Lecture 20: Uncertainty, Rationality, Probability

Craig Innes

Attacking Uncertainty in Planning

Attacking Uncertainty in Planning

- **Conformant Planning**
- **Contingency planning**
- **Online planning & replanning**

Uncertainty Issues: Rarely a guaranteed action

A_{90} — *Leave home 90 minutes before flight and drive at reasonable speed*

"Plan A_{90} will get us to the airport in time...

Uncertainty Issues: Rarely a guaranteed action

A_{90} — *Leave home 90 minutes before flight and drive at reasonable speed*

> *"Plan A_{90} will get us to the airport in time...as long as the car doesn't break down, and I don't get into an accident, and the road isn't closed, and no meteorite hits the car, and even then..."*

Uncertainty Issues: Rarely a guaranteed action

A_{90} — *Leave home 90 minutes before flight and drive at reasonable speed*

> *"Plan A_{90} will get us to the airport in time...as long as the car doesn't break down, and I don't get into an accident, and the road isn't closed, and no meteorite hits the car, and even then..."*

A_{180} — *Leave home 180 minutes before flight and drive at reasonable speed*

> *"Plan A_{180} will get us to the airport in time as long as the car doesn't break down, and I don't get into an accident, and the road isn't closed, and no meteorite hits the car, and even then..."*

Rational decision making

The right thing to do — the **rational decision** *— depends on both the relative importance of various goals and the likelihood that, and degree to which, they will be achieved*

Uncertainty Issues: Logical Qualification

$Toothache \implies Cavity$

Uncertainty Issues: Logical Qualification

$$Toothache \implies Cavity$$

$$Toothache \implies Cavity \lor GumProblem \lor Abscess \ldots$$

Uncertainty Issues: Logical Qualification

$Toothache \implies Cavity$

$Toothache \implies Cavity \lor GumProblem \lor Abscess \dots$

$Cavity \implies Toothache$

Uncertainty Issues: Logical Qualification

$Toothache \implies Cavity$

$Toothache \implies Cavity \lor GumProblem \lor Abscess \ldots$

$Cavity \implies Toothache$

Problems

- Laziness
- Theoretical Ignorance
- Practical Ignorance

$$Toothache \implies Cavity$$

$$Toothache \implies Cavity$$

"Given that the patient has a toothache, there's an 80% chance she has a cavity"

$$Toothache \implies Cavity$$

"Given that the patient has a toothache, there's an 80% chance she has a cavity"

"Given that the patient has a toothache and a history of gum disease, theres a 40% chance she has a cavity"

$$Toothache \implies Cavity$$

"Given that the patient has a toothache, there's an 80% chance she has a cavity"

"Given that the patient has a toothache and a history of gum disease, theres a 40% chance she has a cavity"

"Given that i've done a thorough inspection of the patient's mouth, the probability they have a cavity is almost 0"

Ontological / Epistemological Commitments

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional Logic	facts	true/false/unknown
First-Order Logic	facts, objects, relations	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$

Ontological / Epistemological Commitments

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional Logic	facts	true/false/unknown
First-Order Logic	facts, objects, relations	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy Logic	facts w truth degree $\in [0, 1]$	known interval value

Decision Theory

A_{90} — *Leave home 90 minutes before flight and drive at reasonable speed*

95% chance of getting to airport on time

A_{180} — *Leave home 180 minutes before flight and drive at reasonable speed*

99% chance of getting to airport on time

$$\text{Decision Theory} = \text{Probability Theory} + \text{Utility Theory}$$

Principle of Maximum Expected Utility

An agent is rational if and only if it chooses the action that yields the highest expected utility, **averaged over all possible outcomes of the action.**

Allais Paradox

A : 80% chance of £4000

B : 100% chance of £3000

Allais Paradox

A : 80% chance of £4000

B : 100% chance of £3000

C : 20% chance of £4000

D : 25% chance of £3000

Allais Paradox

A : 80% chance of £4000

B : 100% chance of £3000

C : 20% chance of £4000

D : 25% chance of £3000

If you chose B, then chose C. You are **irrational**

$$U(3000) > 0.8 * U(4000)$$

$$0.25 * U(3000) < 0.2 * U(4000)$$
$$= U(3000) < 0.8 * U(4000)$$

Ambiguity Aversion

An Urn contains:

- $\frac{1}{3}$ **Red** balls
- $\frac{2}{3}$ some mix of **Black** and **Yellow** balls

A : £100 if you draw a red ball

B : £100 if you draw a black ball

Ambiguity Aversion

An Urn contains:

- $\frac{1}{3}$ **Red** balls
- $\frac{2}{3}$ some mix of **Black** and **Yellow** balls

A : £100 if you draw a red ball

B : £100 if you draw a black ball

C : £100 if you draw a red or yellow ball

D : £100 if you draw a black or yellow ball

Ambiguity Aversion

An Urn contains:

- $\frac{1}{3}$ **Red** balls
- $\frac{2}{3}$ some mix of **Black** and **Yellow** balls

A : £100 if you draw a red ball

B : £100 if you draw a black ball

C : £100 if you draw a red or yellow ball

D : £100 if you draw a black or yellow ball

Most people prefer A over B, but *also* prefer D over C!

