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informatics

Introduction to Quantum Computing

Lecture 4: Postulate IV – Measurement

Raul Garcia-Patron Sanchez

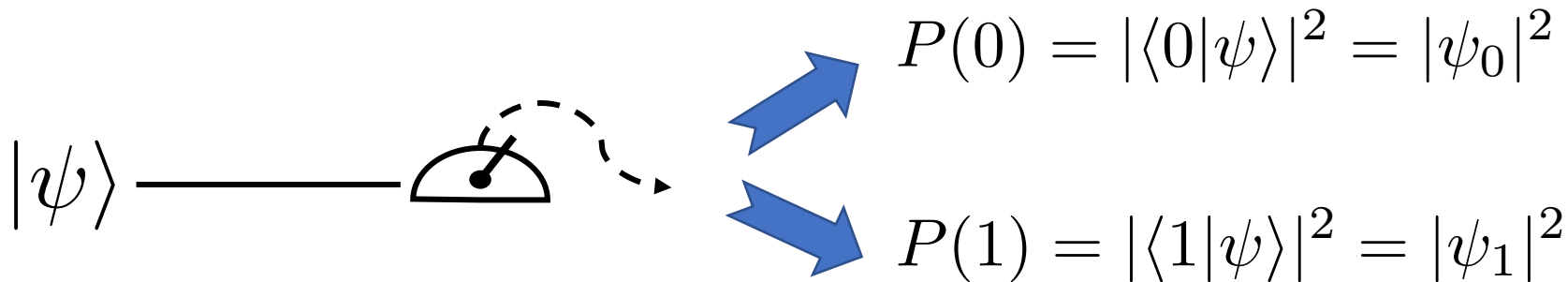
Computational basis

$$\mathcal{H}_{\mathcal{Q}} = \text{Span}\{|0\rangle, |1\rangle\}$$

- $\forall |\psi\rangle, \exists \psi_0$ and $\psi_1 : |\psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle$
- $\langle 0|1\rangle = 0$ (Orthogonal basis)
- $\| |0\rangle \| = \| |1\rangle \| = 1$ (Normalized basis)

This ensure logical 0 and 1 is an orthonormal basis of a Hilbert space of dim 2.

Computational basis measurement



The amplitudes of the quantum state on the logical basis (0 and 1) are associated with the outcome probabilities of the computational basis measurement (logical 0 or 1).



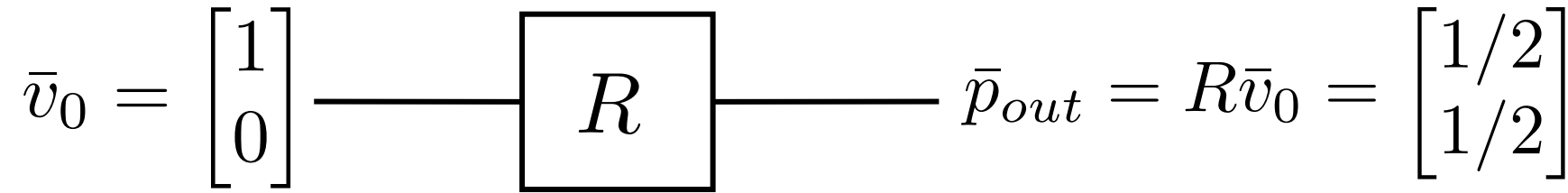
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The effect of measurement



