# Introduction to Quantum Computing
# Lecture 23: Measurement-Based Quantum Computing (MBQC) I

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# This Lecture

1. Measurement-Based Quantum Computing:

   What, Why & How

2. The $J(\theta)$ quantum gate

3. MBQC as Universal Model of Quantum Computation

**Measurement-Based Quantum Computing:**

**What, Why & How**

**Circuit.** Basic mechanism:

- Evolve unitarily qubits through a circuit by applying on the qubits the gates one-by-one
- Measure (read-out) at the end to convert quantum information to classical
- Resource Cost: **Number of Gates**

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**MBQC.** (also known as **one-way quantum computer**).
Basic mechanism:

- Start with a large (generic) entangled state consisting of multiple qubits
- Make single-qubit measurements in suitably chosen bases (depending on the computation).
  Single-qubit measurements are easy to perform.
- Resource Cost: **Entanglement "consumed"**

# MBQC: Why

- For **certain quantum hardware and architectures** is easier to implement (e.g. photonic)
- Has alternative **ways to treat fault-tolerance** and error correction (potentially advantageous)
- Certain **applications** are easier in MBQC (see later Lecture for crypto related)
- **Foundationally** a different perspective (e.g. the role of contextuality or certain complexity theoretic implications can be better seen in MBQC).

# MBQC: How (The General Idea)

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**General MBQC "Ingredients":**

1. Large entangled quantum state with many qubits (**resource state**) – "consumed" during the computation.
   Easy to prepare and same for different computations.
2. Perform computation by **single qubit measurements** (easy to implement).

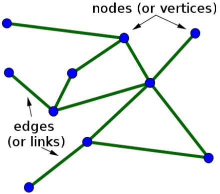
# MBQC: What do we need?

- What **resource state** and what **measurements** are needed to implement a universal set of gates?
- How to combine those gates for **universal computation**?
- Does the order of measurements matters? Can we parallelise some of these measurements?
- How to include an (unknown) quantum state $|\psi_0\rangle$ as input?

  (Ans: Entangle this state at one side of the resource. Then measure **all** qubits, one-by-one.)

### A: Resource States

- Entangled states used are called **graph states**.
- Given graph $G = (V, E)$ with vertices $V$ and edges $E$



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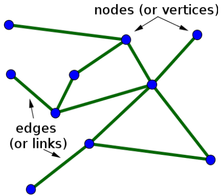


- Place at each vertex a qubit at $|+\rangle$
- For each edge apply $\wedge Z$ to entangle the vertices

Resulting state: $|G\rangle = \prod_{(a,b)\in E} \wedge Z^{(a,b)} |+\rangle^{\otimes V}$

# Resource States and Measurements

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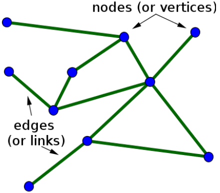


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- For each edge apply $\wedge Z$ to entangle the vertices

  Resulting state: $|G\rangle = \prod_{(a,b)\in E} \wedge Z^{(a,b)} |+\rangle^{\otimes V}$
  Note: $\wedge Z$'s commute, so order does not matter

**Remarks**:

- If the graph used is subset of $d$-dimensional lattice the state are also known as **cluster states**.
- Graph states are highly entangled between **all** qubits. Entanglement remains after measuring some qubits
- Entanglement is "consumed" during the computation $\Rightarrow$ **resource** of the computation.

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  Is called **one-way quantum computation**, since the resource is consumed during the computation $\Rightarrow$ non-reversible.