Knowledge of Preferences

Ice Cream Shop Serves two ice cream flavours:

- Vanilla (£2)
- Durian (£1.50)

Principle of Maximum Expected Utility

An agent is rational if and only if it chooses the action that yields the highest expected utility, **averaged over all possible outcomes of the action.**

Decision Theory = *Probability Theory* + *Utility Theory*

Probability Theory: Basic Terminology

Random Variable: A part of the world whose status is unknown

Domain: The values a random variable can take on

$$\mathit{Cavity} : \langle \mathit{true}, \mathit{false} \rangle$$

$$(\mathit{Cavity} = \mathit{true} \wedge \mathit{Toothache} = \mathit{false})$$

Probability Theory: Basic Terminology

Atomic event: A complete specification of the state of the world.

Probability Theory: Basic Terminology

Atomic event: A complete specification of the state of the world.

$$Cavity, Toothache, GumDisease$$

$$\langle Cavity = true, Toothache = true, GumDisease = true\rangle$$
$$\langle Cavity = true, Toothache = true, GumDisease = false\rangle$$
$$\vdots$$
$$\langle Cavity = false, Toothache = false, GumDisease = false\rangle$$

Probability Theory: Basic Terminology

Unconditional / Prior probability: degree of belief in a proposition in *absence of any other information*.

$$P(Cavity = true) = 0.1$$
$$P(cavity) = 0.1$$

Probability distribution: probabilities of all values of a random variable

$$P(Weather) = \langle 0.7, 0.2, 0.1 \rangle$$

$$P(Weather = sunny) = 0.7$$
$$P(Weather = rain) = 0.2$$
$$P(Weather = cloudy) = 0.1$$

Probability Theory: Basic Terminology

Joint probability distribution (JPD) $P(X, Y)$: The cross-product of individual distributions X and Y.

Probability Theory: Basic Terminology

Joint probability distribution (JPD) $P(X, Y)$: The cross-product of individual distributions X and Y.

Conditional probability $P(a|b)$: the probability of a given that all we know is b

$$P(cavity|toothache) = 0.8$$

Conditional Probabilities

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

Product Rule: $P(a \wedge b) = P(a|b)P(b)$

Axioms of Probability (Kolmogorov's Axioms)

- $0 \leq P(a) \leq 1$, for any proposition a
- $P(true) = 1, P(false) = 0$
- $P(a \vee b) = P(a) + P(b) - P(a \wedge b)$

$$\mathbf{P}(\neg \mathbf{a}) = \mathbf{1} - \mathbf{P}(\mathbf{a})$$

$$\mathbf{P(\neg a) = 1 - P(a)}$$

$$\begin{aligned}
P(\neg a \vee a) &= P(true) \\
&= \color{red}{1} \\
&= P(\neg a) + P(a) - P(\neg a \wedge a) \\
&= P(\neg a) + P(a) - 0 \\
&= \color{red}{P(\neg a) + P(a)} \\
P(\neg a) &= 1 - P(a)
\end{aligned}$$

Any JPD over finite variables sums to 1

For variable D with domain $\langle d_1, \ldots d_n \rangle$, $\sum_{i=1}^{n} P(D = d_i) = 1$

Any JPD over finite variables sums to 1

For variable D with domain $\langle d_1, \ldots d_n \rangle$, $\sum_{i=1}^{n} P(D = d_i) = 1$

$$
\begin{aligned}
P(D = d_1 \vee \cdots \vee D = d_n) &= P(true) \\
&= \ldots
\end{aligned}
$$

Extra: Why Probability Theory?

Why **not** have an agent that believes:

$$P(a) = 0.4, P(b) = 0.3, P(a \wedge b) = 0.0, P(a \vee b) = 0.8$$

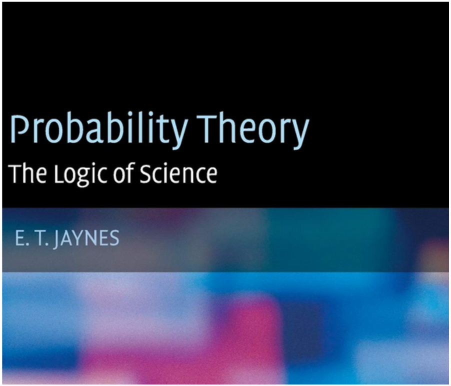
Extra: Why Probability Theory?

Don't be a sucker at gambling (aka "De Finetti's Theorem"):

> *If an agent has some degree of belief in a proposition a, then the agent should be able to state odds at which it is indifferent to a bet for or against a.*
>
> *If one's degrees of belief do not accurately reflect the world, then one would expect to lose money over the long run to an opposing agent whose beliefs more accurately reflect the state of the world.*

Extra: Why Probability Theory?

Probability should behave like logic



Summary

- Insufficiency of logic to model uncertainty
- Rational Decision making
- Probability Theory - random Variables, atomic events, distributions, conditional probability
- Axioms of Probability