Random coins vs quantum coins

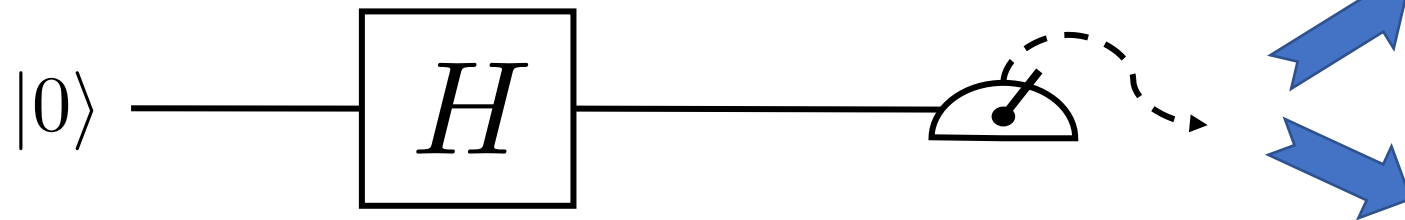
- Classical coin flip



$$R = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$



- Quantum random number generator circuit



$$P(0) = |\psi_0|^2 = 1/2$$

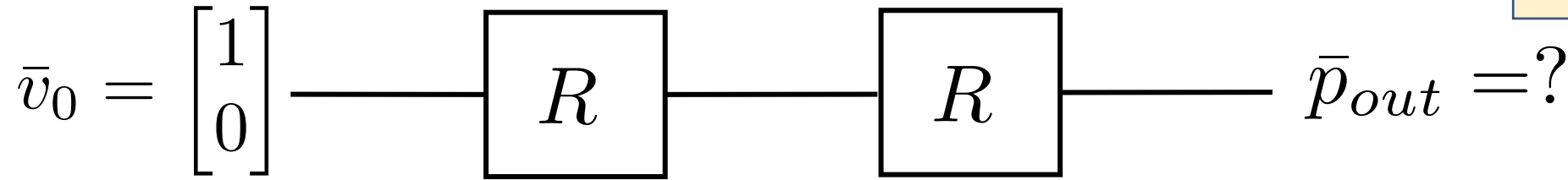
$$P(1) = |\psi_1|^2 = 1/2$$



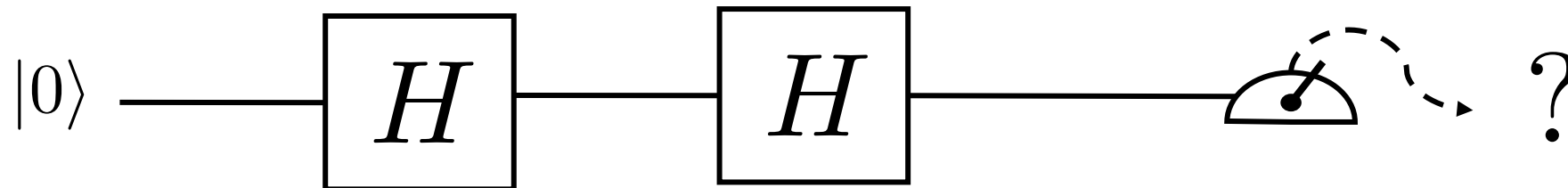
Random coins vs quantum coins: concatenation

- Classical coin flip

$$R = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$



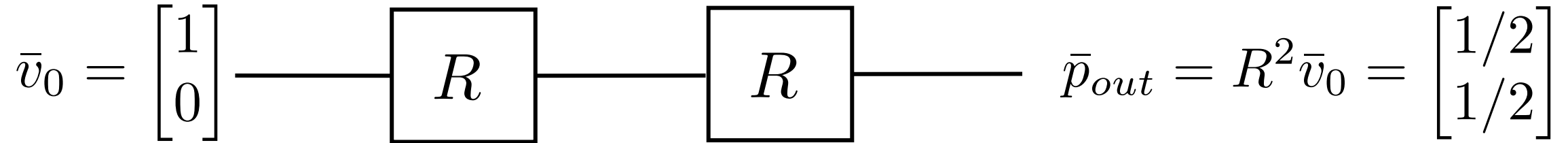
- Quantum random number generator circuit