### B: Measurements

- **Single-qubit** measurements
  **Subscript** denotes qubit measured
  **Superscript** denotes basis of measurement
- Bases used:

  $M_j^\theta = \{|+_\theta\rangle, |-_\theta\rangle\}$ for all $\theta$ and $M_j^z = \{|0\rangle, |1\rangle\}$

  Recall that $|\pm_\theta\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm e^{i\theta}|1\rangle)$

  The role of the $Z$ measurement will be explained later
- Measurements have binary outcome, for qubit $j$ we denote the one outcome $s_j = 0$ and the second $s_j = 1$

- Measurement outcomes are random. To achieve deterministic outcome (unitary), we need to adapt the measurement angles to "cancel" the randomness of previous measurements.
- The (partial) **order** of measurements and **adaptivity** will be explored in the next lecture.
- Here we see how to obtain in MBQC the "$J(\theta)$" **universal gate-set**, up to certain "corrections"

# Some Useful Background

1. **The $|\pm_\theta\rangle$-basis.** For all $\theta$ we define:

$$|+_\theta\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\theta}|1\rangle\right), \quad |-_\theta\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - e^{i\theta}|1\rangle\right)$$

Note: $\{|+_\theta\rangle, |-_\theta\rangle\}$ is a basis and $\theta = 0$ is the $|\pm\rangle$-basis.

$$|0\rangle = \frac{1}{\sqrt{2}}(|+_\theta\rangle + |-_\theta\rangle), \quad |1\rangle = \frac{1}{\sqrt{2}}e^{-i\theta}(|+_\theta\rangle - |-_\theta\rangle)$$

# Some Useful Background

2. **The** $J(\theta)$ **universal gate-set**:
   The set of quantum gates $\{\wedge Z, J(\theta) \text{ for all } \theta\}$ is universal

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   Recall: $R(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$

   We define the Hadamard rotated phase gate:

$$J(\theta) = HR(\theta) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & e^{i\theta} \\ 1 & -e^{i\theta} \end{bmatrix}$$

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- Any single-qubit unitary gate can be decomposed as:

$$U = J(0)J(\theta_1)J(\theta_2)J(\theta_3)$$

  for some $\theta_1, \theta_2, \theta_3$

- For universal set we need a two-qubit gate: $\wedge Z$

3. **The $|H\rangle$ maximally entangled state**:
Recall the controlled-Z gate ($\wedge Z$)

$$\wedge Z |i\rangle |j\rangle = (-1)^{ij} |i\rangle |j\rangle$$

is symmetric w.r.t. inputs (unlike $\wedge X |i\rangle |j\rangle = |i\rangle |i \oplus j\rangle$)
We define:

$$|H\rangle := \wedge Z |+\rangle \otimes |+\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$$

This state is maximally entangled:

$$|H\rangle = \frac{1}{\sqrt{2}}(|0\rangle |+\rangle + |1\rangle |-\rangle) = \frac{1}{\sqrt{2}}(|+\rangle |0\rangle + |-\rangle |1\rangle)$$

$$|H\rangle = (\mathbb{I} \otimes H) |\Phi^+\rangle = (H \otimes \mathbb{I}) |\Phi^+\rangle = \wedge Z |+\rangle \otimes |+\rangle$$

Note1: $\wedge Z$ acts on $|+\rangle$'s entangles qubits symmetrically
Note2: The $|H\rangle$ is a two-qubit state not to be confused with the Hadamard operator $H$.

**The $J(\theta)$ quantum gate**

# Expressing an MBQC computation

It is called **“Measurement Pattern”**

- **Resource State:**

  - A graph with labelled vertices (qubits)
  - Set of vertices that are **input** and **output** of the computation

  Unless stated otherwise: inputs are on the left-hand side; outputs are on the right-hand side (and are not-measured)

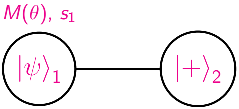
- **Measurements:**

  Angles that each qubit is measured are denoted on the vertex

  In general, angles need to be adaptively corrected. Denoted angles are the “default” un-corrected ones (see next lecture)

# The $J(\theta)$ single-qubit gate

**Gate Teleportation:** We start with unknown state $|\psi\rangle_1 = a\,|0\rangle_1 + b\,|1\rangle_1$ plugged in the following MBQC pattern:




