Introduction to Quantum Computing
Pennylane: circuits and teleportation

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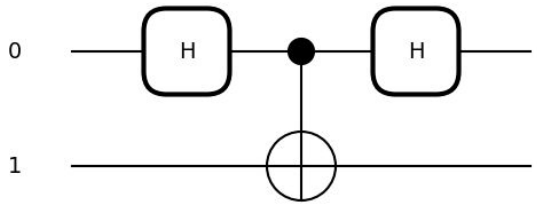
Exercise 1: Circuits

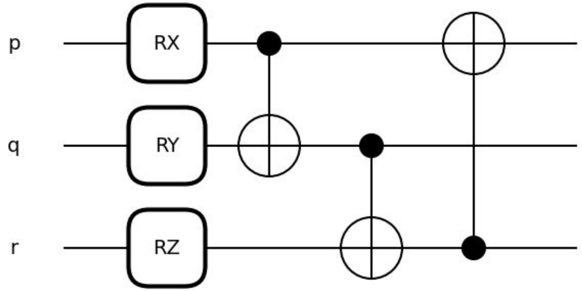
```
import pennylane as pl
from pennylane import numpy as np
import matplotlib.pyplot as plt
```




```
def circuit():
    # Call pl.Hadamard and pl.CNOT to build and return this circuit
    pl.
    return pl.

print(pl.draw(circuit)())
```

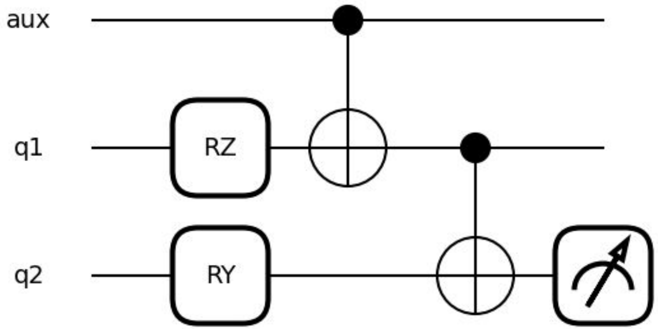
Exercise 1: Parameterised circuits




```
def circuit(alpha,beta,gamma):
   # Call pl.RX, pl.RY, pl.RZ, and pl.CNOT to build and return this circuit
   pl.
   return pl.

pl.drawer.use_style("black_white")
pl.draw_mpl(circuit)(np.pi/4, np.pi/8, 0)
```

Exercise 3: Executing circuits




```
device = pl.device('default.qubit', wires=['aux', 'q1', 'q2'], shots=[1,10,100,1000])
def circuit(alpha, beta):
   # Build and return this circuit, using pl.expval and pl.PauliZ to measure qubit 2 in the Z basis
   pl.
   return pl.

qnode = pl.QNode(circuit,device)
qnode(np.pi/4, np.pi/4)
```

Exercise 4: Circuit depth

```
device = pl.device('default.qubit', wires=3)

@pl.qnode(device)
def circuit(x, y):
   # Build any circuit and inspect its depth
   return pl.
pl.draw_mpl(circuit)(0,0)
print(pl.specs(circuit)(0,0)['resources'])

@pl.compile
@pl.qnode(device)
def circuit(x, y):
   # Build any circuit and inspect its depth, now using @pl.compile decorator
   return pl.
pl.draw_mpl(circuit)(0,0)
print(pl.specs(circuit)(0,0)['resources'])
```

Quantum teleportation

Suppose there are two researchers named Alice and Bob, and Alice wants to send her quantum state to Bob. The quantum teleportation protocol enables Alice to do exactly this in a very elegant manner, and it can be described in four steps:

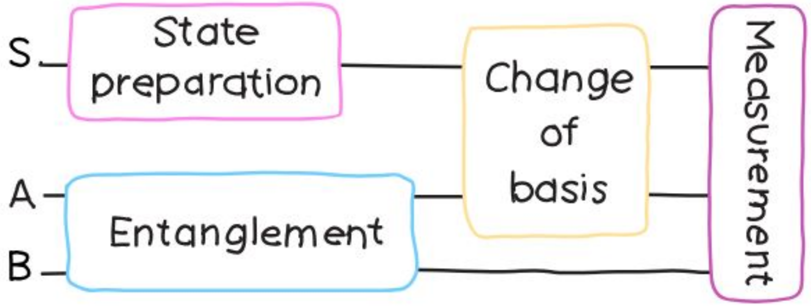
1. State preparation: Alice initializes her qubit to the state she wishes to teleport.