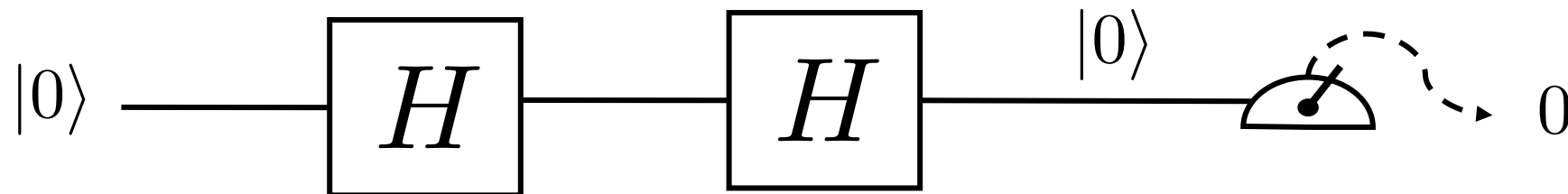
Random coins vs quantum coins: concatenation

- Two classical coin flips

$$R = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$



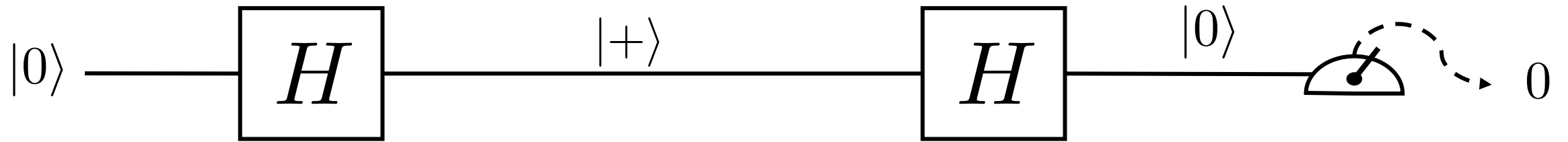
- Two "quantum coin flips"



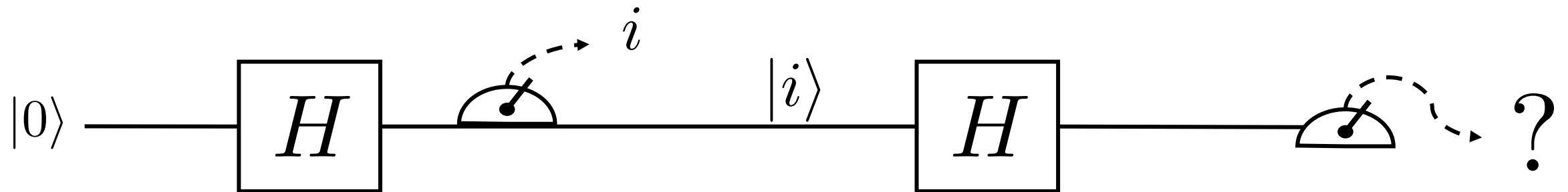
$$H^2 = I$$

To measure or not to measure....

- Two "quantum coin flips"