The $J(-\theta)$-gate MBQC pattern

The total state after entangling ($\wedge Z$) becomes:

$$|\phi\rangle_{12} := \wedge Z_{12}\left(|\psi\rangle_1 \otimes |+\rangle_2\right) = a\,|0+\rangle_{12} + b\,|1-\rangle_{12}$$

## The $J(\theta)$ single-qubit gate

- To see the effect of the measurement $M_1^\theta$, we express **qubit 1** (that is to be measured) in the corresponding $|\pm_\theta\rangle$ basis (see expansion of $|0\rangle, |1\rangle$ in this basis):

$$\begin{aligned} |\phi\rangle_{12} &= \frac{a}{\sqrt{2}}(|+_\theta\rangle_1 + |-_\theta\rangle_1)|+\rangle_2 + \frac{b}{\sqrt{2}}e^{-i\theta}(|+_\theta\rangle_1 - |-_\theta\rangle_1)|-\rangle_2 \\ &= \frac{1}{\sqrt{2}}|+_\theta\rangle_1 (a|+\rangle_2 + be^{-i\theta}|-\rangle_2) + \\ &\quad + \frac{1}{\sqrt{2}}|-_\theta\rangle_1 (a|+\rangle_2 - be^{-i\theta}|-\rangle_2) \end{aligned} \tag{1}$$

- We can re-express now the state of **qubit 2** in each of the two terms in the RHS of Eq **??**

# The $J(\theta)$ single-qubit gate

- We note that the first term can be written as:

$$HR(-\theta)\left(a|0\rangle + b|1\rangle\right) = H\left(a|0\rangle + be^{-i\theta}|1\rangle\right) = a|+\rangle + be^{-i\theta}|-\rangle$$

- and that the second term can be written as:

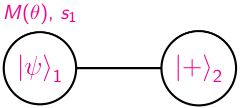
$$XHR(-\theta)\left(a|0\rangle + b|1\rangle\right) = Xa|+\rangle + Xbe^{-i\theta}|-\rangle = a|+\rangle - be^{-i\theta}|-\rangle$$

- We therefore have:

$$|\phi\rangle_{12} = |+_\theta\rangle_1 (X_2)^0 J(-\theta)_2 |\psi\rangle_2 + |-_\theta\rangle_1 (X_2)^1 J(-\theta)_2 |\psi\rangle_2$$

- We can see that measuring **qubit 1** in the $M_1^\theta$-basis we end-up with **qubit 2** being at the state $X^{s_1} J(-\theta) \left|\psi\right\rangle$, where $s_1$ is the outcome of qubit's 1 measurement.
- **Interpretation:** We have teleported the state $\left|\psi\right\rangle_1$ to **qubit 2**, and in the same time we have applied on it, the gate $J(-\theta)$ along with an extra operation $X^{s_1}$ that depends on the previous measurement outcome
- To restore "determinism" we need to "cancel" the gate $X^{s_1}$, something that is possible by adapting the measurement angles (**see next lecture**)

# The $J(\theta)$ single-qubit gate: Summary




The $J(-\theta)$-gate MBQC pattern

The above measurement pattern results to:

$X^{s_1} J(-\theta)\,|\psi\rangle_2 = X^{s_1} HR(-\theta)\,|\psi\rangle_2$

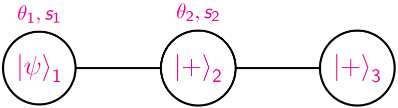
**Examples:**

- $\theta = 0$: Output $X^{s_1} H\,|\psi\rangle_2$
- $\theta = \pi$: Output $X^{s_1} HZ\,|\psi\rangle_2$
- $\theta = \pi/2$: Output $X^{s_1} HR(-\pi/2)\,|\psi\rangle_2 = X^{s_1} H \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} |\psi\rangle_2$

