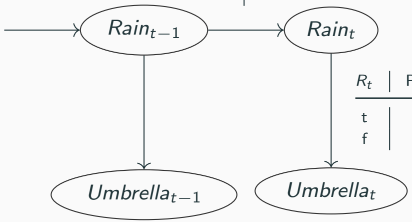
Lecture 28: Dynamic Bayesian Networks

Last Lecture

Efficient inference for Temporal Models (HMMs)

We have technically already seen Dynamic Bayesian Networks

$P(R_0) = \langle 0.5, 0.5 \rangle$



R_{t-1}	$P(R_t)$
t	0.7
f	0.3

R_t	$P(U_t)$
t	0.9
f	0.2

Example: Battery-Powered Robot

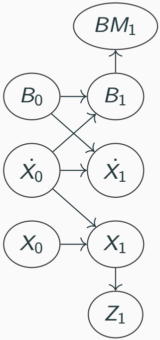
B_t – Battery Level

BM_t – Batter Meter Reading

X_t – $\langle x, y \rangle$ position

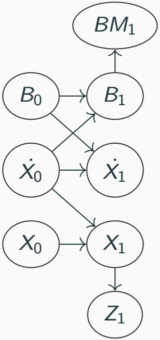
$\dot{X}_t$ – velocities

Z_t – GPS readings



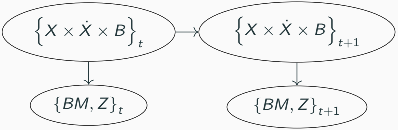
Inference Strategy 1: HMM-Conversion

$$P(X_4 | z_{1:4}, bm_{1:4})$$



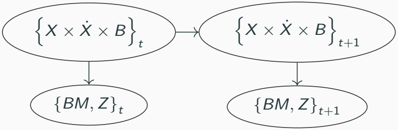
Inference Strategy 1: HMM-Conversion

$$P(X_4 | z_{1:4}, bm_{1:4})$$



Inference Strategy 1: HMM-Conversion

$$P(X_4|z_{1:4}, bm_{1:4})$$

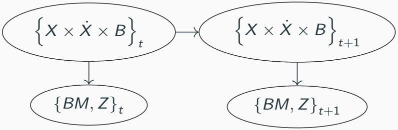


n – number of variables

d – domain size of largest variable

Inference Strategy 1: HMM-Conversion

$$P(X_4|z_{1:4}, bm_{1:4})$$



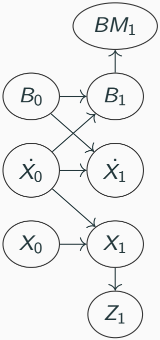
n – number of variables

d – domain size of largest variable

$$O(d^{2n})$$

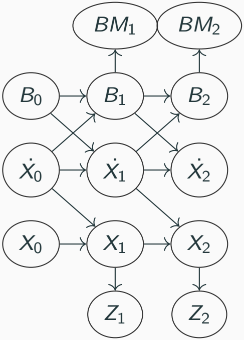
Inference Strategy 2: Unrolling

$$P(X_4 | z_{1:4}, bm_{1:4})$$



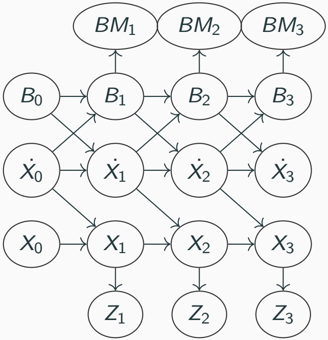
Inference Strategy 2: Unrolling

$$P(X_4 | z_{1:4}, bm_{1:4})$$



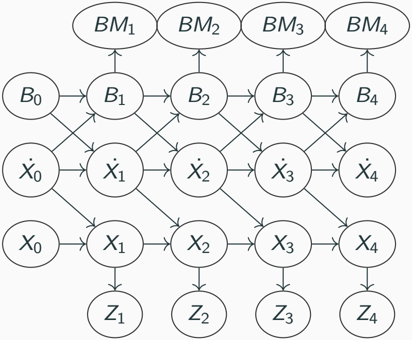
Inference Strategy 2: Unrolling

$$P(X_4 | z_{1:4}, bm_{1:4})$$

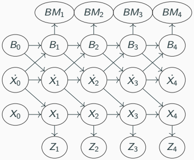


Inference Strategy 2: Unrolling

$$P(X_4|z_{1:4}, bm_{1:4})$$



Inference Strategy 2: Unrolling



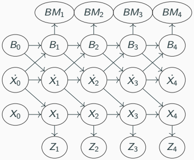
$$P(X_4|z_{1:4}, bm_{1:4})$$

$$= \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(x_{0:3}, \dot{x}_{0:3}, b_{0:4}, X_4 | z_{1:4}, bm_{1:4})$$

