Raul Garcia-Patron
Jakub Adamski

Tutorial 1

Problem 1: Quantum Operations

The *Hadamard* gate plays a very prominent role in quantum computation:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

a. Prove that H is unitary, i.e. that it satisfies $UU^\dagger = U^\dagger U = I$.

b. Prove that H is it own inverse by showing $H^2 = I$ where I is the identity operator.

c. Calculate the action of the operator on the vectors:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad |+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad |-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Problem 2: Pauli matrices

Consider the four Pauli matrices:

$$I, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

a. Prove that for each Pauli matrix σ_i we have $\sigma_i^2 = I$ and $\sigma_i^\dagger = \sigma_i$.

b. Show that the Pauli matrices are unitary matrices.

c. Show that $Y = iXZ$.

d. Show that $HXH = Z$ and $HZH = X$.

Problem 3: Measurement

Consider two quantum states $|L\rangle$ and $|R\rangle$ (eigenvalues of Pauli Y operator):

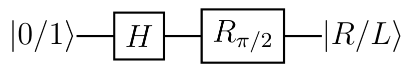
$$|R\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$$
$$|L\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i\,|1\rangle)$$

a. Consider the general quantum state:

$$|\psi\rangle = \psi_0\,|0\rangle + \psi_1\,|1\rangle$$

What are the probabilities of outcome $|R\rangle$ and $|L\rangle$ if we measure $|\psi\rangle$?

b. Show that the states $|L\rangle$ and $|R\rangle$ can be generated from $|0\rangle$ and $|1\rangle$ using the following circuit:



where

$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}.$$

c. What circuit will allow implementing a measurement on the $|L\rangle$ and $|R\rangle$ basis if our hardware only allows for measurement in the computational basis? Use H and R_θ gates.