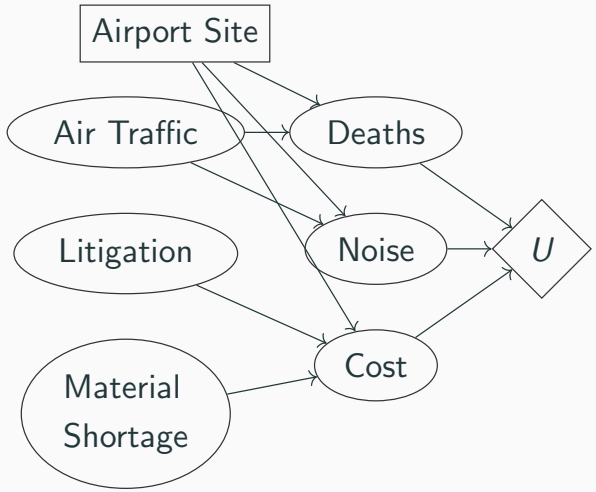
Lecture 29: Markov Decision Processes

A Map of the Landscape

Benign	Chaotic
Fully Observable	Partially Observable
Deterministic	Stochastic
Episodic	Sequential
Static	Dynamic
Discrete	Continuous
Single Agent	Multi-Agent
Passive Reasoning	Agent can Act
Fixed Goal	Quantified Preferences



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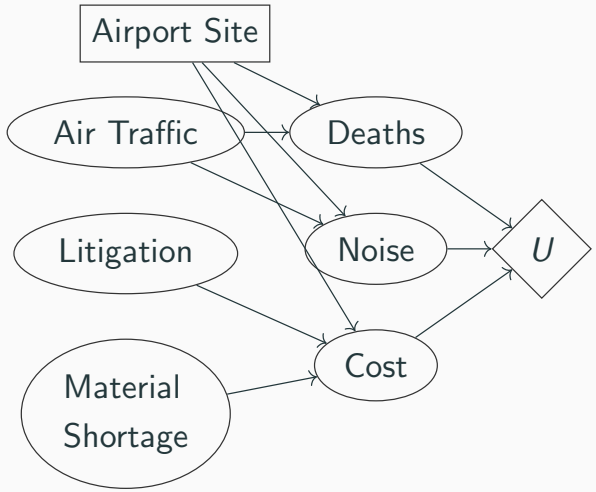
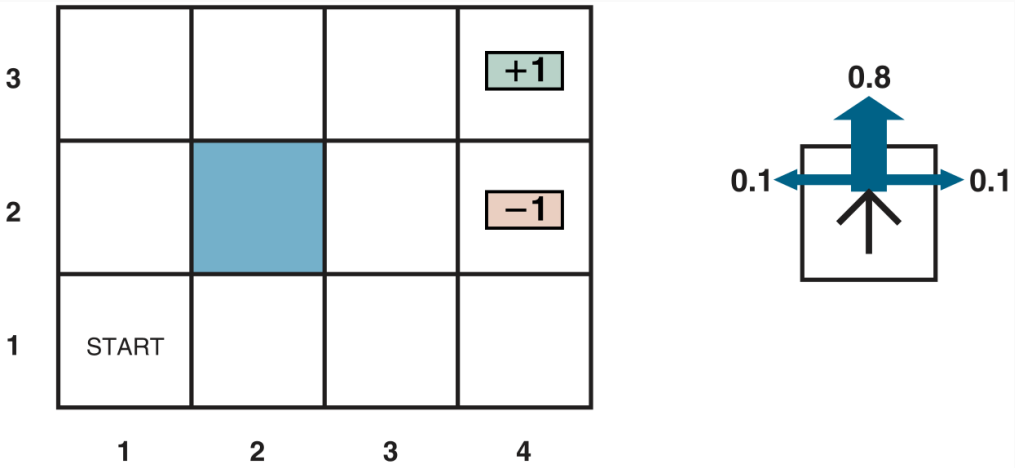
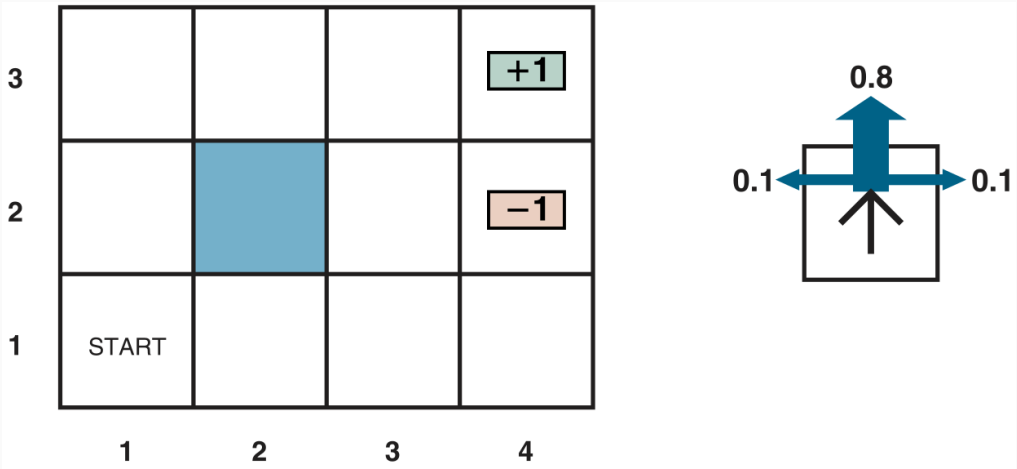


