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Tutorial 3

IQC 2024-25
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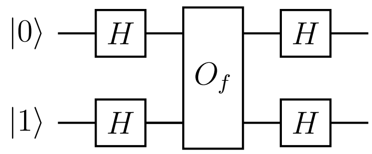
Problem 1: Deutsch Algorithm

Consider the following circuit:



a. The oracle O_f is a two-qubit gate that maps $|x\rangle |y\rangle \to |x\rangle |y \oplus x\rangle$. By comparing to what we have seen in the lectures, what is the classical function implemented by the oracle O_f, i.e. $f(x)$? Do you think $f(x)$ will be balanced or constant?

b. What is the circuit of O_f?

c. Compute the two-qubit output state of this circuit and the probability of getting an outcome 0 when measuring the upper qubit.

d. Having found the probability of getting outcome 0 on the upper qubit, conclude whether the function is balanced or constant. Justify your answer.

Problem 2: Phase kick-back

Suppose you have the balanced function $f : \{0,1\}^2 \to \{0,1\}$ such that:

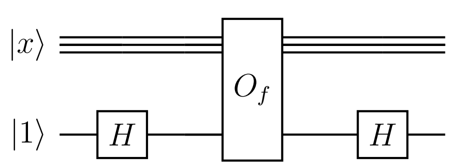
$$f(0,0) = 0, f(0,1) = 1$$
$$f(1,0) = 1, f(1,1) = 0$$

a. A classical oracle C_f returns for a given query input $\bar{x}$ the value $f(\bar{x})$ of the Boolean function $f : \{0,1\}^n \to \{0,1\}$. We have seen that for every classical oracle there exist a quantum oracle O_f satisfying

$$O_f |\bar{x}\rangle |q\rangle = |\bar{x}\rangle |q \oplus f(\bar{x})\rangle .$$

Provide a circuit of 3 qubits implementing O_f for the function given above.

b. In the lecture, we have seen that the Oracle O_f can be used to implement a phase-kickback unitary U_f acting on the address qubits. Show that the circuit below



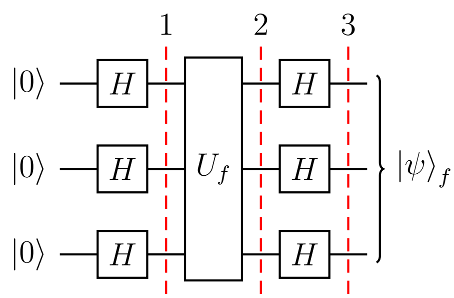
is equivalent to applying the phase kick-back U_f to the address register:

$$U_f |x\rangle = (-1)^{f(x)} |x\rangle$$

C. What is the size of the address register in the Deutsch algorithm in Problem 1?

Problem 3: Bernstein-Vazirani Algorithm

Consider the function $f(x) = ax \mod 2$ with the string a being $a = `111'$. The goal is to find a with a single call to the phase kick-back U_f. Consider the quantum circuit implementing the Bernstein-Vazirani algorithm:



a.Write the quantum state at stage 1 of the figure above, i.e. after the first layer of parallel Hadamard gates.

b. What transformation does the oracle U_f perform on the state $|x\rangle$, where$x$ is a string of 3 bits encoding the computational basis of 3 qubits?

c. Calculate the state of the composite system at stage 2 of the circuit.

d. Derive the action of a layer of three Hadamard gates (Walsh-Hadamard transform) on a computational state $|x_1x_2x_3\rangle$ of three qubits.

e. Provide the quantum state at stage 3 of the computation.

f. Suppose that we perform a measurement. What is the probability of the output being the $|000\rangle$ state? What would be the probability of obtaining $|111\rangle$?