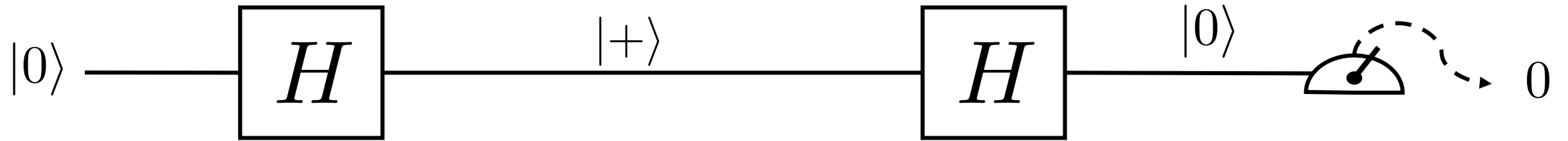


- Two "quantum coin flips" with intermediate measurement

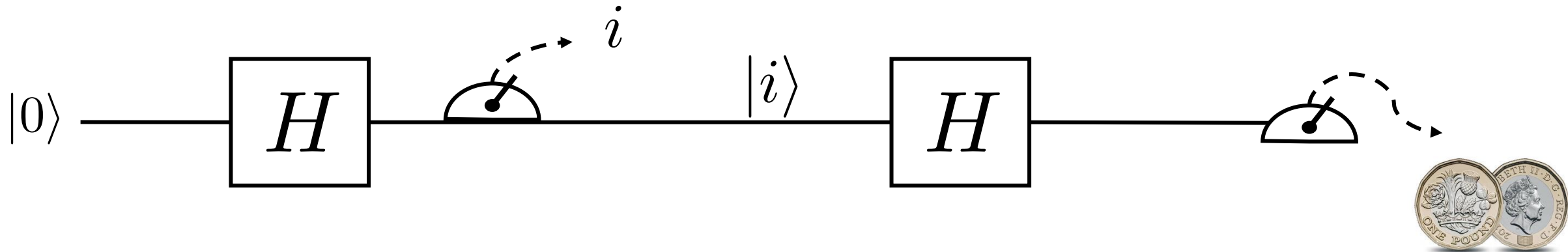


To measure or not to measure....

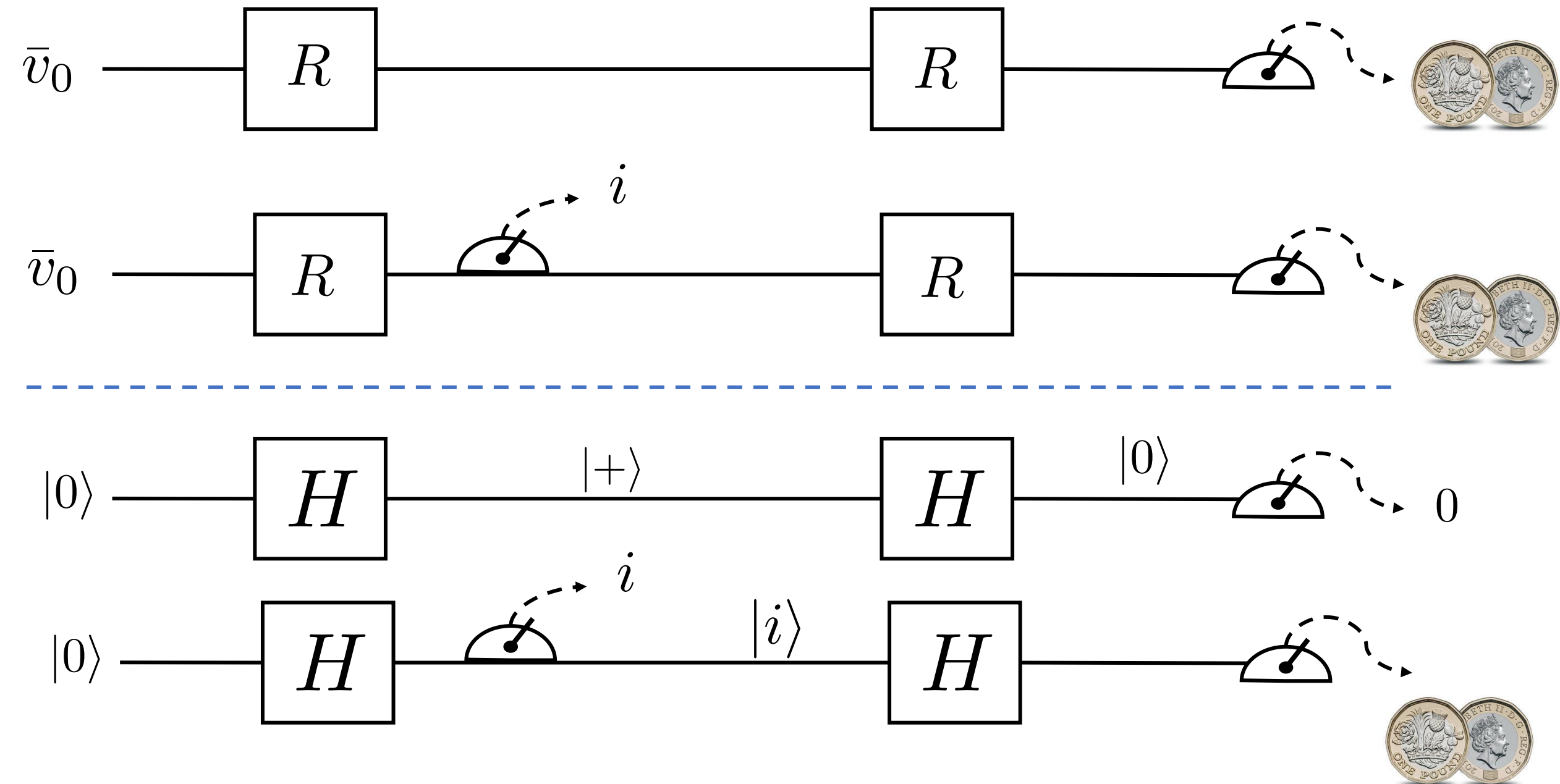
- Two "quantum coin flips"