A Sequential Decision Problem



A Sequential Decision Problem

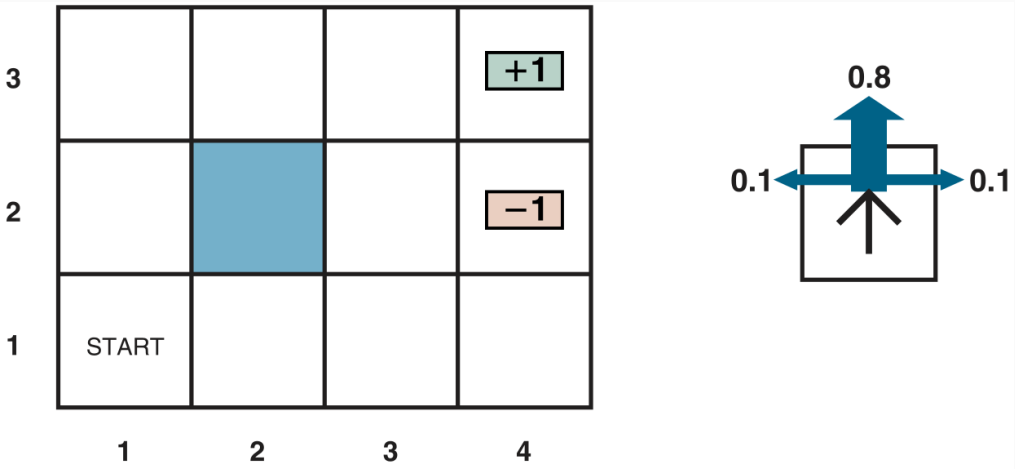


S_0 – initial state

$P(s'|s,a) = T(s,a,s')$ – transition model

$R(s)$ – reward function

A Sequential Decision Problem



S_0 – initial state

$P(s'|s,a) = T(s,a,s')$ – transition model

$R(s)$ – reward function

$$U_h([s_0, s_1, s_2, \ldots])$$

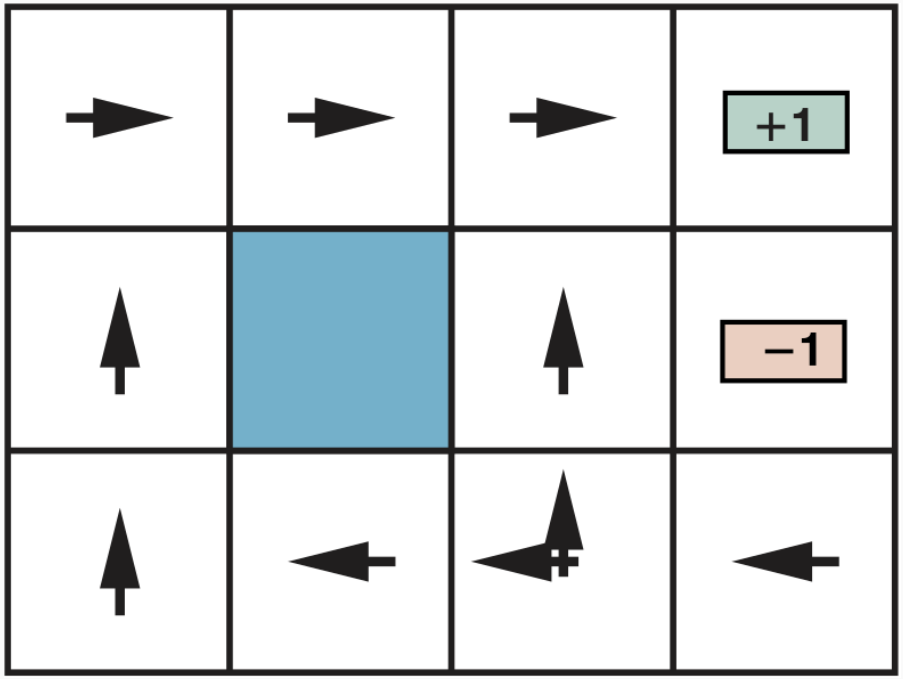
Markov Decision Process

$$MDP = \langle S_0, \mathcal{A}, T(s, a, s'), R(s) \rangle$$

