Problem 1

Recall from the lectures the definition of the $J$ gate as:

$$J(\theta) = H R(\theta) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\theta} \\ 1 & -e^{i\theta} \end{pmatrix}$$

Additionally, recall that any single qubit gate can be decomposed using $J$ gates:

$$U = J(0)J(\theta_1)J(\theta_2)J(\theta_3),$$

for some $\theta_1, \theta_2, \theta_3$.

a. Using Eq. (1) express the general single-qubit unitary $U$ as a matrix that its elements depend on $\theta_1, \theta_2, \theta_3$.

b. Using the matrix form for $U$ derived in the first part of the question (known as the $J$-decomposition), express the gates $Z$ and $X$ in the form of Eq. (1), i.e. find the corresponding angles $\theta_1, \theta_2, \theta_3$.

c. Using this decomposition, find a five-qubit measurement pattern that implements the gate $Z$ and the same for the gate $X$.

Note: This is not the simplest way to implement $X, Z$ with an MBQC measurement pattern.
Can you guess, by inspection, simpler measurement patterns for $X, Z$?

Problem 2

Consider the following MBQC graph with the input state $|\psi\rangle_1$ and the output on qubit 3.

$\phi_{1,s_1} \quad \phi_{2,s_2}$

$|\psi\rangle_1 \quad |+\rangle_2 \quad |+\rangle_3$

Find the angles $\phi_1$ and $\phi_2$ so that the MBQC graph is equivalent to an application of the rotation $\theta$ gate:

$R(\theta)$

Problem 3

Find a measurement pattern that implements the following quantum circuit. You need to give the graph and default measurement angles that implement the said circuit, while you can ignore the “corrections”.
Problem 4

Consider the following MBQC graph state. Assume that the input is the following product state $|\psi\rangle_1 |\chi\rangle_2$, and that the outputs are the qubit 5 and the qubit 6. The flow of the measurement pattern is the standard one, goes horizontally from left to right, i.e. $f(i) = i + 2$. The measurement pattern is defined with the following “default” measurement angles: $\phi_1 = 0, \phi_2 = \pi, \phi_3 = -\pi/4$ and $\phi_4 = 0$.

a. What two-qubit unitary does the above measurement pattern implement?

b. Assume that we perform the above measurement pattern, and in the corresponding measurements we first get the outcomes: $s_1 = 1, s_2 = 1$.

Find the sets of vertices $S_z(3)$ and $S_x(3)$ of $Z$ and $X$ corrections for qubit 3.

Find the corrected measurement angle $\phi'_3$ that the third qubit should be measured.