**MBQC as Universal Model of Quantum Computation**

# Composing single-qubit measurement patterns

How to apply consecutively two $J(\theta)$-gates:




The $(J(-\theta_2)J(-\theta_1))$-gate MBQC pattern

Operators (or measurements) acting on **different** subsystems commute (can be performed in arbitrary order)

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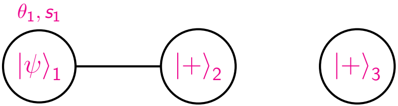


Step 1: entangling the two first qubits

Operators (or measurements) acting on **different** subsystems commute (can be performed in arbitrary order)

We can break the pattern of the figure to two steps:

1. Consider **qubit 1** and **qubit 2** alone: Prepare these qubits, entangle them and measure **qubit 1** (see previous example)

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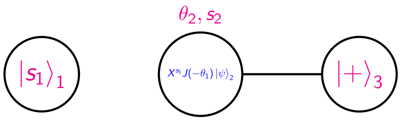


Step 2: after measuring qubit 1

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We can break the pattern of the figure to two steps:

1. Consider **qubit 1** and **qubit 2** alone: Prepare these qubits, entangle them and measure **qubit 1** (see previous example)
2. Then prepare **qubit 3** entangle **qubit 2** with **qubit 3** and measure **qubit 2**.

   Step 2 is again the $J(-\theta)$-gate but has as input **qubit 2** in the state produced in step 1.

# Composing single-qubit measurement patterns

In more details the two steps:

1. The input was $|\psi\rangle_1$, measurement angle $\theta_1$ and outcome $s_1$:

$$|s_1\rangle \otimes X_2^{s_1} H_2 R_2(-\theta_1)\,|\psi\rangle_2 = |s_1\rangle \otimes X_2^{s_1} J_2(-\theta_1)\,|\psi\rangle_2$$

2. The input was $X_2^{s_1} J_2(-\theta_1)\,|\psi\rangle_2$ (we can ignore qubit 1 now that is measured), measurement angle $\theta_2$ and outcome $s_2$:

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The "corrections" $X^{s_1}, X^{s_2}$ will be dealt at next lecture in the general case.

Now note that the output (**qubit 3**) is now at the state $|\psi\rangle$ with the gates $J(-\theta_2)J(-\theta_1)$ applied.

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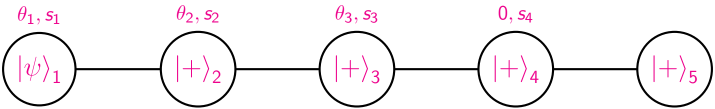
- (Standard) Teleportation: Case $\theta_1 = \theta_2 = 0$:
$X_3^{s_2} H_3 X_3^{s_1} H_3\,|\psi\rangle_3 = X_3^{s_2} Z_3^{s_1} H_3 H_3\,|\psi\rangle = X_3^{s_2} Z_3^{s_1}\,|\psi\rangle$

# General Single-Qubit Gate

**Any single-qubit gate** can be implemented repeating the $J(\theta)$ pattern since any single-qubit unitary can be written using three angles $\theta_1, \theta_2, \theta_3$

$$U = J(0)J(-\theta_3)J(-\theta_2)J(-\theta_1)$$




A general single-qubit gaete: MBQC pattern

- This pattern results to (step by step):

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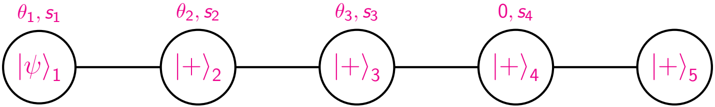


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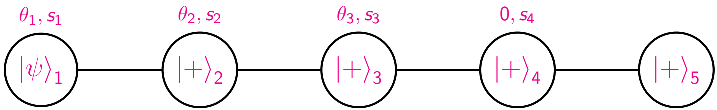