$\pi(s) -$ **policy**

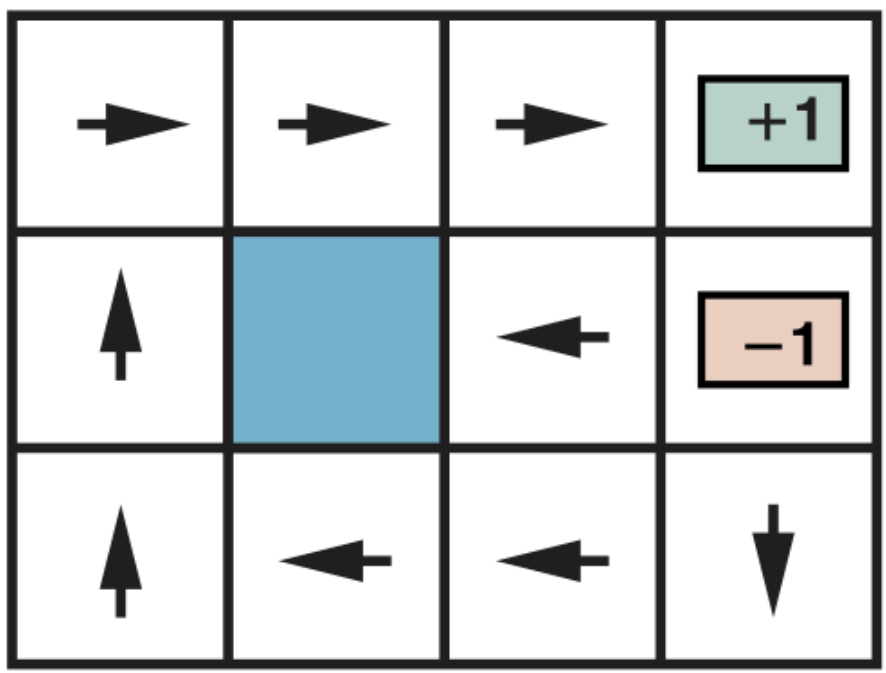
$\pi^*(s) -$ **optimal policy** (the one that yields highest expected utility)

Reward function affects optimal policy



$$R(s) = -0.04$$

Reward function affects optimal policy



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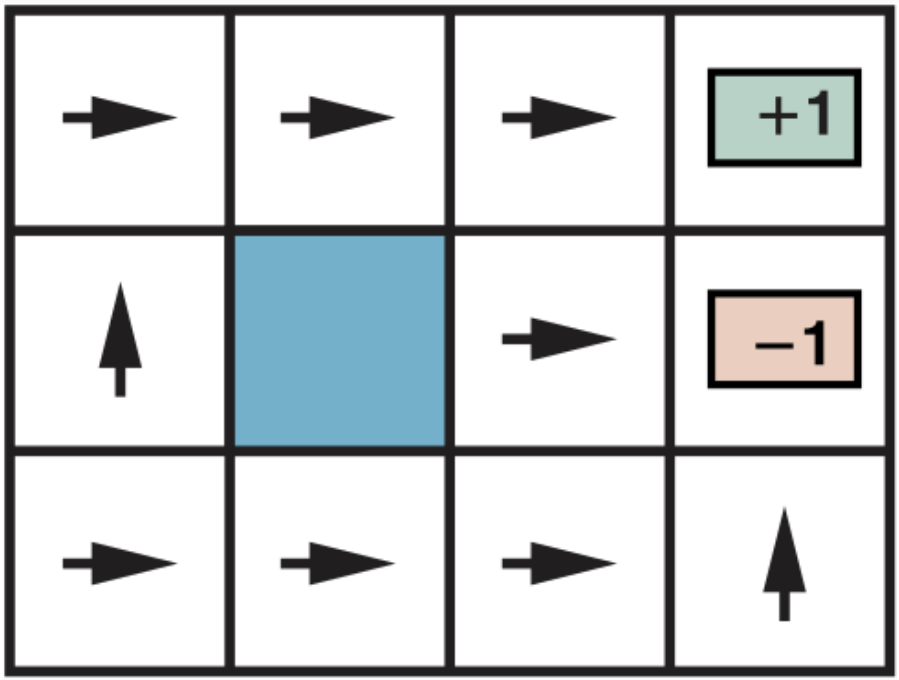
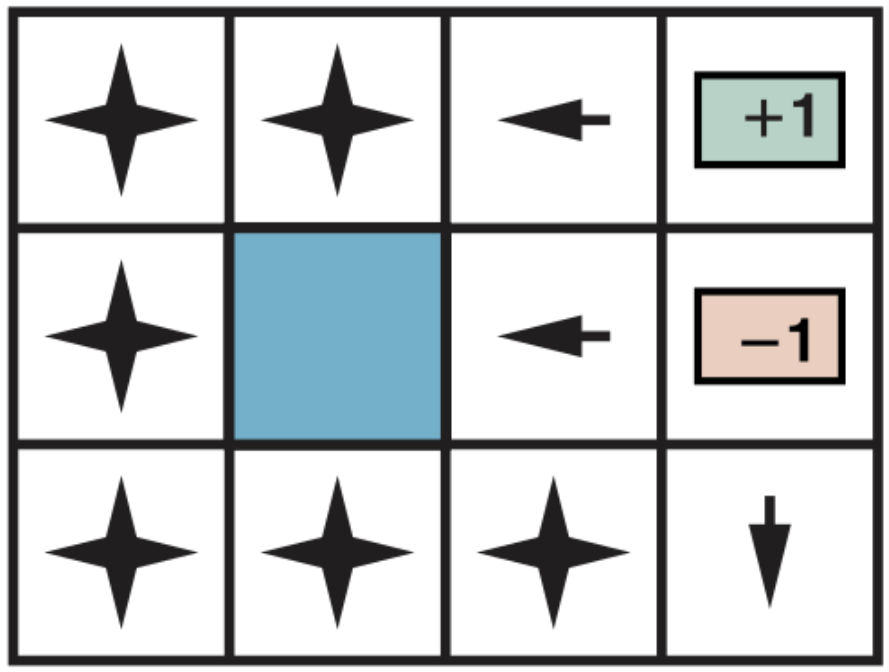
Reward function affects optimal policy



