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Tutorial 0

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Problem 1: Complex Numbers

Consider the two complex numbers $v_1 = 1 + i$ and $v_2 = 1 - 2i$ where $i^2 = -1$.

a. Calculate the complex numbers $z_1 = v_1 + v_2$ and $z_2 = v_1 - v_2^*$ where z^* denotes the complex conjugate of the complex number z.

b. Let $w = 1 - i$. Calculate wz_1 and $(z_2 w)^*$.

c. Calculate the norm of v_1 and v_2.

Problem 2: Inner-product and orthonormal bases

a. Consider the quantum states $|R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$, $|L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$,

1. Write $\langle R|$ and $\langle L|$ in vector notation.
2. Prove that both $|R\rangle$ and $|L\rangle$ are normalized, i.e. $\sqrt{\langle R|R\rangle} = \sqrt{\langle L|L\rangle} = 1$
3. Are $|R\rangle$ and $|L\rangle$ orthogonal?
4. Show that $|R\rangle$ and $|L\rangle$ satisfy all the conditions of an orthonormal basis of $\mathcal{H} = \mathbb{C}^2$.

Problem 3: Matrices and operators

a.

1. One of the most important linear operators in quantum computing is the *Hadamard operator* defined as:
$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$
Find what is the action of the operator on the vector $|v\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$.

2. Consider two of the Pauli matrices:
$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
Calculate XZ and ZX. Compare the two calculations.

b.

1. Show that for finite-size matrices $(A^\dagger)^\dagger = A$ always holds.

2. Prove that for two general matrices A and B we have $(AB)^\dagger = B^\dagger A^\dagger$.

3. Prove that the Hadamard operator defined above is a self-adjoint operator.

c. Compute the eigenvalues and eigenvectors of X and Z.

Optional: More complex numbers

a. Use the *Euler equation*, i.e. $e^{i\theta} = \cos\theta + i\sin\theta$, to calculate $e^{i\pi}$ and $e^{2i\pi/4}$.

b. Let $z = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$. First calculate $|z|$ and then use the Euler equation to obtain ϕ so that $z = |z|e^{i\phi}$.