$$= \alpha \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(x_{0:3}, \dot{x}_{0:3}, b_{0:4}, X_4, z_{1:4}, bm_{1:4})$$

$$= \alpha \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(b_0)P(\dot{x}_0)P(x_0)P(b_1|b_0)P(\dot{x}1|b_0, \dot{x}_0)P(x_1|x_0, \dot{x}_0)P(z_1|x_1)\ldots$$

Inference Strategy 2: Unrolling



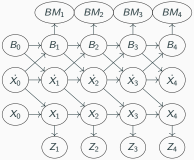
$$P(X_4|z_{1:4}, bm_{1:4})$$

$$= \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(x_{0:3}, \dot{x}_{0:3}, b_{0:4}, X_4|z_{1:4}, bm_{1:4})$$

$$= \alpha \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(x_{0:3}, \dot{x}_{0:3}, b_{0:4}, X_4, z_{1:4}, bm_{1:4})$$

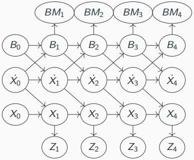
$$= \alpha \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(b_0)P(\dot{x}_0)P(x_0)P(b_1|b_0)P(\dot{x}1|b_0, \dot{x}_0)P(x_1|x_0, \dot{x}_0)P(z_1|x_1)\ldots$$

Inference Strategy 2: Unrolling

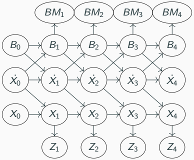


$$
\begin{aligned}
&P(X_4|z_{1:4}, bm_{1:4}) \\
&= \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(x_{0:3}, \dot{x}_{0:3}, b_{0:4}, X_4|z_{1:4}, bm_{1:4}) \\
&= \alpha \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(x_{0:3}, \dot{x}_{0:3}, b_{0:4}, X_4, z_{1:4}, bm_{1:4}) \\
&= \alpha \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(b_0)P(\dot{x}_0)P(x_0)P(b_1|b_0)P(\dot{x}1|b_0, \dot{x}_0)P(x_1|x_0, \dot{x}_0)P(z_1|x_1)\dots
\end{aligned}
$$

Inference Strategy 2: Unrolling



$$
\begin{aligned}
&P(X_4 | z_{1:4}, bm_{1:4}) \\
&= \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(x_{0:3}, \dot{x}_{0:3}, b_{0:4}, X_4 | z_{1:4}, bm_{1:4}) \\
&= \alpha \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(x_{0:3}, \dot{x}_{0:3}, b_{0:4}, X_4, z_{1:4}, bm_{1:4}) \\
&= \alpha \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(b_0) P(\dot{x}_0) P(x_0) P(b_1 | b_0) P(\dot{x}1 | b_0, \dot{x}_0) P(x_1 | x_0, \dot{x}_0) P(z_1 | x_1) \ldots
\end{aligned}
$$

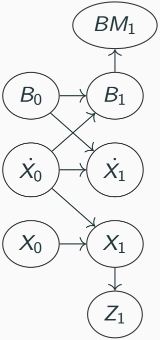


$$
\begin{aligned}
&P(X_4 | z_{1:4}, bm_{1:4}) \\
&= \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(x_{0:3}, \dot{x}_{0:3}, b_{0:4}, X_4 | z_{1:4}, bm_{1:4}) \\
&= \alpha \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} P(x_{0:3}, \dot{x}_{0:3}, b_{0:4}, X_4, z_{1:4}, bm_{1:4}) \\
&= \alpha \sum_{x_{0:3}, \dot{x}_{0:4}, b_{0:4}} \underbrace{P(b_0)}_{f_1(B_0)} \underbrace{P(\dot{x}_0)}_{f_2(\dot{X}_0)} \underbrace{P(x_0)}_{f_3(X_0)} \underbrace{P(b_1 | b_0)}_{f_4(B_0, B_1)} \underbrace{P(\dot{x}1 | b_0, \dot{x}_0)}_{f_5(\dot{X}_0, B_0, \dot{X}_1)} \underbrace{P(x_1 | x_0, \dot{x}_0)}_{f_6(\dot{X}_0, X_0, X_1)} \underbrace{P(z_1 | x_1)}_{f_7(X_1)} \ldots
\end{aligned}
$$

More Efficient / Approximate DBN Inference?