$$R(s) = -1.15$$

Reward function affects optimal policy



$R(s) = 2.0$

Finite vs Infinite Utilities

Infinite Horizon Utility Function:

$$U_h([s_1, s_2, \ldots])$$

Finite Horizon Utility Function:

$$\forall k, U_h([s_1, s_2, \ldots s_N, \ldots, s_{N+k}]) = U_h([s_1, s_2, \ldots s_N]) \quad \text{for fixed N}$$

Additive Reward and Infinite Horizons

$$U_h([s_0, s_1, s_2, \ldots]) = R(s_1) + R(s_2) + \ldots$$

Discounted Reward Formulation

$$U_h([s_0, s_1, s_2, \ldots]) = R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \gamma^3 R(s_3) + \ldots$$

$$= \sum_{t=0}^{\infty} \gamma^t R(s_t)$$

$$\leq \sum_{t=0}^{\infty} \gamma^t R_{max}$$

$$= \frac{R_{max}}{1-\gamma}$$

Discounted Reward Formulation

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\end{aligned}
$$

How good is a given state?

$$U^{\pi}(s) = E\left[\sum_{t=0}^{\infty} \gamma^t R(S_t)\right]$$

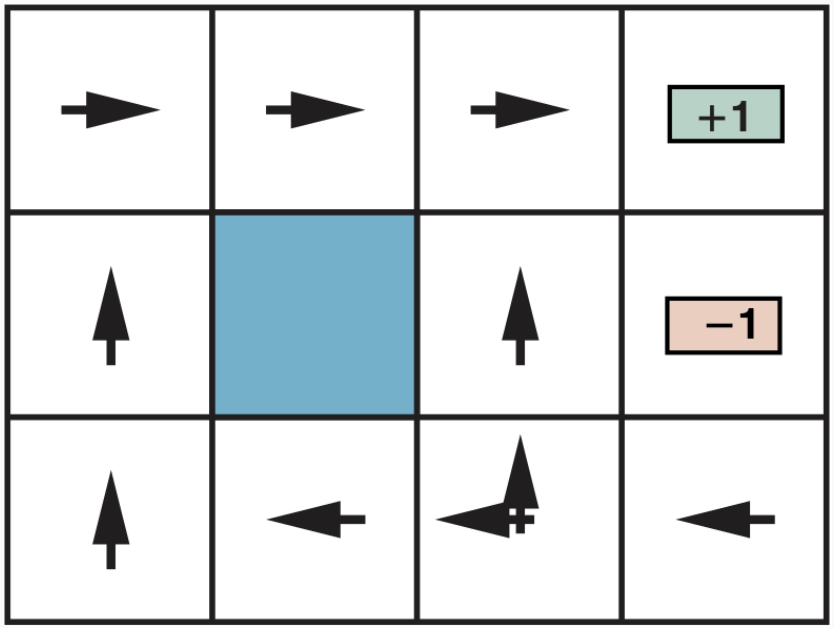
$$\pi_s^* = \underset{\pi}{argmax}\ U^{\pi}(s)$$

How good is a given state?

$$U^{\pi}(s) = E\left[\sum_{t=0}^{\infty} \gamma^t R(S_t)\right]$$

$$\pi_s^* = \underset{\pi}{argmax}\ U^{\pi}(s)$$

$$\gamma = 1, R(s) = 0.04$$



(a) $\pi(s)$

0.8516	0.9078	0.9578	+1
0.8016		0.7003	−1
0.7453	0.6953	0.6514	0.4279

(b) $U^{\pi}(s)$

Finding the Optimal Policy: The Bellman Equation

$$\pi^*(s) = \underset{\pi}{argmax}\ U^{\pi}(s)$$

$$\pi^*(s) = \underset{a}{argmax} \sum_{s'} P(s'|s,a) U^{\pi^*}(s')$$

Bellman Equation: $U^{\pi^*}(s) = R(s) + \gamma \max_a \sum_{s'} P(s'|s,a) U^{\pi^*}(s')$

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Bellman Equations

$$\textbf{Bellman Equation: } U^{\pi^*}(s) = R(s) + \gamma \max_a \sum_{s'} P(s'|s,a) U^{\pi^*}(s')$$

Q: Problem with n states, how many bellman equations?