- Two "quantum coin flips" with intermediate measurement



In quantum world observing the system can change its dynamics!!





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Arbitrary basis qubit measurement

Raul Garcia-Patron Sanchez



- Spanning set of \mathcal{H} : set of vectors $|v_1\rangle, |v_2\rangle, \dots, |v_n\rangle$

$$\forall |\psi\rangle \in \mathcal{H} : |\psi\rangle = \sum_{i=1}^n \psi_{v_i} |v_i\rangle \quad \text{where amplitudes are given by: } \psi_{v_i} = \langle \psi | v_i \rangle$$

- Linearly independent: $\nexists a_1, a_2, \dots, a_n \neq 0$ complex numbers

$$a_1 |v_1\rangle + \dots + a_n |v_n\rangle = 0$$

- Basis: $\text{Span}\{|v_i\rangle\} = \mathcal{H} \Leftrightarrow n = d$
+ Lin. ind.

- Orthonormal: $\forall i, j \in \{1, \dots, d\}, \langle v_i | v_j \rangle = \delta_{i,j}$

- Has an associated measurement

Example

$$|+\rangle = \frac{1}{2}(|0\rangle + |1\rangle)$$

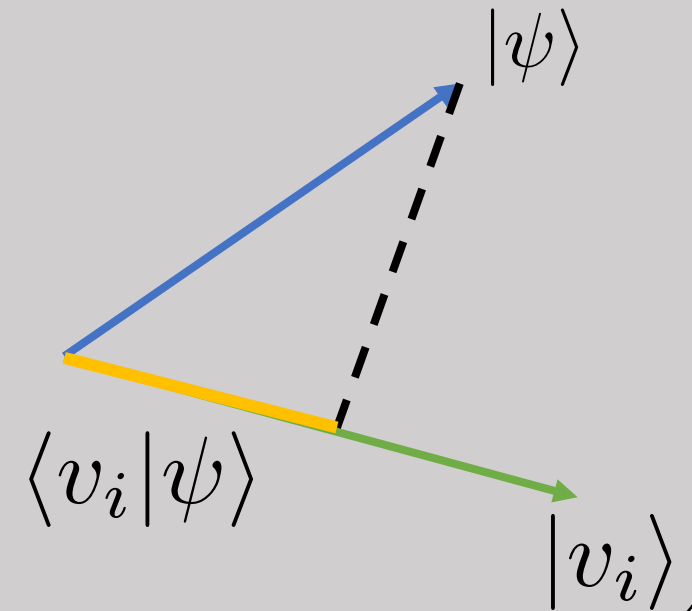
$$|-\rangle = \frac{1}{2}(|0\rangle - |1\rangle)$$

Measurement of orthonormal basis

Any orthonormal basis $\{|v_i\rangle\}$ that span \mathcal{H} has an associated measurement

