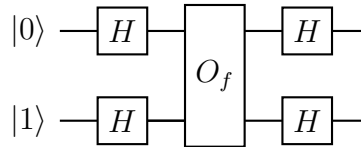


## Problem 1: Deutsch Algorithm

Consider the following circuit:



- a. The oracle  $O_f$  is a two-qubit gate that maps  $|x\rangle |y\rangle \rightarrow |x\rangle |y \oplus x\rangle$ . By comparing to what we have seen in the lectures, what is the classical function implemented by the oracle  $O_f$ , i.e.  $f(x)$ ? Do you think  $f(x)$  will be balanced or constant?
- b. What is the circuit of  $O_f$ ?
- c. Compute the two-qubit output state of this circuit and the probability of getting an outcome 0 when measuring the upper qubit.
- d. Having found the probability of getting outcome 0 on the upper qubit, conclude whether the function is balanced or constant. Justify your answer.

## Problem 2: Phase kick-back

Suppose you have the balanced function  $f : \{0, 1\}^2 \rightarrow \{0, 1\}$  such that:

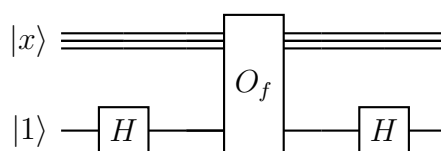
$$\begin{aligned} f(0, 0) &= 0, f(0, 1) = 1 \\ f(1, 0) &= 1, f(1, 1) = 0 \end{aligned}$$

- a. A classical oracle  $C_f$  returns for a given query input  $\bar{x}$  the value  $f(\bar{x})$  of the Boolean function  $f : \{0, 1\}^n \rightarrow \{0, 1\}$ . We have seen that for every classical oracle there exist a quantum oracle  $O_f$  satisfying

$$O_f |\bar{x}\rangle |q\rangle = |\bar{x}\rangle |q \oplus f(\bar{x})\rangle.$$

Provide a circuit of 3 qubits implementing  $O_f$  for the function given above.

- b. In the lecture we have seen that the Oracle  $O_f$  can be used to implement a phase-kickback unitary  $U_f$  acting on the address qubits. Show that the circuit below



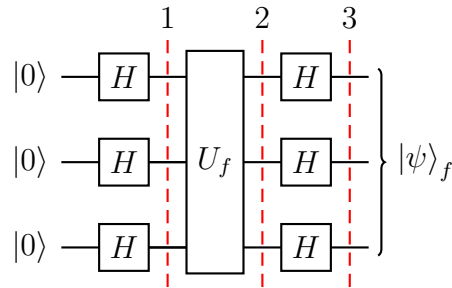
is equivalent to applying the phase kick-back  $U_f$  to the address register:

$$U_f |x\rangle = (-1)^{f(x)} |x\rangle$$

C. What is the size of the address register in the Deutsch algorithm in Problem 1?

### Problem 3: Bernstein-Vazirani Algorithm

Consider the function  $f(x) = ax \pmod 2$  with the string  $a$  being  $a = '111'$ . The goal is to find  $a$  with a single call to the phase kick-back  $U_f$ . Consider the quantum circuit implementing the Bernstein-Vazirani algorithm:



- a. Write the quantum state at stage 1 of the figure above, i.e. after the first layer of parallel Hadamard gates.
- b. What transformation does the oracle  $U_f$  perform on the state  $|x\rangle$ ,  $x$  is string of 3 bits encoding the computational basis of 3 qubits?
- c. Calculate the state of the composite system at stage 2 of the circuit.
- d. Derive the action of a layer of three Hadamard gates (Walsh-Hadamard transform) on a computational state  $|x_1x_2x_3\rangle$  of three qubits.
- e. Provide the quantum state at stage 3 of the computation.
- f. Suppose that we perform a measurement. What is the probability the output being the  $|000\rangle$  state? What would be the probability of obtaining  $|111\rangle$ ?