Bellman Equations

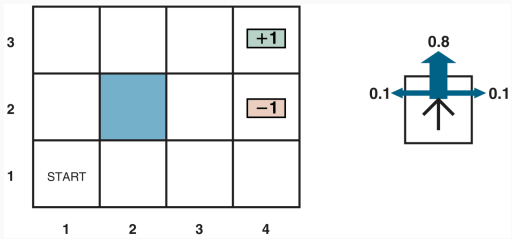
$$U^{\pi^*}(s_{\langle 1,1\rangle}) = R(s_{\langle 1,1\rangle}) + \gamma \max_a \sum_{s'} P(s'|s_{\langle 1,1\rangle}, a) U^{\pi^*}(s')$$

$$U^{\pi^*}(s_{\langle 1,2\rangle}) = R(s_{\langle 1,2\rangle}) + \gamma \max_a \sum_{s'} P(s'|s_{\langle 1,2\rangle}, a) U^{\pi^*}(s')$$

$$U^{\pi^*}(s_{\langle 1,3\rangle}) = R(s_{\langle 1,3\rangle}) + \gamma \max_a \sum_{s'} P(s'|s_{\langle 1,3\rangle}, a) U^{\pi^*}(s')$$

$$U^{\pi^*}(s_{\langle 1,4\rangle}) = R(s_{\langle 1,4\rangle}) + \gamma \max_a \sum_{s'} P(s'|s_{\langle 1,4\rangle}, a) U^{\pi^*}(s')$$

$\ldots$



Bellman Equations

$$U^{\pi^*}(s_{\langle 1,1\rangle}) = \underbrace{R(s_{\langle 1,1\rangle})}_{\textit{reward}} + \gamma \max_a \sum_{s'} \underbrace{P(s'|s_{\langle 1,1\rangle}, a)}_{\textit{transition}} \underbrace{U^{\pi^*}(s')}_{\textit{recursive}}$$

$$U^{\pi^*}(s_{\langle 1,2\rangle}) = R(s_{\langle 1,2\rangle}) + \gamma \max_a \sum_{s'} P(s'|s_{\langle 1,2\rangle}, a) U^{\pi^*}(s')$$

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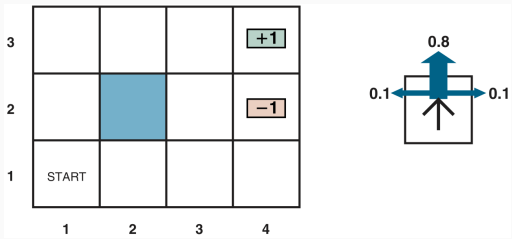


Value Iteration

Setup: Set Utility guesses to arbitrary values

$$U_0(s_{\langle 1,1 \rangle}) \leftarrow 0, U_0(s_{\langle 1,2 \rangle}) \leftarrow 0, \ldots$$

Repeat: One-step update Utilities using bellman equation

$$U_{i+1}(s_{\langle 1,1 \rangle}) \leftarrow R(s_{\langle 1,1 \rangle}) + \gamma \max_a \sum_{s'} P(s'|s_{\langle 1,1 \rangle}, a) U_i(s')$$

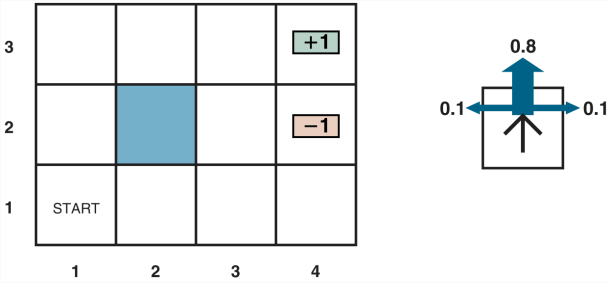
$$\ldots$$

$$U_{i+1}(s_{\langle 3,4 \rangle}) \leftarrow R(s_{\langle 3,4 \rangle}) + \gamma \max_a \sum_{s'} P(s'|s_{\langle 3,4 \rangle}, a) U_i(s')$$

Until: $U_{i+1}(s) \approx U_i(s)$ **for all** s

Example Update

$$
\begin{aligned}
U_{i+1}(s_{\langle 1,1\rangle}) \leftarrow -0.04 + \gamma \max[& \\
& 0.8U_i(s_{\langle 1,2\rangle}) + 0.1U_i(s_{\langle 2,1\rangle}) + 0.1U_i(s_{\langle 1,1\rangle}), && \textit{(up)} \\
& 0.9U_i(s_{\langle 1,1\rangle}) + 0.1U_i(s_{\langle 1,2\rangle}), && \textit{(left)} \\
& 0.9U_i(s_{\langle 1,1\rangle}) + 0.1U_i(s_{\langle 2,1\rangle}), && \textit{(down)} \\
& 0.8U_i(s_{\langle 2,1\rangle}) + 0.1U_i(s_{\langle 1,2\rangle}) + 0.1U_i(s_{\langle 1,1\rangle}) && \textit{(right)} \\
]
\end{aligned}
$$



