knowledge base (either using a truth table or a diagram showing the possible models):

1. Assuming that there is a pit in square $(2,2)$ show that the wumpus is not in square $(2,2)$.
2. Assuming that there is a stench in $(1,1)$ and that there is not a wumpus in square $(1,1)$ show that there is either a wumpus in $(1,2)$ or a wumpus in $(2,1)$. (Assume the grid begins at (1,1) and ignore the off-grid squares in your rules).
3. Assuming that there is a breeze in square $(2,2)$ and that there is not a pit in squares $(1,2)$, $(2,1)$ or $(3,2)$, show that there is a pit in square $(2,3)$.

Answers

2.2.1 Propositional Rules

1. $\neg(W_{i,j} \wedge P_{i,j})$
2. $B_{i,j} \Rightarrow P_{i+1,j} \vee P_{i-1,j} \vee P_{i,j+1} \vee P_{i,j-1}$
3. $S_{i,j} \Leftrightarrow W_{i,j} \vee W_{i+1,j} \vee W_{i-1,j} \vee W_{i,j+1} \vee W_{i,j-1}$

2.2.2 Entailment

Either use the diagramatic representation of wumpus world states (as shown in the lecture notes and Russel and Norvig) or construct a truth table for the proposition you are trying to show and demonstrate that each model for the KB is a model of the proposition. Note that if you are using a truth table that you only need to consider the rows in which the propositions in the KB are true.

1. $KB \Leftrightarrow \neg(W_{2,2} \wedge P_{2,2}) \wedge P_{2,2}$
   $\alpha \Leftrightarrow \neg W_{2,2}$
2. $KB \Leftrightarrow S_{1,1} \wedge (S_{1,1} \Leftrightarrow W_{1,1} \vee W_{2,1} \vee W_{1,2}) \wedge \neg W_{1,1}$
   $\alpha \Leftrightarrow W_{2,1} \vee W_{1,2}$
3. $KB \Leftrightarrow (B_{2,2} \Rightarrow P_{1,2} \vee P_{2,1} \vee P_{3,2} \vee P_{2,3}) \wedge B_{2,2} \wedge \neg P_{1,2} \wedge \neg P_{2,1} \wedge \neg P_{3,2}$
   $\alpha \Leftrightarrow P_{2,3}$