- 3rd Year Course: Introduction to Mobile Robotics
- Book: Probabilistic Robotics — Thrun, Burgard and Fox
- Paper: Dynamic Bayesian networks: Representation, inference and learning

Modelling Failure: Noise



B domain: $[0, 1, 2, 3, 4, 5]$

$$P(BM_t = i|B = j) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
\begin{array}{lllllllllll}
B_t: & [& 5, & 5, & 5, & 5, & 4, & 4, & 4, & 3, & 3] \\
BM_t: & [& 5.5, & 4.3, & 5.4, & 4.3, & 3.5, & 3.6, & 3.0, & 3.2, & 2.4]
\end{array}
$$

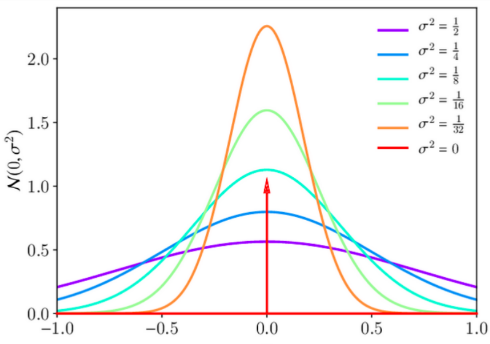
B_t :	[	5,	5,	5,	5,	4,	4,	4,	3,	3]
BM_t :	[	5.5,	4.3,	5.4,	4.3,	3.5,	3.6,	3.0,	3.2,	2.4]

$$f(BM_t = bm_t | B_t = b_t)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(bm_t - b_t)^2}{2\sigma^2}}$$

B_t :	[5,	5,	5,	5,	4,	4,	4,	3,	3]
BM_t :	[5.5,	4.3,	5.4,	4.3,	3.5,	3.6,	3.0,	3.2,	2.4]

$$f(BM_t = bm_t | B_t = b_t)$$
$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(bm_t - b_t)^2}{2\sigma^2}}$$



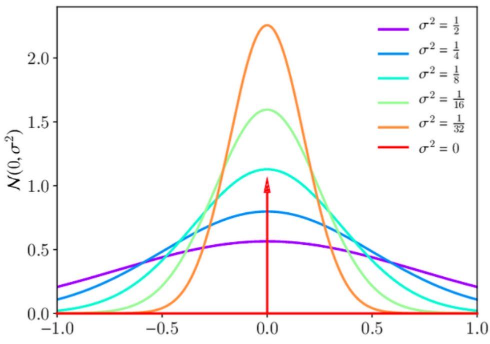
$$
\begin{array}{llllllllll}
B_t : & [\ 5, & 5, & 5, & 5, & 4, & 4, & 4, & 3, & 3] \\
BM_t : & [\ 5.5, & 4.3, & 5.4, & 4.3, & 3.5, & 3.6, & 3.0, & 3.2, & 2.4]
\end{array}
$$

$$
f(BM_t = bm_t | B_t = b_t)
$$

$$
= \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(bm_t - b_t)^2}{2\sigma^2}}
$$

$$
P(b_t - n * \sigma \leq BM_t \leq b_t + n * \sigma)
$$

$$
= \int_{b_t - n*\sigma}^{b_t + n*\sigma} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(BM_t - b_t)^2}{2\sigma^2}}
$$



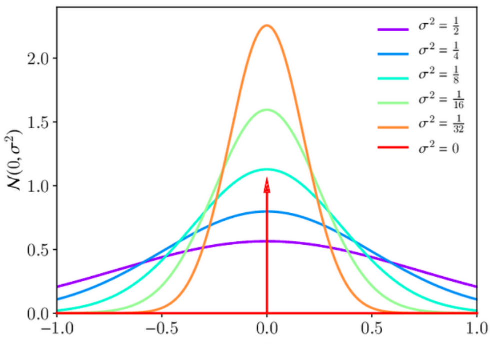
$$
\begin{array}{llllllllll}
B_t : & [\ 5, & 5, & 5, & 5, & 4, & 4, & 4, & 3, & 3] \\
BM_t : & [\ 5, & 4, & 5, & 4, & 4, & 4, & 3, & 3, & 2]
\end{array}
$$

$$f(BM_t = bm_t | B_t = b_t)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(bm_t - b_t)^2}{2\sigma^2}}$$

$$P(b_t - n * \sigma \leq BM_t \leq b_t + n * \sigma)$$

$$= \int_{b_t - n*\sigma}^{b_t + n*\sigma} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(BM_t - b_t)^2}{2\sigma^2}}$$