Value Iteration in Action

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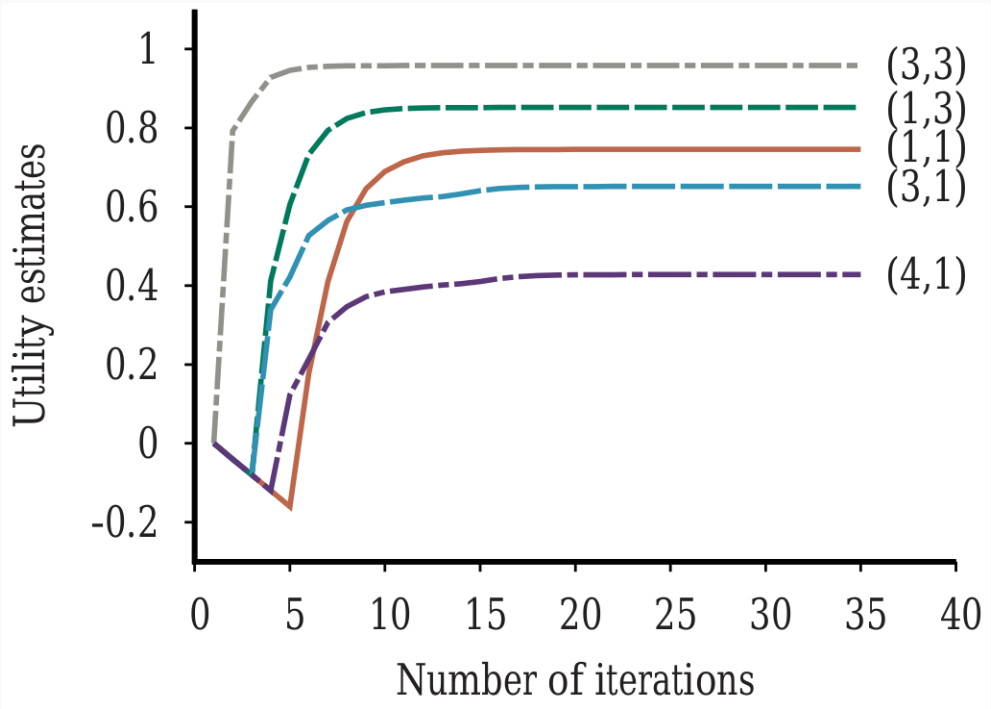
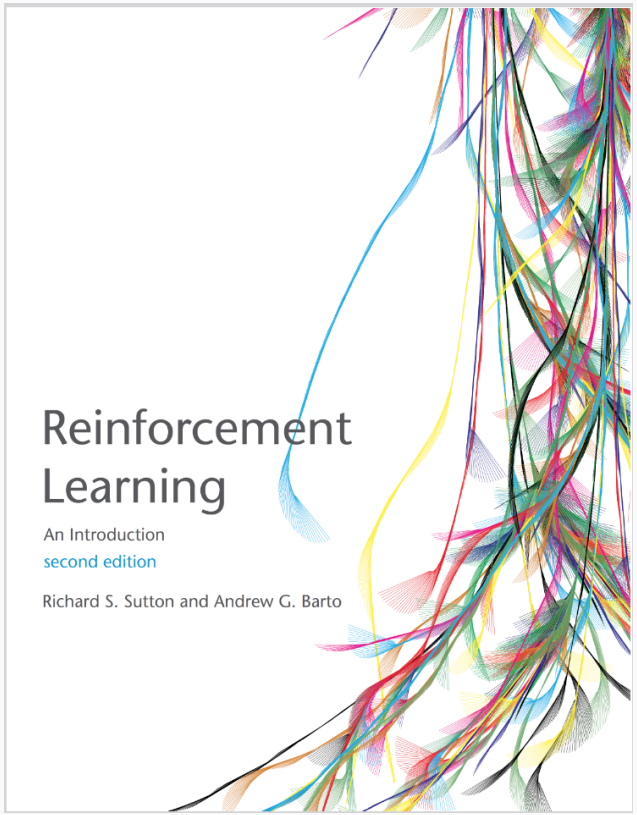


Wait! Thats it?