2. Shared entanglement: A Bell state is created and distributed to Alice and Bob (one qubit each).

3. Change of basis: Alice converts her two qubits from the Bell basis to the computational basis.

4. Measurement: Alice measures her two qubits, then tells Bob how to convert his qubit to obtain the desired state. Note that it is only quantum *information* being teleported, and not a physical particle.

An overview of the protocol can be seen here:



No-cloning theorem

You might be wondering why we need to teleport a state at all. Can't Alice just make a copy of it and send the copy to Bob? It turns out that copying arbitrary states is *prohibited*, which you can understand using something called the **no-cloning theorem**. The proof is surprisingly straightforward. Suppose we would like to design a circuit (unitary transformation) U that can perform the following action:

$$U(|\psi\rangle \otimes |s\rangle) = |\psi\rangle \otimes |\psi\rangle,$$
$$U(|\varphi\rangle \otimes |s\rangle) = |\varphi\rangle \otimes |\varphi\rangle,$$

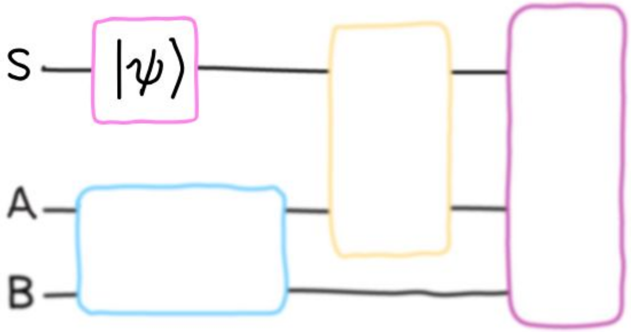
where $|\psi\rangle$ and $|\varphi\rangle$ are arbitrary, normalized single-qubit states, and $|s\rangle$ is some arbitrary, normalized starting state. We will now prove that no such U exists!

First, let's take the inner product of the left-hand sides of the two equations:

$$(\langle\psi| \otimes \langle s|)U^\dagger U(|\varphi\rangle \otimes |s\rangle) = \langle\psi|\varphi\rangle \langle s|s\rangle$$

Since $\langle s|s\rangle$ equals 1, this evaluates to $\langle\psi|\varphi\rangle$. Next, we compare the inner product of the right-hand sides of the two equations: $(\langle\psi|\varphi\rangle)^2$. These inner products must be equal, and they are only equal if they are a value that squares to itself. The only valid values for the inner product then are 1 and 0. But if the inner product is 1, the states are the same; on the other hand, if the inner product is 0, the states are orthogonal. Therefore, we can't clone arbitrary states!

Quantum teleportation: state preparation



Teleportation involves three qubits. Two of them are held by Alice, and the third by Bob. We'll denote their states using subscripts:

1. $|\cdot\rangle_S$, Alice's first qubit that she will prepare in some arbitrary state
2. $|\cdot\rangle_A$, Alice's auxiliary (or "ancilla") qubit that she will entangle with Bob's qubit for communication purposes
3. $|\cdot\rangle_B$, Bob's qubit that will receive the teleported state

