Raul Garcia-Patron
Petros Wallden
Ioannis Kolotouros

Tutorial 8

IQC 2022-23
November 23, 2023

Problem 1

Recall from the lectures the definition of the J gate as:

$$J(\theta) = HR(\theta) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\theta} \\ 1 & -e^{i\theta} \end{pmatrix}$$

Additionally, recall that any single qubit gate can be decomposed using J gates:

$$U = J(0)J(\theta_1)J(\theta_2)J(\theta_3), \tag{1}$$

for some θ_1, θ_2, θ_3.

a. Using Eq. (1) express the general single-qubit unitary U as a matrix that its elements depend on $\theta_1, \theta_2, \theta_3$.

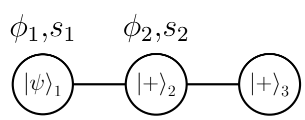
b. Using the matrix form for U derived in the first part of the question (known as the J-decomposition), express the gates Z and X in the form of Eq. (1), i.e. find the corresponding angles $\theta_1, \theta_2, \theta_3$.

c. Using this decomposition, find a five-qubit measurement pattern that implements the gate Z and the same for the gate X.

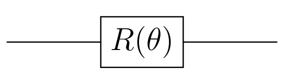
Note: This is not the simplest way to implement X, Z with an MBQC measurement pattern. Can you guess, by inspection, simpler measurement patterns for X, Z?

Problem 2

Consider the following MBQC graph with the input state $|\psi\rangle_1$ and the output on qubit 3.

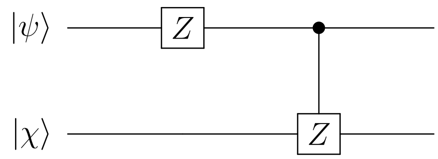


Find the angles ϕ_1 and ϕ_2 so that the MBQC graph is equivalent to an application of the rotation θ gate:



Problem 3

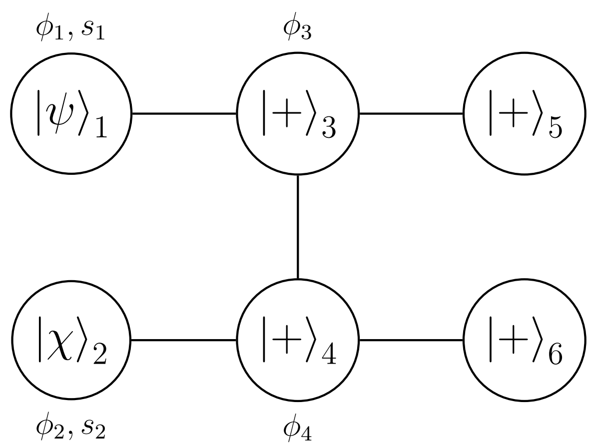
Find a measurement pattern that implements the following quantum circuit. You need to give the graph and default measurement angles that implement the said circuit, while you can ignore the "corrections".



Hint: You need the second qubit, while the first qubit implements the gate Z, to do nothing i.e. implement the identity gate I.

Problem 4

Consider the following MBQC graph state. Assume that the input is the following product state $|\psi\rangle_1 |\chi\rangle_2$, and that the outputs are the qubit 5 and the qubit 6. The flow of the measurement pattern is the standard one, goes horizontally from left to right, i.e. $f(i) = i + 2$. The measurement pattern is defined with the following "default" measurement angles: $\phi_1 = 0, \phi_2 = \pi, \phi_3 = -\pi/4$ and $\phi_4 = 0$.



a. What two-qubit unitary does the above measurement pattern implement?

b. Assume that we perform the above measurement pattern, and in the corresponding measurements we first get the outcomes: $s_1 = 1$, $s_2 = 1$.

Find the sets of vertices $S_z(3)$ and $S_x(3)$ of Z and X corrections for qubit 3.

Find the corrected measurement angle ϕ'_3 that the third qubit should be measured.