Courses

- Robot and Reinforcement Learning



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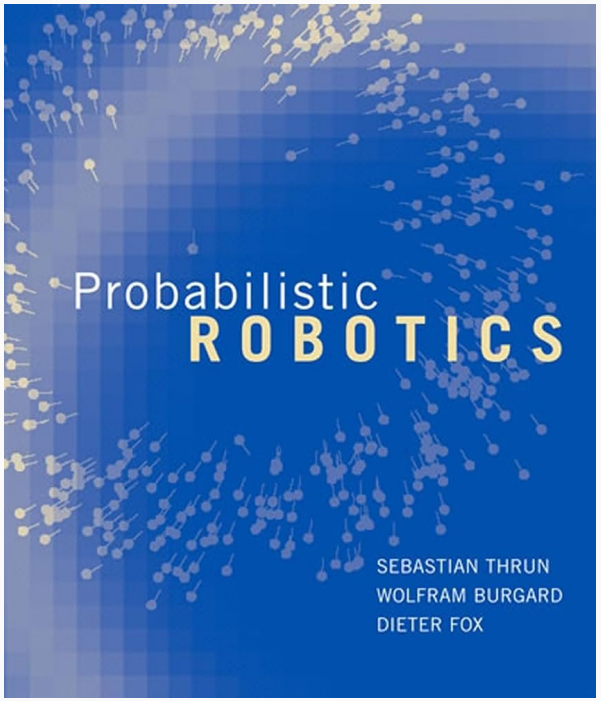
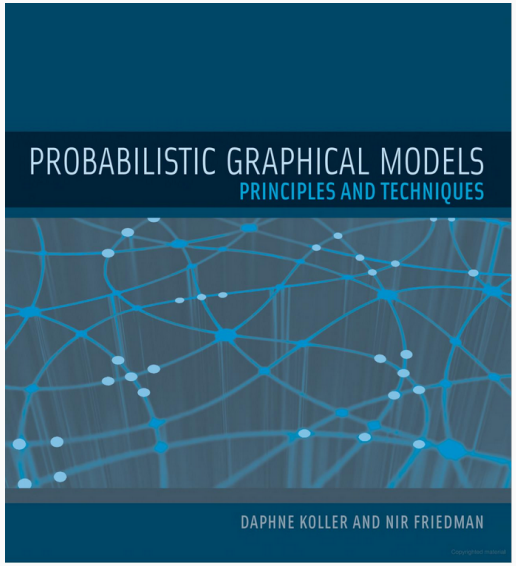
Continuous, Stochastic, Partially Observable, Sequential, . . .