Modelling Failure: Transient Failure

$$
\begin{array}{ll}
B_t: & [\ 5,\ 5,\ 5,\ 5,\ 5,\ 5,\ 5,\ 5,\ 5] \\
BM_t: & [\ 5,\ 5,\ 5,\ 5,\ 0,\ 0,\ 5,\ 5,\ 5]
\end{array}
$$

Modelling Failure: Transient Failure

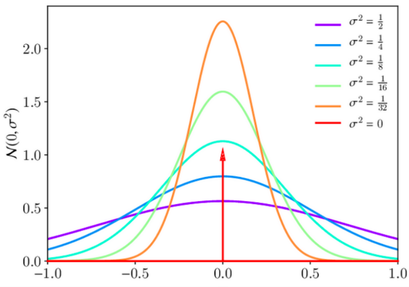
$$\begin{array}{ll} B_t: & [\ 5,\ 5,\ 5,\ 5,\ 5,\ 5,\ 5,\ 5,\ 5] \\ BM_t: & [\ 5,\ 5,\ 5,\ 5,\ 0,\ 0,\ 5,\ 5,\ 5] \end{array}$$

$$P(B_{21}|bm_{1:21})$$
$$= \alpha \underbrace{P(bm_{21}|B_{21})}_{sensor} \sum_{b_{20}} \underbrace{P(B_{21}|b_{20})}_{transition} \underbrace{P(b_{20}|bm_{1:20})}_{recursion}$$

Modelling Failure: Transient Failure

$$
\begin{array}{ll}
B_t: & [\;5,\;5,\;5,\;5,\;5,\;5,\;5,\;5,\;5] \\
BM_t: & [\;5,\;5,\;5,\;5,\;0,\;0,\;5,\;5,\;5]
\end{array}
$$

$$
\begin{aligned}
&P(B_{21} \mid bm_{1:21}) \\
&= \alpha \underbrace{P(bm_{21} \mid B_{21})}_{\text{sensor}} \sum_{b_{20}} \underbrace{P(B_{21} \mid b_{20})}_{\text{transition}} \underbrace{P(b_{20} \mid bm_{1:20})}_{\text{recursion}}
\end{aligned}
$$

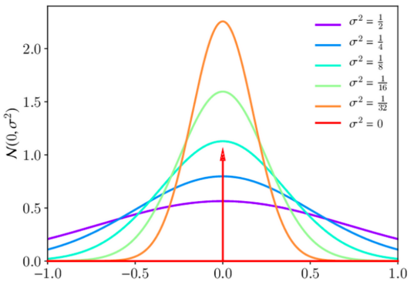


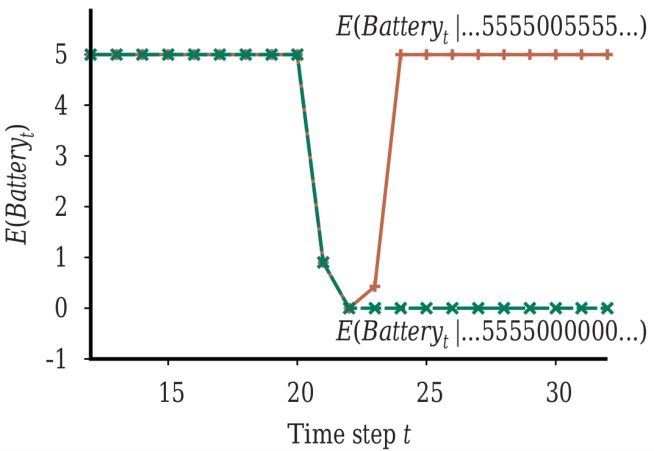
Modelling Failure: Transient Failure

$$
\begin{array}{ll}
B_t: & [\ 5,\ 5,\ 5,\ 5,\ 5,\ 5,\ 5,\ 5,\ 5] \\
BM_t: & [\ 5,\ 5,\ 5,\ 5,\ 0,\ 0,\ 5,\ 5,\ 5]
\end{array}
$$

$$
P(B_{21}|bm_{1:21}) \\
= \alpha \underbrace{P(bm_{21}|B_{21})}_{\text{sensor}} \sum_{b_{20}} \underbrace{P(B_{21}|b_{20})}_{\text{transition}} \underbrace{P(b_{20}|bm_{1:20})}_{\text{recursion}}
$$