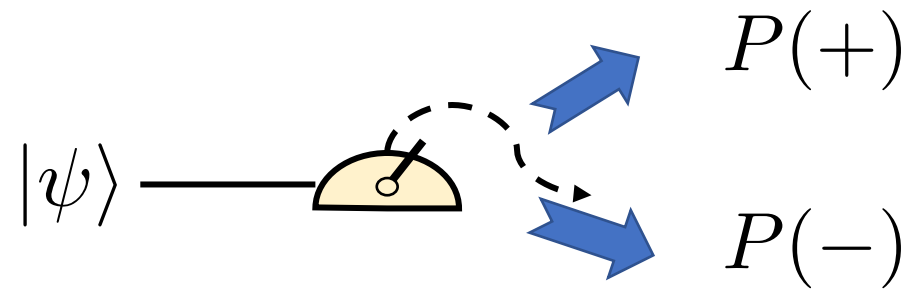
Probability of outcome i reads: $P(i) = |\langle v_i | \psi \rangle|^2$

The quantum state is updated to $|v_i\rangle$



Example: +/- basis

- $|\psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle$



$$\mathcal{H}_Q = \text{Span}\{|+\rangle, |-\rangle\}$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

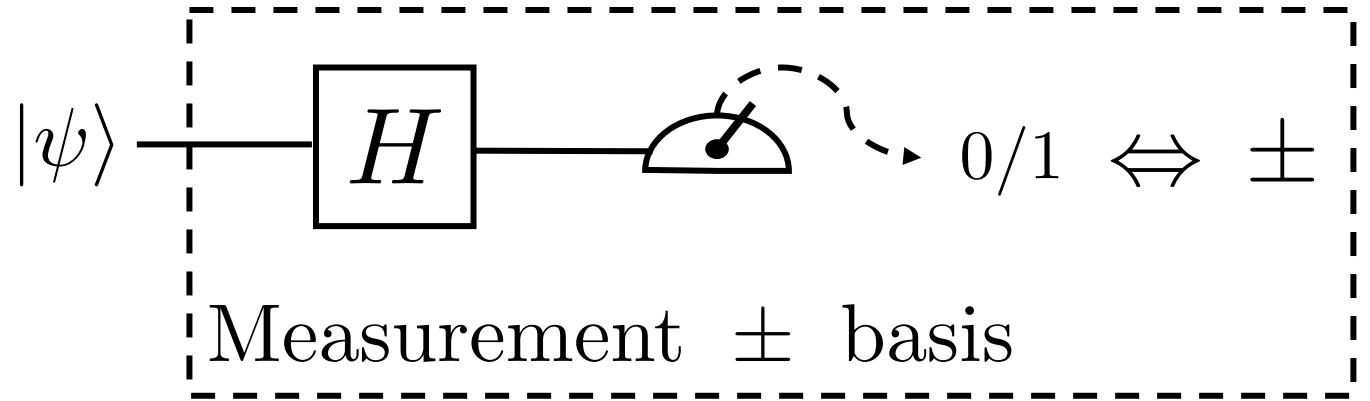
- $P(+)$ = $|\langle +|\psi\rangle|^2 = \left|\frac{1}{\sqrt{2}}(\langle 1| + \langle 0|)(\psi_0|0\rangle + \psi_1|1\rangle)\right|^2 = |\psi_0 + \psi_1|^2/2$

- $P(-)$ = $|\langle -|\psi\rangle|^2 = |\psi_0 - \psi_1|^2/2$

Arbitrary basis measurement

- Measurement basis $\{|\pm\rangle\}$:

$$|\psi\rangle \text{ --- } \text{meter} \text{ --- } \pm \equiv P(\pm) = |\langle \pm | \psi \rangle|^2$$