Courses

- Introduction to Mobile Robotics (MOB)
- Advanced Robotics



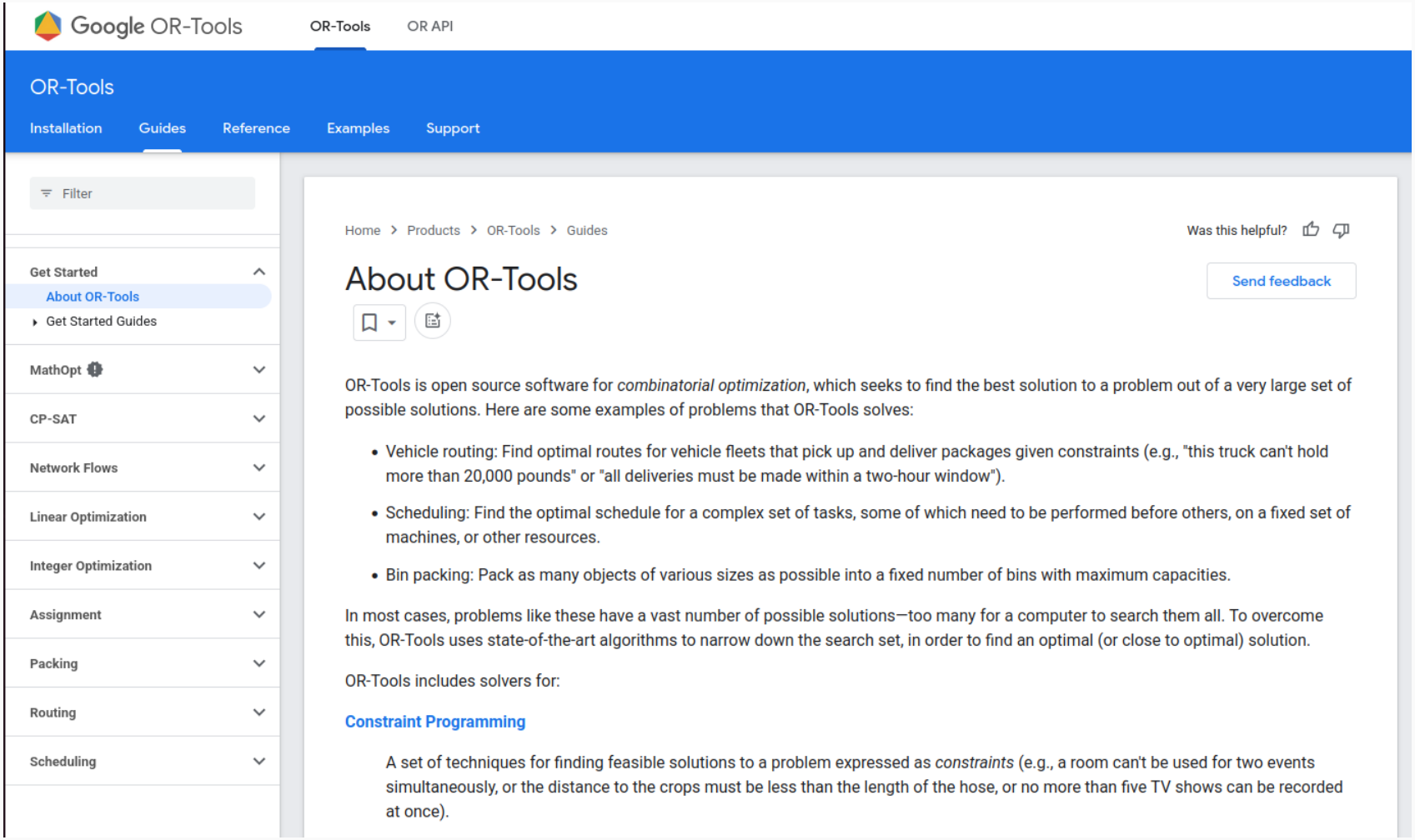
Graphical Models



Courses:

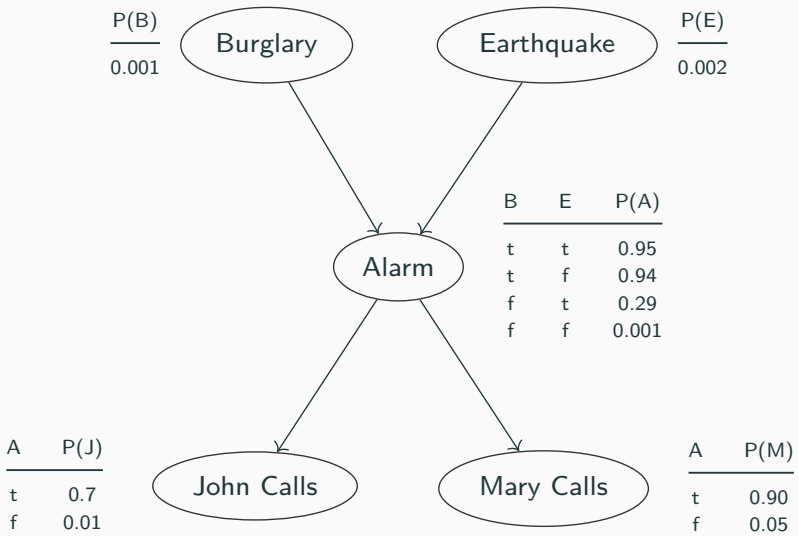
- Probabilistic Modelling and Reasoning (PMR)
- Methods for Causal Inference (MCI)

I liked Constraint Solving / AC3 Stuff



I liked PDDL / The coursework!

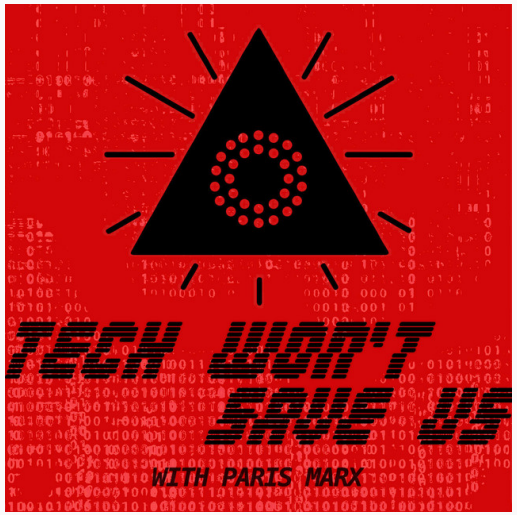
Where did these parameters come from?



Courses

- Machine Learning (MLG)
- Introductory Applied Machine Learning (IAML)
- Machine Learning Practical (MLP)
- Machine Learning Systems (MLS)
- Machine Learning Theory (MLT)
- Machine Learning and Pattern Recognition (MLPR)

I can make an AI agent...but should I?



Courses: Professional Issues, Modelling of Systems for Sustainability, Usable Security and Privacy

The Cynical Exam Prep Slide

- **Examiner Laziness:** Teachers rarely design exam Qs entirely from scratch. You can access the same resources they can (past exam papers are available online; tutorial solutions are out; required reading has exercises chapters...)
- **Not all Qs Created Equal**: The largest marks in compsci exams tend to be for algorithmic questions over bookwork Qs. Do you know how to implement the algorithms that have come up in the course?
- **Homo-Economicus**: Stuck on a 2-mark question? Move on and come back!
- **Worked Example Hinting:** Does a lecture follow up a particular method with a long worked-through example? Sounds like the lecturer really wants to make sure you understand this concept!
- **Pay Attention to "Weird Specifics":** question explicitly instructs you to run through the algorithm "in a particular order", "round to x decimal places", "justify answer with respect to specific definition"? Might be a clue that this question could get really hard if you dont follow this advice!