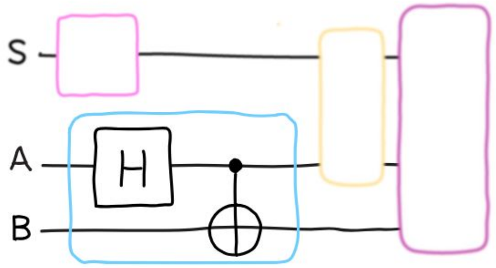
Together, their starting state is:

$$|0\rangle_S|0\rangle_A|0\rangle_B.$$

The first thing Alice does is prepare her first qubit in whichever state $|\psi\rangle$ that she'd like to send to Bob so that their combined state becomes:

$$|\psi\rangle_S|0\rangle_A|0\rangle_B.$$

Quantum teleportation: shared entanglement



The reason why teleportation works is the use of an *entangled state* as a shared resource between Alice and Bob. You can imagine some process that generates a pair of entangled qubits, and sends one qubit to each party. For simplicity (and simulation!), we will represent the entanglement process as part of our circuit.

Entangling the qubits A and B leads to the combined state:

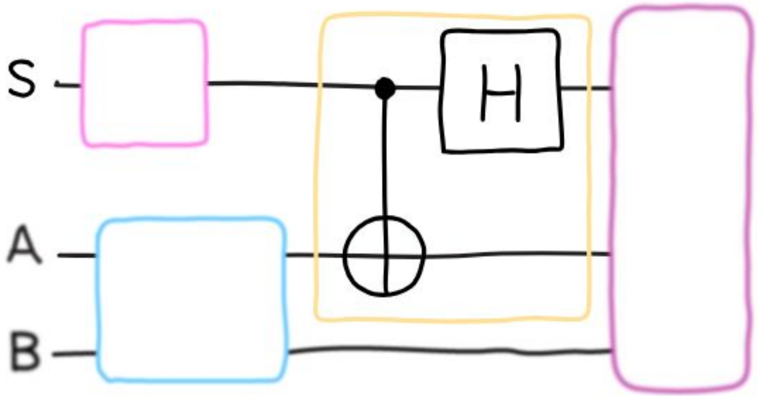
$$\frac{1}{\sqrt{2}}(|\psi\rangle_S|0\rangle_A|0\rangle_B + |\psi\rangle_S|1\rangle_A|1\rangle_B) \quad (1)$$

The AB subsystem is now in what is known as a *Bell state*. There are four maximally entangled two-qubit Bell states, and they form the Bell basis:

$$\begin{aligned}
|\psi_+\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\,, \\
|\psi_-\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)\,, \\
|\phi_+\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)\,, \\
|\phi_-\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)\,.
\end{aligned}$$

In our experiment, because AB started in the $|00\rangle$ state, we create the $|\psi_+\rangle$ Bell state as is shown in equation (1).

Quantum teleportation: change of basis



This is where things get tricky, but also very interesting. The third step of the protocol is to apply a CNOT and a Hadamard to the first two qubits. This is done prior to the measurements, and labelled “change of basis”. But what basis is this? Notice how these two gates are the *opposite* of what we do to create a Bell state. If we run them in the opposite direction, we transform the basis back to the computational one, and simulate a measurement in the Bell basis.

After the basis transform, if we observe the first two qubits to be in the state $|00\rangle$, this would correspond to the outcome $|\psi_+\rangle$ in the Bell basis, $|11\rangle$ would correspond to $|\phi_-\rangle$, etc. Let’s perform this change of basis, one step at a time.

Quantum teleportation: change of basis

Suppose we write our initial state $|\psi\rangle$ as $\alpha|0\rangle + \beta|1\rangle$, with α and β being complex coefficients. Expanding out the terms from (1), we obtain:

$$\frac{1}{\sqrt{2}}(\alpha|000\rangle + \beta|100\rangle + \alpha|011\rangle + \beta|111\rangle)$$

Now let's apply a CNOT between Alice's two qubits:

$$\frac{1}{\sqrt{2}}(\alpha|000\rangle + \beta|110\rangle + \alpha|011\rangle + \beta|101\rangle)$$