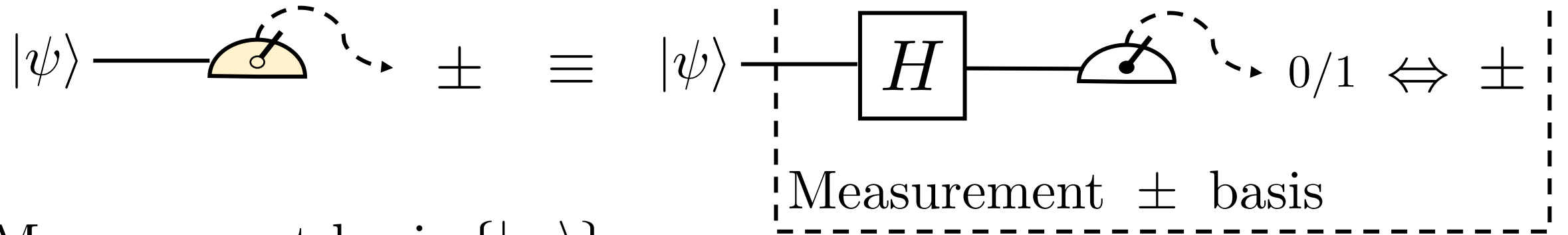
$$\begin{aligned} |+\rangle &= H|0\rangle \\ |-\rangle &= H|1\rangle \\ (A|\psi\rangle)^\dagger &= \langle \psi | A^\dagger \\ H^\dagger &= H \end{aligned}$$



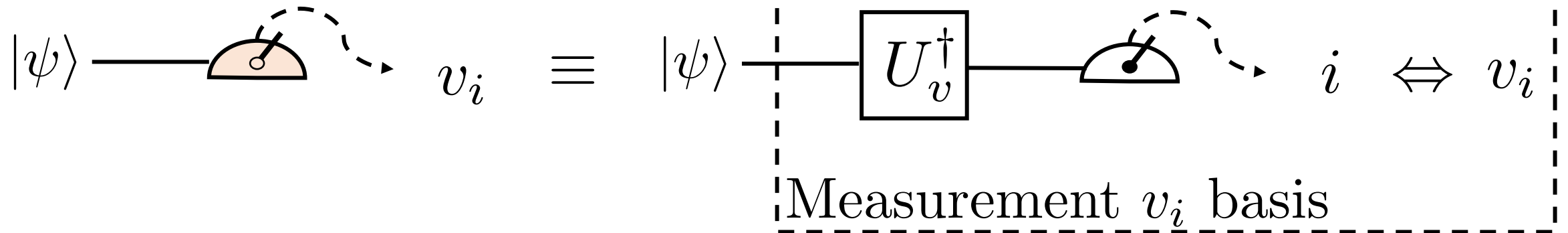
$$\begin{aligned} \langle + | \psi \rangle &= \langle 0 | H | \psi \rangle \\ \langle - | \psi \rangle &= \langle 1 | H | \psi \rangle \end{aligned}$$

Arbitrary basis measurement

- Measurement basis $\{|\pm\rangle\}$:



- Measurement basis $\{|v_i\rangle\}$:



$$\forall \text{ basis } \{|v_i\rangle\}, \exists U_v \text{ s.t. } |v_i\rangle = U_v |i\rangle \quad \langle v_i | \psi \rangle = \langle i | U_v^\dagger | \psi \rangle$$

References

Reading references

1. NC 2.2.3 and 2.2.5

NC \equiv Michael Nielsen and Isaac Chuang, Quantum Computing and Quantum Information
Cambridge University Press (2010)

There is even a more general concept of measurement in N&C called POVM (2.2.6) that we will not cover in this course.

Even more general is the concept of [quantum instrument](#).

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Conferences

QIP, TQC

<https://qipconference.org/>

Workshops/schools/semesters

Simon's Institute for the Theory of Computation

<https://simons.berkeley.edu/>

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References

- Additional references
- **Quantum compute architectures**
 - *Quantum computers*, T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe & J. L. O'Brien, *Nature* **464**, 45 (2010).
- **Popular Science books**
 - *Logicomix: An Epic Search for Truth*, by Apostolos Doxiadis and Christos H. Papadimitriou, Bloomsbury Publishing (2009).
 - *The Golden Ticket, P, NP, and the search for the impossible*, Lance Fortnow, Princeton University Press (2013).
 - *Godel, Escher, Bach: An Eternal Golden Braid*, Douglas Hofstadter, Basic Books (1999).
 - *Quantum Computing for Babies*, Chris Ferrie, Sourcebooks Explore (2018).