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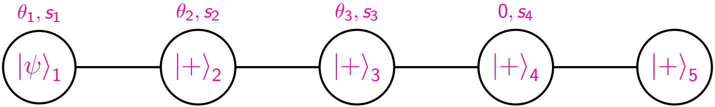


A general single-qubit gaete: MBQC pattern

- This pattern results to (step by step):

$$|s_1\rangle \otimes |s_2\rangle \otimes |s_3\rangle \otimes |s_4\rangle \otimes X^{s_4} J(0) X^{s_3} J(-\theta_3) X^{s_2} J(-\theta_2) X^{s_1} J(-\theta_1) |\psi\rangle$$

# Two Qubit Gates

- What is missing to achieve the universal $J(\theta)$ gate-set is a way to implement the $\wedge Z$-gate.
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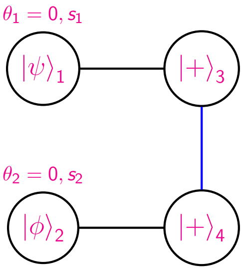
- Information "flows" as qubits are teleported through the measurement pattern
- Entangling should happen without obstructing the "flow" (teleportation path)
- Horizontal $\wedge Z$ is used to teleport information (and gates)
- Vertical $\wedge Z$ is used as the 2-qubit gate.

We will ignore the corrections (assume all $s_i$'s are zero).

We will see step by step the patern:




The $(\wedge Z_{12})\,(H_1 \otimes H_2)$-gate MBQC pattern

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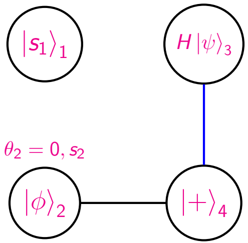


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# Example: Two Qubit Gate

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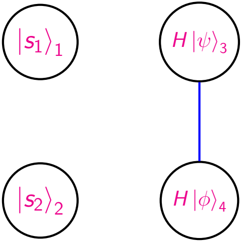


The $(\wedge Z_{12})\,(H_1 \otimes H_2)$-gate MBQC pattern

# Example: details

1. Consider qubit 1 and qubit 3 (the effect of measuring qubit 1).

$$|s_1\rangle_1 \otimes X_3^{s_1} H_3 |\psi\rangle_3$$

2. Consider qubit 2 and qubit 4 (the effect of measuring qubit 1).

$$|s_2\rangle_2 \otimes X_4^{s_2} H_4 |\phi\rangle_4$$

3. We apply a $\wedge Z$ on the qubits 3 and 4.

$$|s_1\rangle_1 \otimes |s_2\rangle \otimes \wedge Z_{34} \left(X_3^{s_1} H_3 |\psi\rangle_3 \otimes X_4^{s_2} H_4 |\phi\rangle_4\right)$$

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- The corrections ($X$'s that depend on measurement outcomes) will be formally treated later (L16)
- The net effect (baring corrections – setting $s_i = 0$) is:

$$(\wedge Z)\,(H \otimes H)\,(|\psi\rangle \otimes |\phi\rangle)$$

- We have measurement pattern for the $J(-\theta)$ gate:

  Input qubit entangled with another one and measured in the $M^{\theta}$ basis

# Summary: MBQC Universality

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# Summary: MBQC Universality

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- **Next Lecture:** formally how to treat "corrections" and resort deterministic application of gates!

# Further Reading


1. One-way Quantum Computation - a tutorial introduction, D. Browne and H. Briegel, arxiv:quant-ph/0603226
2. An introduction to measurement based quantum computation, R. Jozsa, arxiv:quant-ph/0508124
3. Quantum computing with photons: introduction to the circuit model, the one-way quantum computer, and the fundamental principles of photonic experiments, S. Barz, Journal of Physics B: Atomic, Molecular and Optical Physics, Vol 48, Num. 8 (2015).
4. Chapter 7, Semantic Techniques in Quantum Computation – Editors Simon Gay and Ian Mackie