$$
P(b_t - 5 * \sigma \leq BM_t \leq b_t + 5 * \sigma) \approx 0.0000005733
$$

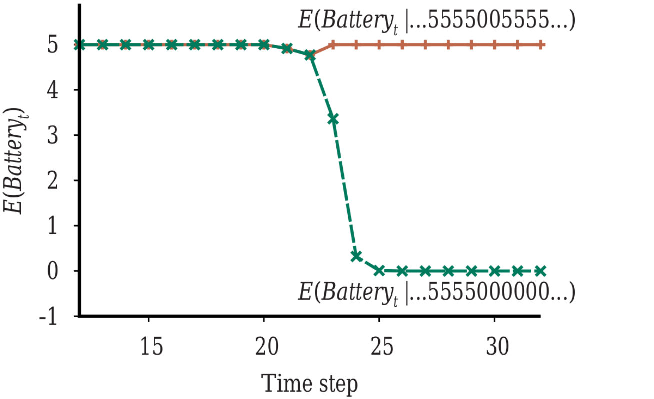



$E(Battery_t\,|\ldots5555005555\ldots)$
$E(Battery_t\,|\ldots5555000000\ldots)$
$E(Battery_t)$
5
4
3
2
1
0
-1
15
20
25
30
Time step t

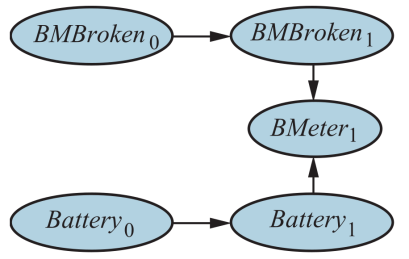
Fix Attempt 1: Include possibility of radical failure

$$P(BM_t = 0 | B_t = 5) = 0.05$$

Fix Attempt 1: Include possibility of radical failure

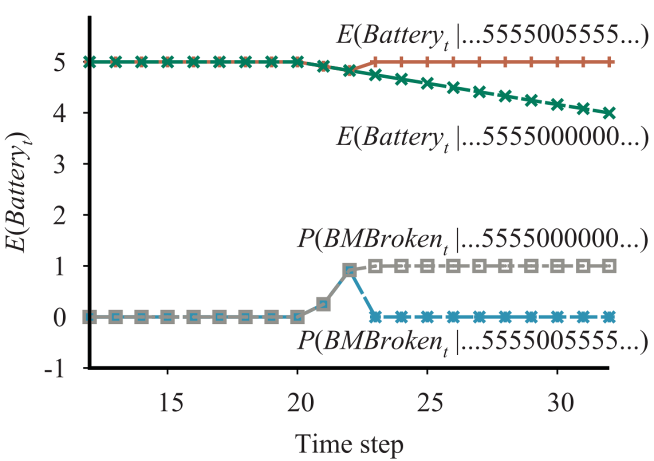


Fix Attempt 2: Persistent Failure Model



$BMBroken_{t-1}$	$BMBroken_t$
t	1.0
f	0.001

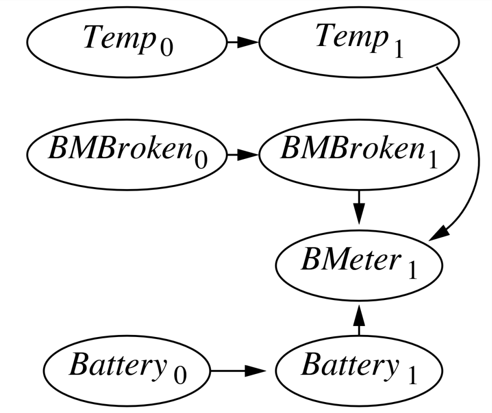
Fix Attempt 2: Persistent Failure Model



Bonus Q: Modelling other failures

From the battery meter manual:

> *"External temperature may affect battery sensor such that null (zero) readings become more likely as the temperature increases"*


$Temp_0$
$Temp_1$
$BMBroken_0$
$BMBroken_1$
$BMeter_1$
$Battery_0$
$Battery_1$

Summary

- DBNs — Unrolling and HMM Conversion
- Modelling Failure
  - Random Noise
  - Transient Failure
  - Persistent Failure

Semester 2 Feedback!

https://forms.office.com/e/71gDVje4j9

EUSA Awards

https://www.eusa.ed.ac.uk/whatson/awards/teachingawards