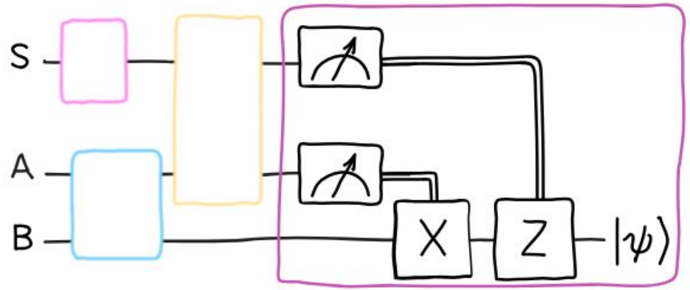
And then a Hadamard on her first qubit:

$$\frac{1}{2}(\alpha|000\rangle + \alpha|100\rangle + \beta|010\rangle - \beta|110\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|001\rangle - \beta|101\rangle).$$

Now we need to do some rearranging. We group the terms based on the first two qubits:

$$\frac{1}{2}|00\rangle(\alpha|0\rangle + \beta|1\rangle) + \frac{1}{2}|01\rangle(\beta|0\rangle + \alpha|1\rangle) + \frac{1}{2}|10\rangle(\alpha|0\rangle - \beta|1\rangle) + \frac{1}{2}|11\rangle(-\beta|0\rangle + \alpha|1\rangle). \qquad (2)$$

Quantum teleportation: measurement and correction



The last step of the protocol involves Alice performing a measurement on her qubits, and telling Bob to perform some operations depending on what she measured. But why exactly do we need to do this? In the previous step, we already performed a basis rotation back to the computational basis, so shouldn't we be good to go? Not quite, but almost!

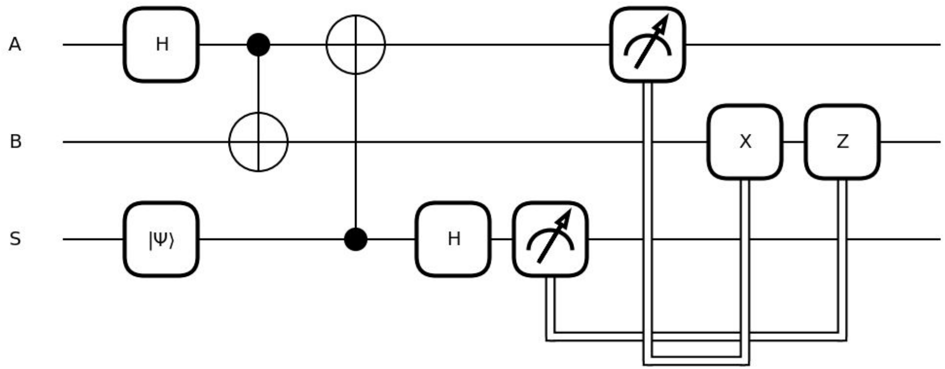
Let's take another look at equation (2). If Alice measures her two qubits in the computational basis, she is equally likely to obtain any of the four possible outcomes. If she observes the first two qubits in the state $|00\rangle$, she would immediately know that Bob's qubit was in the state $\alpha|0\rangle + \beta|1\rangle$, which is precisely the state we are trying to teleport!

If instead she observed the qubits in state $|01\rangle$, she'd still know what state Bob has, but it's a little off from the original state. In particular, we have:

$$\beta|0\rangle + \alpha|1\rangle = X|\psi\rangle.$$

After obtaining these results, Alice could tell Bob to simply apply an X gate to his qubit to recover the original state. Similarly, if she obtained $|10\rangle$, she would tell him to apply a Z gate.

Exercise 5: Teleportation




```python
def state_preparation (state):
    # use pl.StatePrep to prepare wire S in the given state

def entangle_qubits ():
    # use pl.Hadamard and pl.CNOT to create a Bell pair

def basis_rotation ():
    # use pl.Hadamard and pl.CNOT to rotate the basis

def measure_and_update ():
    # use pl.measure for Alice's measurement and pl.PauliX, pl.PauliZ, and pl.cond for Bob's correction

def teleport(state):
    state_preparation (state)
    entangle_qubits ()
    basis_rotation ()
    measure_and_update ()

state = np.array([1 / np.sqrt(2) + 0.3j, 0.4 - 0.5j])
_ = pl.draw_mpl(teleport)(state)
```